Automatica 48 (2012) 1868–1873

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Application of matrix perturbation theory in robust control of large-scale systems ☆

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ARTICLE INFO

Article history:
Received 10 July 2010
Received in revised form
29 January 2012
Accepted 25 February 2012
Available online 22 June 2012

Keywords:
Robust control
Matrix perturbation theory
Decentralized control
Eigenstructure assignment
Robust stability
Large-scale systems

ABSTRACT

The aim of this paper is to develop new results for robust centralized and decentralized control of large-scale systems using *matrix perturbation theory*. A new robustness measure is proposed which is more appropriate for large-scale systems than the existing measures. The use of this new index allows for an evaluation of system eigenvalue sensitivity to perturbations without calculating the eigenvectors and the condition number of the modal matrix. In addition, a novel robust decentralized controller design method is proposed to assign the closed-loop eigenvalues of the large-scale system in a desired region. This method is based on eigenstructure assignment of isolated subsystems. The assignment of overall closed-loop poles in the desired region is guaranteed provided that certain sufficient conditions for the isolated subsystems are satisfied. Simulation results are given to show the efficiency of the new methods.

1. Introduction

Matrix perturbation theory considers how matrix functions such as eigenvalues or singular values change when the matrix is subject to perturbations (Stewart & Sun, 1990). The matrix perturbation analysis can result in defining a perturbation bound. The Bauer–Fike theorem obtained from the matrix perturbation theory has already been used in control theory (Kautsky, Nichols, & Van Dooren, 1985). In this paper, a similar theorem that is used less in control theory is implemented to propose a new robustness index. The new index is more suitable for large-scale systems. In addition, this theorem solves the problem of robust decentralized regional pole assignment.

In many practical control design problems, systems are subject to perturbation and parameter variation. In addition, uncertainties of the dynamics of the system model severely affect system performance and stability of the closed-loop designed based on the nominal model of the system. The risk of instability, however, is reduced if the controller is designed to minimize the sensitivity of the closed-loop eigenvalues to perturbations. In multivariable systems, it is shown that the feedback gain assigning a set of eigenvalue assignment is not unique (D'Azzo & Houpis, 1995). Hence, one of the eigenvalue sensitivity measures introduced in the literature can be minimized to select the feedback (see Duan, 1992, Kautsky et al., 1985 and Liu & Patton, 1998).

Among the eigenvalue sensitivity measures, the condition number of the modal matrix of the closed-loop system is the most popular measure. However, implementation of this index imposes some difficulties (Duan, 1992; Lam & Yan, 1996), mainly due to the calculation of the modal matrix and particularly for large matrices. Therefore, other robustness indices based on the Frobenius norm are introduced. The Frobenius condition number of the closed-loop eigenvector matrix is proposed in the literature as an alternative index. However, this index is more conservative than the spectral condition number. The closed-loop eigenvector calculation requires high computation effort and the result is unreliable for large matrices. This study suggests a new robustness index which uses the Frobenius norm and does not need the computation of closed-loop eigenvectors. This new index is applicable to centralized controller design for large-scale systems. In the case of state feedback design, the new robustness index leads to the index proposed by Dickman (1987).

Decentralized control of the large-scale systems is the second topic considered in this paper. In practical applications, a centralized controller is difficult to design and costly to implement

☆ The material in this paper was partially presented at the 16th IFAC World Congress, July 3–8, 2005, Prague, Czech Republic. This paper was recommended for publication in revised form by Associate Editor Tong Zhou under the direction of Editor Roberto Tempo.

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doi:10.1016/j.automatica.2012.05.061

for large-scale systems. As a result, decentralized control theory emerged in the 1960s and developed to a pole of attraction in the system and control community thereafter (Lunze, 1992; Šiljak, 1991). Decentralized stabilization is an active field of research for large-scale systems. In 1973, Wang and Davison introduced the notation of fixed-modes which is a natural generalization of uncontrollable modes and unobservable modes in centralized control systems (Wang & Davison, 1973). Based on this concept, they proposed a necessary and sufficient condition for the existence of local controllers to stabilize the overall system.

In Grosdidier and Morari (1986), structured singular value interaction measure is used as a tool for decentralized controller design. The interaction measure considers the stability and performance of the closed-loop system. This method provides a sufficient condition in terms of the subsystem design constraints under which the aggregation of the stable subsystem design yields an overall stable design. Thereafter, an explicit stability condition for 2-block systems was proposed by Nett and Uthgenannt (1988). However, it is assumed in both of these methods that the initial system is square, stable and minimum phase and for systems of high dimensions, it requires very complicated computations. Recently, a novel sufficient condition for the decentralized stabilization of large-scale systems has been proposed by Labibi, Lohmann, Khaki Sedigh, and Jabedar Maralani (2000). In this method, a sufficient condition for the stability is stated in terms of the maximum eigenvalues of the Hermitian parts of the state matrix in each isolated closed-loop subsystem and the interaction matrix.

A sufficient condition for robust stabilization of large-scale systems is given in Labibi, Lohmann, Khaki Sedigh, and Jabedar Maralani (2003) based on the eigenstructure assignment methodology introduced in Duan (1992). It is shown in Labibi et al. (2003) that the robust stability of the overall closed-loop system is guaranteed by appropriately assigning the eigenstructure of each isolated subsystem. However, only the exponential stability of the closed-loop system is considered and the paper does not discuss the pole assignment problem in a desirable region.

