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Brief paper

A decentralized optimal LQ state observer based on an augmented Lagrangian approach☆

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ABSTRACT

This paper is devoted to the design of a decentralized optimal batch LQ state observer for state estimation of large-scale interconnected systems, well suited for implementation on a sensor network. The here-proposed approach relies on both the use of an augmented Lagrangian formulation and a price-decomposition–coordination algorithm. The state estimation of an open-channel hydraulic system illustrates the effectiveness of this approach and is used to provide a comparison with alternative methods.

1. Introduction

The design of decentralized or distributed state estimation or control algorithms is an important research topic which has attracted a great interest for more than 30 years (Farina, Farrari-Trecate, & Scattolini, 2010; Hess & Rantzer, 2010; Ishizaki, Sakai, Kashima, & Imura, 2011; Khan & Moura, 2008; Looze, Houpt, Sandell, & Athans, 1978; Maestre, Giselsson, & Rantzer, 2010; Mansouri, Boutat-Baddas, Darouach, & Messaoud, 2010; Martensson & Rantzer, 2011; Sanders, Tacker, & Linton, 1978; Siljak, 1991; Stankovic, Stankovic, & Stipanovic, 2009). On the one hand, estimation of large-scale complex systems is of the greatest importance for monitoring applications in various engineering fields, such as power grids, road traffic networks, environmental systems (open-channel hydraulic systems, water supply networks, weather, etc.). On the other hand, sensor networks (Akyildiz, Su, Sankarasubramaniam, & Cayirci, 2002) are now recognized for being well suited for measurement, monitoring, tracking of distributed physical phenomena, such as environmental phenomena (weather, seismic events, wildfires, air/soil/river pollution (Ghanem, Guo, Hassard, Osmond, & Richards, 2004), sound or vibration monitoring, etc.) or complex industrial systems. Sensor networks are defined as a collection of embedded sensors with communication capabilities. Large scale deployment of such ad hoc networks mainly relies on the availability of cheap embedded sensors. For that reason, it is necessary to consider algorithmic architectures capable of performing low complexity computations distributed on each sensor node. Here the attention is paid to sensor network-based monitoring applications for large-scale systems, in which local measurements are made and a local state estimation with low computational complexity is performed by each sensor agent while some computation results are provided to the other agents connected to it. Furthermore a systematic way to manage decentralized state estimation through the application of dual optimization theory and the deterministic interpretation of the continuous-time Kalman filter as described in Bornard, Celle-Couenne, and Gilles (1995) is provided. Unlike the approaches in Looze et al. (1978); Mansouri et al. (2010), the proposed approach provides an exact decentralized solution to the centralized Kalman filter and does not require strong structural properties of the system. The here-proposed approach is related to some previous works in Maestre et al. (2010), where dual decomposition is used. However the authors in Maestre et al. (2010) only consider the case of coupling of the subsystems via the measurement outputs. The here-proposed approach also differs from approaches based on consensus (see Khan & Moura, 2008; Stankovic et al.,

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2009), since the local systems are never some overlapping subsystems. It can be compared to the approach proposed in Farina et al. (2010) since decentralization leads to the solution of distributed small-size moving horizon estimators without overlapping. However to the best of our knowledge, such an augmented Lagrangian formulation is used for the first time. It will be also shown that the proposed observer algorithm can be used to derive a moving horizon state observer. The paper is now organized as follows: In Section 2, the state estimation problem is stated. The decentralized design, based on both the use of an augmented Lagrangian formulation and a relaxation technique, together with some backgrounds on dual optimization, is then presented in Section 3. Section 4 will be devoted to the derivation of a moving horizon observer based on the decentralized algorithm. Section 5 proposes an environmental state estimation application that illustrates the effectiveness of the approach. In this section, the proposed approach will be also compared to alternative approaches. Finally, the paper ends with some conclusions.

2. Problem statement

We consider the problem of designing a decentralized state observer for large-scale interconnected linear systems defined by:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + D_i \sum_{j=1}^{N} F_{ij} x_j(t), \tag{1}$$

$$y_i(t) = C_i x_i(t), \quad i = 1, \ldots, N, \tag{2}$$

where the full state vector $x \in R^n$ and the full input vector $u \in R^m$ are partitioned as $x = (x_1^T, x_2^T, \ldots, x_N^T)^T$ and $u = (u_1^T, u_2^T, \ldots, u_N^T)^T$ respectively, where $x_i \in R^{n_i}$ and $u_i \in R^{m_i}$, and each subsystem admits a measurement output $y_i \in R^{p_i}$. The overall system is assumed to be observable.

System (1)–(2) may be equivalently defined as the following algebraic–differential system:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + D_i s_i(t), \tag{3}$$

$$s_i(t) - \sum_{j=1}^{N} F_{ij} x_j(t) = 0, \tag{4}$$

$$y_i(t) = C_i x_i(t), \quad i = 1, \ldots, N, \tag{5}$$

where the s_i's represent some slack variables. We seek to derive an optimal Linear–Quadratic (LQ) state observer with a finite horizon by solving the following optimal output tracking problem, based on the knowledge of $y_i(t)$, $u_i(t)$, $\forall t \in I = [0, T]$:

