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Brief paper

# Smooth switching LPV controller design for LPV systems☆

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ABSTRACT

This paper presents a method to design a smooth switching gain-scheduled linear parameter varying (LPV) controller for LPV systems. The moving region of the gain-scheduling variables is divided into a specified number of local subregions as well as subregions for the smooth controller switching, and one gain-scheduled LPV controller is assigned to each of the local subregions. For each switching subregion, a function interpolating two local LPV controllers associated with its neighborhood subregions is designed to satisfy the constraint of smooth transition of controller system matrices. The smooth switching controller design problem amounts to solving a feasibility problem which involves nonlinear matrix inequalities. To find a solution to the feasibility problem, an iterative descent algorithm which solves a series of convex optimization problems is proposed. The usefulness of the proposed controller design method is demonstrated with a control example of a flexible ball-screw drive system.



## 1. Introduction

Analysis and design for linear parameter varying (LPV) systems have attracted a considerable amount of attention over the last two decades. A number of gain-scheduled LPV controller design methods for LPV systems have been developed, and various successful control applications have been reported. See e.g. Apkarian and Adams (1998), Apkarian and Gahinet (1995), Mohammadpour and Scherer (2012), Packard (1994), Rugh and Shamma (2000), Shamma and Athans (1990) and White, Zhu, and Choi (2013) and references therein.

A common issue of LPV controller design methods based on robust control theory is the inherent conservatism of the designed controllers. As a means to reduce the conservatism and to improve the closed-loop performance achieved by a single LPV controller, a switching LPV controller design method was proposed in Lu and Wu (2004) and Lu, Wu, and Kim (2006) by using the multiple parameter-dependent Lyapunov functions. In this method, the moving region of the gain-scheduling variables is divided into subregions, and each subregion is associated with one local LPV controller. To ensure the performance even with controller switching, two strategies for switching, that is, the hysteresis switching strategy and the average dwell time strategy, were investigated. The advantage of utilizing the switching LPV controller in some industrial practices has been demonstrated in Hanifzadegan and Nagamune (2014), Hu and Yuan (2008), Lu et al. (2006) and Postma and Nagamune (2012).

A potential drawback of the developed LPV switching controllers is that it may not provide a smooth transient response after switching of controllers. A nonsmooth transient response can lead to mechanical damage, fatigue loading, or signal saturations, and therefore, it is undesirable in practical applications. To cover this drawback, there has been extensive research on a bumpless transfer of controllers switching in Edwards and Postlethwaite (1998), Graebe and Ahlen (1996), Kinnaert, Delwiche, and Yamé (2009), Pour Safaei, Hespanha, and Stewart (2012), Turner and Walker (2000), Yamé and Kinnaert (2004), and Zaccarian and Teel (2005) and interpolation of controllers in Bendtsen, Stoustrup, and Trangbaek (2005), Bianchi and Sanchez Pena (2011), Chen (2012, 2013), Chen, Wu, and Chuang (2010), Hencey and Alleyne (2010) and Liberzon (2003). With a bumpless transfer, continuity of a plant input is ensured at the switching time for linear time-invariant (LTI) controllers by different techniques. Some examples of bumbles transfer methods are to force an off-line controller to track an on-line controller signal with anti-windup bumpless transfer in Edwards and Postlethwaite (1998), Graebe and Ahlen (1996) and Yamé and Kinnaert (2004), to match of switching controller signals

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by an optimal linear quadratic minimization in Turner and Walker (2000), to follow an ideal control signal target at switching time by minimizing an $L_2$-gain bound in Zaccarian and Teel (2005), and to set states of an off-line controller before switching to assure a continuous control signal for a set of linear single-input–single-output controllers in Kinnaert et al. (2009) and for multi-input–multi-output controllers in Pour Safaei et al. (2012).

On the other hand, in the interpolation approaches, local controllers are blended to guarantee the closed-loop and switching stability (Liberzon, 2003). For instance, an observer-based multivariable gain-scheduled controller has been developed in Bendtsen et al. (2005) by linear interpolation of local controllers. As another attempt, a supervisory controller has been synthesized in Hencey and Alleyne (2010), where control signals coming from local LTI controllers for different control objectives are interpolated to provide quadratic stability and performance guarantees. Moreover, in Bianchi and Sanchez Pena (2011), interpolation of LTI controllers has been presented to generate a gain-scheduled controller with preserving the closed-loop quadratic stability and transient performance. Furthermore, in Chen et al. (2010), a method was developed to design a smooth switching LPV controller by interpolating LPV controllers in overlapping regions of any two adjacent subregions. Its applications have been presented in Chen (2012, 2013).

Among all of the previous work, the method in Chen (2012, 2013) and Chen et al. (2010) directly deals with smooth switching of the LPV controllers. Instead of switching controllers instantaneously, this method interpolates the controllers with a fixed function in an overlapping region. As a consequence of employing a fixed function for controller interpolation, the control signal may not be differentiable at the boundaries of overlapping regions, which can degrade signals' smoothness. The other disadvantage of this approach is the lack of a measure to quantitatively evaluate smoothness of controller switching. Moreover, the method in Chen (2012, 2013) and Chen et al. (2010) is limited to the state feedback case, and to the case of one-dimensional space of the scheduling variable.