This paper presents a novel methodology to design decentralized controllers based on matrix perturbation theory. The new method can be applied to non-square, non-minimum phase and unstable systems. Using the new approach, one can assign the closed-loop poles within a "specific region" with minimum sensitivity to parameter perturbations, while the aforementioned methods may only guarantee the stability of the system. This is achieved by designing local controllers for each isolated subsystem such that certain sufficient conditions are satisfied.

2. Mathematical background

This section provides background material from matrix perturbation theory. The notation $\lambda_i(A)$ is used throughout this paper to represent the i-th eigenvalue of the matrix A. Also, a block diagonal matrix A is represented by $\mathrm{diag}\{A_i\}$ where A_i's are diagonal blocks for $i = 1, \ldots, L$ and L is the number of diagonal blocks. A square matrix A is called normal if $A^H A = AA^H$, where H denotes the conjugate transpose of the matrix. The following theorem shows how the eigenvalues of a matrix are perturbed when its elements are subject to perturbations.

Theorem 2.1. *Let*

$$Q^H AQ = D + N, \tag{1}$$

be a Schur decomposition of $A \in \mathbb{C}^{n\times n}$ *where* $D = \mathrm{diag}\{\lambda_1, \ldots, \lambda_n\}$, $N \in \mathbb{C}^{n\times n}$ *is a strictly upper triangular matrix and Q is an appropriate unitary matrix. Suppose that* $E \in \mathbb{C}^{n\times n}$ *is an arbitrary matrix. If* $\mu \in \eta(A+E)$ *and p is the smallest positive integer such that* $|N|^p = 0$, *then*

$$\min_{\lambda\in\eta(A)} |\mu - \lambda| \le \max(\theta, \theta^{1/p}), \tag{2}$$

where $\theta = \|E\|_2 \sum_{k=0}^{p-1} \|N\|_2^k$. *Here,* $\eta(.)$ *represents the set of the eigenvalues of* $(.)$ *and the matrix* $|N|$ *is the element-wise absolute value of N (see Golub & Van Loan, 1996).*

The following lemma expresses a useful property of normal matrices.

Lemma 2.2. *Matrix* $A \in \mathbb{C}^{n\times n}$ *is normal if and only if in the Schur decomposition of the matrix A, the matrix N is equal to zero (see Golub & Van Loan, 1996).*

Finally, the following theorem gives another measure to examine whether or not a matrix is normal.

Theorem 2.3. *Matrix* $A \in \mathbb{C}^{n\times n}$ *is normal if and only if* $\|A\|_F^2 = \sum_{i=1}^n |\lambda_i(A)|^2$, *where* $\|A\|_F$ *is the Frobenius norm of A.*

The proof of this theorem is straightforward and is omitted.

3. Problem formulation

Consider a linear multivariable system with the following state space description

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{3}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector and $y \in \mathbb{R}^p$ is the output vector. It is assumed that (A, B) and (A, C) are controllable and observable, respectively. The aim of the robust centralized pole assignment is to find an output feedback $u(t) = Ky(t)$ such that the closed-loop eigenstructure is as insensitive as possible to the system parameter variations. The first problem considered in the following sections is to find a measure of the sensitivity suitable for large-scale systems.

The second problem studied in this paper is the designing of robust decentralized controllers. Assume that the large-scale system $G(s)$ in (3) is decomposed into L linear time-invariant subsystems $G_i(s)$, described by

$$G_i : \begin{cases} \dot{x}_i(t) = A_{ii}x_i(t) + B_{ii}u_i(t) + \displaystyle\sum_{j=1,j\ne i}^{L} A_{ij}x_j(t) \\ y_i(t) = C_{ii}x_i(t), \end{cases} \tag{4}$$

where $A_{ii} \in \mathbb{R}^{n_i\times n_i}$ $B_{ii} \in \mathbb{R}^{n_i\times m_i}$, $C_{ii} \in \mathbb{R}^{p_i\times n_i}$, $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $y_i \in \mathbb{R}^{p_i}$, $\sum_{i=1}^L n_i = n$, $\sum_{i=1}^L m_i = m$, and $\sum_{i=1}^L p_i = p$. It is assumed that all (A_{ii}, B_{ii}) and (A_{ii}, C_{ii}) are controllable and observable, respectively. Also, all B_{ii} and C_{ii} are of full column and row rank, respectively. In (4), the term $\sum_{j=1,j\ne i}^L A_{ij}x_j$ represents the interactions between the i-th subsystem and the other subsystems. It is desired to design a local dynamic output feedback

$$U_i(s) = K_i(s)(R_i(s) + Y_i(s)), \tag{5}$$

for each isolated subsystem

$$G_{di} = \begin{cases} \dot{x}_i(t) = A_{ii}x_i(t) + B_{ii}u_i(t) \\ y_i(t) = C_{ii}x_i(t). \end{cases} \tag{6}$$

In (5), $K_i(s)$ is the transfer function of the controller and $U_i(s)$, $R_i(s)$ and $Y_i(s)$ are the Laplace transforms of $u_i(t)$, $r_i(t)$ and $y_i(t)$, respectively. Here, $r_i(t)$ is the i-th reference input to the i-th subsystem. The objective is to design a controller $K_i(s)$ for G_{di} such