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Brief paper

Reduced order extended command governor[☆]

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ABSTRACT

Extended command governors (ECGs) are add-on schemes that modify set-point commands as necessary to ensure that imposed state and control constraints are not violated by closed-loop systems designed for set-point tracking. In this paper, we propose a reduced order ECG for systems with dynamics decomposable into slow and fast state variables. We demonstrate that ECG implementation can be based on slow states only, thus reducing the computational complexity. This is achieved by introducing additional constraints, and by slightly tightening the original constraints. We demonstrate that the proposed ECG maintains the response properties of the conventional ECG, including the convergence to the nearest feasible command in finite time in the case of constant reference commands. The results are also shown to apply to conventional command governors. For the case when the reduced order state is not directly measured, a formulation of the result in the presence of a state observer is developed.

1. Introduction

Reference governors (RGs), command governors (CGs), and extended command governors (ECGs) are control schemes that are appended to asymptotically stable closed-loop systems to enforce pointwise-in-time state and control constraints. All three governors take the form shown in Fig. 1. Whenever it is possible to set $v(t) = r(t)$ subject to the constraints, this is done. Otherwise, $v(t)$ is determined by a specific rule that assures constraint satisfaction. The rules employ maximum constraint admissible sets for the state of the closed-loop system with constant reference commands. Under reasonable assumptions, both the RG (Bemporad, 1998; Gilbert & Kolmanovsky, 1999; Gilbert, Kolmanovsky, & Tan, 1995) and CG (Bemporad, Casavola, & Mosca, 1997; Casavola, Mosca, & Angeli, 2000; Casavola, Mosca, & Papini, 2004) exhibit properties of recursive constraint feasibility, finite-settling time for constant or nearly constant reference commands, and convergence to an attractor set, when applied to systems with set-bounded disturbances.

The ECG, introduced in Gilbert and Ong (2011), determines if a command is constraint-admissible by testing whether this command, combined with the output of an asymptotically stable auxiliary system, does not cause subsequent constraint violation. The state of the auxiliary system and the command to the closed-loop system are determined by solving a quadratic programming problem. The constraints in this problem are induced by the maximum constraint admissible set for the system extended by the state of the auxiliary dynamics. Compared to the RG or the CG, the ECG enlarges the maximal constrained domain of attraction, while retaining the key response properties of the RG and the CG.

This paper contributes a method for reducing the computational complexity of the ECG. The reduced order ECG method uses model order reduction by exploiting decomposition based on fast and slow dynamics. The ideas are similar to those used in the reduced order RG of Kalabić, Kolmanovsky, Buckland, and Gilbert (2012), but more complex because of the need to consider the state of the auxiliary system. Because of these auxiliary states, model order reduction is even more important for making the ECG computationally tractable; since the auxiliary dynamics are appended to the already present system dynamics, reduction to a lower order allows their design to be based on a system with fewer variables and therefore simpler. Model order reduction directly contributes to lower complexity by decreasing the number of state variables needed for the implementation of the ECG. In the case of the RG, we have shown a three-fold reduction in computational complexity (measured in memory allocation) in applying the RG to a practical turbocharged gasoline engine control problem (Kalabić et al., 2012), and we have also demonstrated handling of infinite dimensional models based on reduced order RG theory. The approach in Garone and Tedesco (2011) is another example of order

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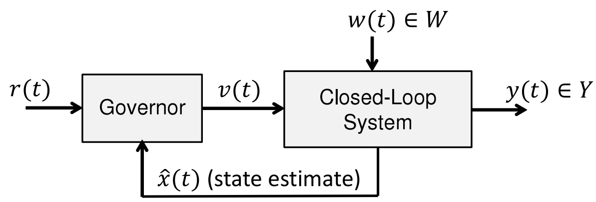

Fig. 1. A schematic of the ECG as applied within a control algorithm.

reduction in predictive control, but is distinctly different from the one here. Our approach decomposes the system into two subsystems and allows us to decrease the order of the ECG by using only the state of the first subsystem to develop the ECG algorithm. There is a trade-off in the order reduction. The errors in the system approximation must be suitably controlled and this is done by tightening constraints.