$$\min_{v_i(t), \hat{s}_i(t), t \in I} \frac{1}{2} \sum_{i=1}^{N} \int_0^T \{\|\hat{y}_i(t) - y_i(t)\|^2_{R_i^{-1}} + \|v_i(t)\|^2_{Q_i^{-1}}\} dt$$
$$+ \frac{1}{2} \sum_{i=1}^{N} \|\hat{x}_i(0) - \underline{x}_i\|^2_{M_i^{-1}} \tag{6}$$
$$s.t.\, \dot{\hat{x}}_i(t) = A_i \hat{x}_i(t) + B_i u_i(t) i + E_i v_i(t) + D_i \hat{s}_i(t),$$
$$\hat{y}_i(t) = C_i \hat{x}_i(t),$$
$$\hat{s}_i(t) = \sum_{j=1}^{N} F_{ij} \hat{x}_j(t),$$

where Q_i, R_i and M_i, $i = 1, \ldots, N$ are some symmetric positive-definite matrices of adequate dimensions, and E_i's are also matrices of appropriate dimensions. $\underline{x}_i$ is the a priori most likely value of $x_i(0)$. This problem is an equivalent formulation to the continuous Kalman filter design problem which seeks the minimum variance state estimate of the overall state $\hat{x}$ of the system (see Bornard et al., 1995; Kwakernaak & Sivan, 1972), where Q_i is interpreted as the covariance matrix of a zero mean gaussian noise affecting the state equation of x_i through matrix E_i, and R_i is the covariance of a zero mean gaussian noise affecting the measurement output y_i. M_i is the covariance matrix of random initial state $x_i(0)$.

3. State observer design

The optimal output tracking problem (6) is closely related to the classical optimal trajectory tracking problem, except that the cost penalizes the initial states rather than the terminal states. For that reason, it appears to be very convenient to reverse time by considering the change $(t \to T - t)$ and interval $I' = [T, 0]$, and thus turning the optimal output tracking problem into:

$$\min_{\tilde{v}_i(t), \tilde{s}_i(t), i \in I'} \frac{1}{2} \sum_{i=1}^{N} \int_0^T \{\|C_i \tilde{x}_i(t) - \tilde{y}_i(t)\|^2_{R_i^{-1}}$$
$$+ \|\tilde{v}_i(t)\|^2_{Q_i^{-1}}\} dt + \frac{1}{2} \sum_{i=1}^{N} \|\tilde{x}_i(T) - \underline{x}_i\|^2_{M_i^{-1}}$$
$$s.t.$$
$$\dot{\tilde{x}}_i(t) = -A_i \tilde{x}_i(t) - B_i \tilde{u}_i(t) - E_i \tilde{v}_i(t) - D_i \tilde{s}_i(t),$$
$$\tilde{s}_i(t) = \sum_{j=1}^{N} F_{ij} \tilde{x}_j(t), \tag{7}$$

where $\tilde{x}_i(t) = \hat{x}_i(T - t)$, $\tilde{u}_i(t) = u_i(T - t)$, $\tilde{v}_i(t) = v_i(T - t)$ and $\tilde{s}_i(t) = \hat{s}_i(T - t)$. Let us now consider an *augmented Lagrangian* formulation of the problem as follows (in the spirit of Cohen & Zhu, 1984; Georges, 2006 for instance):

$$L_c(\tilde{v}, \tilde{s}, \tilde{\mu}) = \frac{1}{2} \sum_{i=1}^{N} \int_0^T \{\|C_i \tilde{x}_i(t) - \tilde{y}_i(t)\|^2_{R_i^{-1}}$$
$$+ \|\tilde{v}_i(t)\|^2_{Q_i^{-1}}\} dt + \frac{1}{2} \sum_{i=1}^{N} \|\tilde{x}_i(T) - \underline{x}_i\|^2_{M_i^{-1}}$$
$$+ \sum_{i=1}^{N} \int_0^T < \tilde{\mu}_i(t) + \frac{c}{2} \left[\tilde{s}_i(t) - \sum_{j=1}^{N} F_{ij} \tilde{x}_j(t) \right],$$
$$\tilde{s}_i(t) - \sum_{j=1}^{N} F_{ij} \tilde{x}_j(t) > dt \tag{8}$$
$$s.t.$$
$$\dot{\tilde{x}}_i(t) = -A_i \tilde{x}_i(t) - B_i \tilde{u}_i(t) - E_i \tilde{v}_i(t) - D_i \tilde{s}_i(t),$$

where $c > 0$ is the coefficient of the augmented Lagrangian, which should be chosen large enough to ensure the existence of a saddle-point. $\tilde{v}$, $\tilde{s}$, $\tilde{\mu}$ denote the vectors of all the $\tilde{v}_i$'s, $\tilde{s}_i$'s and $\tilde{\mu}_i$'s, respectively. $\langle ., . \rangle$ represents the usual scalar product. μ is the vector of Lagrange multipliers associated to the interconnection constraints. *The interest of considering an augmented Lagrangian rather than the ordinary one may be found in the fact the problem is not strongly convex with respect to the slack variables* s_i. The augmented Lagrangian formulation may be viewed as a regularization technique which strongly convexifies the problem and therefore ensures the existence of a saddle-point of the constrained problem for non-strongly convex problems. As a consequence, the relaxation algorithms, such as the Uzawa algorithm (Arrow, Hurwicz, & Uzawa, 1958), will converge by using a constant step gradient method, rather than a gradient method with "small steps" of type σ,[2] which tend to zero as the number of dual iterations tends to

[2] Steps ρ^k with $\rho^k \geq 0$, $\sum_{i=0}^{+\infty} \rho^k = +\infty$ and $\sum_{i=0}^{+\infty} (\rho^k)^2 < +\infty$.