The paper is organized into 6 sections of which this is the first. Section 2 reviews the theory of the ECG. Section 3 introduces the results of the reduced order version of the ECG. Section 4 states the main theorem which is proven in the Appendix. For the case where not all slow states are measured, the treatment of observer errors is considered in Section 5. Concluding remarks are made in Section 6.

Standard mathematical notation is used throughout. $\mathbb{R}$ is the set of real numbers and $\mathbb{Z}_+$ is the set of non-negative integers. The matrix $A^{\mathrm{T}} \in \mathbb{R}^{m\times n}$ is the transpose of $A \in \mathbb{R}^{n\times m}$; $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix; $Q \succ 0$ denotes a symmetric positive definite matrix; $\mathcal{B}_n = \{x \in \mathbb{R}^n : \|x\| \le 1\}$ is the unit ball corresponding to a norm, $\|\cdot\|$. For $Q \succ 0$, let $\|x\|_Q^2 = x^{\mathrm{T}}Qx$. The sets int U and bd U are respectively the interior and boundary of $U \subset \mathbb{R}^n$. For $Q \in \mathbb{R}^{m\times m}$, $QU := \{QU : u \in U\}$. For $V \in \mathbb{R}^n$, the sets $U \oplus V := \{u + v : u \in U, v \in V\} \subset \mathbb{R}^n$ and $U \sim V := \{z \in \mathbb{R}^n : z + v \in U,\ \forall v \in V\}$ are respectively the Minkowski sum and Minkowski (or Pontryagin) difference. The notation $x(t + k|t)$ denotes the predicted value at time $t + k$ assuming the prediction is made at time t.

2. Extended command governor

The ECG, like the RG and CG, is applied to asymptotically stable closed-loop systems to prevent them from violating specified pointwise-in-time constraints. Let the closed-loop system and its hard constraints be represented by,

$$x(t+1) = Ax(t) + Bv(t) + B_w w(t), \tag{1}$$

$$y(t) = Cx(t) + Dv(t) + D_w w(t) \in Y, \quad t \in \mathbb{Z}_+, \tag{2}$$

where $x(t) \in \mathbb{R}^n$, $v(t) \in \mathbb{R}^m$, $w(t) \in \mathbb{R}^\ell$, $y(t) \in \mathbb{R}^p$, $A \in \mathbb{R}^{n\times n}$ is asymptotically stable, and (C, A) is an observable pair. Disturbance sequences are represented by $w(\cdot) \in \mathcal{W}$ where its elements satisfy, for all $t \in \mathbb{Z}_+$, the condition $w(t) \in W$. It is assumed that $0 \in W$ and W is compact. The hard constraints are $y(t) \in Y$ and must be satisfied for all $t \in Z_+$ and $w(\cdot) \in \mathcal{W}$. The set $Y \subset \mathbb{R}^p$ is a polyhedron. We note that the subsequent theory only requires that Y is convex and closed. However, we assume Y is polyhedral because it allows explicit computational procedures.

The output of the ECG is given by,

$$v(t) = U(x(t), r(t)), \tag{3}$$

where the function $U : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m$ is evaluated algorithmically. Specifically, at the current time t, $v(t)$ is based on the auxiliary system,

$$\bar{x}(t+1) = \bar{A}\bar{x}(t), \tag{4}$$

$$v(t) = \bar{C}\bar{x}(t) + \rho(t), \tag{5}$$

where $\bar{A}$ is chosen to be asymptotically stable and $(\bar{C}, \bar{A})$ is observable, $\bar{x}(t) \in \mathbb{R}^{\bar{n}}$ is the auxiliary state and $\rho(t) \in \mathbb{R}^m$ is the steady-state offset. Note that the output of the auxiliary system (4)–(5) is the constraint admissible control and to see how it is exploited requires additional definitions and assumptions. Combining (1)–(2) and (4)–(5) and assuming $\rho(t) \equiv \rho$,

$$\begin{bmatrix} \tilde{x}(t+1) \\ \rho(t+1) \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{B} \\ 0 & I_m \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \rho(t) \end{bmatrix} + \begin{bmatrix} \tilde{B}_w \\ 0 \end{bmatrix} w(t), \tag{6}$$

$$y(t) = \begin{bmatrix} \tilde{C} & D \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \rho(t) \end{bmatrix} + D_w w(t) \in Y, \tag{7}$$

where,

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ \bar{x}(t) \end{bmatrix}, \qquad \tilde{A} = \begin{bmatrix} A & B\bar{C} \\ 0 & \bar{A} \end{bmatrix},$$

$$\tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \qquad \tilde{B}_w = \begin{bmatrix} B_w \\ 0 \end{bmatrix}, \qquad \tilde{C} = \begin{bmatrix} C & D\bar{C} \end{bmatrix}.$$

The maximal constraint admissible set for (6)–(7) is,

$$O_\infty^{aug} := \{(x(0), \bar{x}(0), \rho(0)) : (6)\text{–}(7) \text{ are satisfied for all } t \in \mathbb{Z}_+ \text{ and } w(\cdot) \in \mathcal{W}\}. \tag{8}$$

Define,

$$\Pi(x) := \{(\bar{x}, \rho) : (x, \bar{x}, \rho) \in O_\infty^{aug}\}. \tag{9}$$

Under appropriate conditions, both O_∞^{aug} and $\Pi(x)$ exist and can be determined algorithmically. See Gilbert and Ong (2011) for details. Roughly stated, the appropriate conditions correspond to slightly tightening the constraint $y(t) \in Y$ in steady-state. Since O_∞^{aug} is polyhedral,

$$O_\infty^{aug} = \{(x, \bar{x}, \rho) : H_x x + H_{\bar{x}}\bar{x} + H_r\rho \le h\}, \tag{10}$$

$$\Pi(x) = \{(\bar{x}, \rho) : H_{\bar{x}}\bar{x} + H_r\rho \le h - H_x x\}. \tag{11}$$

To determine $U(x, r)$, let,

$$\|(\bar{x}, \rho)\|^2 := \|\bar{x}\|_{\bar{S}}^2 + \|\rho\|_S^2, \tag{12}$$

where $\bar{S} \in \mathbb{R}^{\bar{n}\times\bar{n}}$ and $S \in \mathbb{R}^{m\times m}$ satisfy the conditions: $S \succ 0$, $\bar{S} \succ 0$, $\bar{A}^{\mathrm{T}}\bar{S}\bar{A} - \bar{S} \prec 0$. Since $\bar{A}$ is asymptotically stable, there exists an $\bar{S}$ satisfying the Lyapunov-like condition. Let the pair,

$$(\bar{x}_{op}, \rho_{op}) = \underset{(\bar{x},\rho)\in\Pi(x)}{\arg\min}\ \|(\bar{x}, \rho - r)\|^2. \tag{13}$$

Then,

$$U(x, r) := \bar{C}\bar{x}_{op} + \rho_{op}. \tag{14}$$

It is now possible to state the main results of Gilbert and Ong (2011); to do this we need some standard definitions and assumptions. The main assumptions have already been stated: A and $\bar{A}$ are asymptotically stable, $(\bar{C}, \bar{A})$ is an observable pair, and S and $\bar{S}$ are positively definite matrices such that $\bar{S}$ and $\bar{A}$ satisfy the Lyapunov-like condition above; furthermore, W is compact and contains 0, and Y is closed and convex. Let,

$$F_\infty(r_s) = \{\Gamma r_s\} \oplus F_\infty, \qquad \Gamma := (I_n - A)^{-1}B,$$
$$F_\infty = \lim_{t\to\infty} F_t, \qquad F_t = \bigoplus_{i=0}^{t-1} A^i B_w W,\ F_0 = \{0\}. \tag{15}$$

The set $F_\infty(r_s)$ is the attractor set for (1) when $v(t) \equiv r_s$ (Gilbert & Kolmanovsky, 1999; Gilbert & Ong, 2011; Kerrigan, 2000; Kolmanovsky & Gilbert, 1998). Specifically, $F_\infty(r_s)$ is compact and