In this paper, we propose a method to design a smooth switching LPV controller. The method is novel in the sense that, in contrast to the previous work in Bendtsen et al. (2005), Bianchi and Sanchez Pena (2011), Hencey and Alleyne (2010) and Liberzon (2003), we design LPV controllers as local controllers and interpolate smoothly between them instead of LTI controllers interpolation. Also, different from the method in Chen (2012, 2013) and Chen et al. (2010), we consider the controller design problem for output feedback systems, and for both one- and two-dimensional spaces of gain-scheduling variables. In addition, unlike Chen (2012, 2013) and Chen et al. (2010), we introduce a matrix norm of the controller system matrix as a measure of "smoothness" for controller switching. Also, we utilize adjustable interpolation functions to improve the smoothness, and provide a higher order differentiable control signal by taking them into account in a controller design problem. The controller design problem is formulated to satisfy both the standard $L_2$-gain condition of the closed-loop system and the smoothness condition of the switching controllers. Since the formulated problem boils down to a feasibility problem with nonlinear matrix inequalities in general, we develop an algorithm in which Lyapunov variables, the local controllers and the interpolating functions are optimized alternately to find a feasible solution.

## 2. A smooth switching LPV controller design problem

In this section, we will formulate a smooth switching LPV controller design problem to be dealt with in this paper. For the formulation, we introduce the following notation for interval sets. Consider an interval set in the set of real numbers $\mathbb{R}$:

$$\Theta := \left\{\theta \in \mathbb{R} : \underline{\theta} \le \theta \le \overline{\theta}\right\}. \tag{1}$$

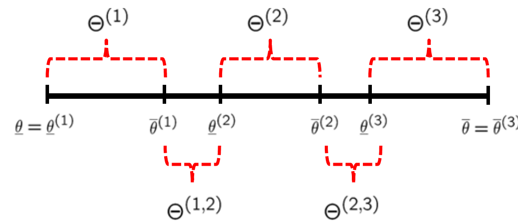

**Fig. 1.** Subintervals for switching control ($J = 3$).

This interval set will become a region where a parameter $\theta$ of an LPV system is assumed to vary in time in the controller design problem to be formulated later. For the switching LPV controller design purpose, as shown in Fig. 1, we divide the interval $\Theta$ into $(2J-1)$-number of subintervals:

$$\begin{aligned} \Theta^{(j)} &:= \left\{\theta \in \mathbb{R} : \underline{\theta}^{(j)} \le \theta \le \overline{\theta}^{(j)}\right\}, \quad j \in \mathbb{N}_J, \\ \Theta^{(j,j+1)} &:= \left\{\theta \in \mathbb{R} : \overline{\theta}^{(j)} \le \theta \le \underline{\theta}^{(j+1)}\right\}, \quad j \in \mathbb{N}_{J-1}, \end{aligned} \tag{2}$$

where the set of natural numbers up to $J$ is denoted by

$$\mathbb{N}_J := \{1, \ldots, J\}. \tag{3}$$

### 2.1. Description of an LPV plant and an LPV controller

We will consider a standard LPV generalized plant described by

$$\begin{cases} \dot{x}(t) = A(\theta(t))x(t) + B_1(\theta(t))w(t) + B_2(\theta(t))u(t), \\ z(t) = C_1(\theta(t))x(t) + D_{11}(\theta(t))w(t) + D_{12}(\theta(t))u(t), \\ y(t) = C_2(\theta(t))x(t) + D_{21}(\theta(t))w(t), \end{cases} \tag{4}$$

where the vectors $x \in \mathbb{R}^n$, $w \in \mathbb{R}^{r_1}$, $u \in \mathbb{R}^{r_2}$, $z \in \mathbb{R}^{p_1}$ and $y \in \mathbb{R}^{p_2}$ are the state, the exogenous input, the control input, the performance output and the measured output, respectively. We assume that the parameter $\theta$ is one-dimensional, that $\theta$ is measurable in real time, and that it varies in time within an interval $\Theta$ in (1) with a constraint on the rate of change of $\theta$:

$$\theta(t) \in \Theta, \qquad \dot{\theta}(t) \in V, \quad \forall t \ge 0, \tag{5}$$

where the interval $V$ is defined by

$$V := \left\{v \in \mathbb{R} : \underline{v} \le v \le \overline{v}\right\}, \tag{6}$$

for given real values $\underline{v}$ and $\overline{v}$ satisfying $\underline{v} \le \overline{v}$.

To the LPV generalized plant (4), we connect the output feedback gain-scheduling LPV controller. The parameter-varying system matrix of the LPV controller is denoted by $K(\theta) \in \mathbb{R}^{(n_k+r_2)\times(n_k+p_2)}$, where $n_k$ is the dimension of the controller state vector. Then, it is well-known (see e.g. Gahinet & Apkarian, 1994) that the closed-loop system matrix is an affine function of $K(\theta)$, and expressed by

$$\begin{bmatrix} \dot{x}_{cl}(t) \\ z(t) \end{bmatrix} = (X_0(\theta) + X_L(\theta)K(\theta)X_R(\theta)) \begin{bmatrix} x_{cl}(t) \\ w(t) \end{bmatrix}, \tag{7}$$

where $x_{cl}$ is the state vector of the closed-loop system, and the matrices $X_0$, $X_L$ and $X_R$ can be acquired from Gahinet and Apkarian (1994, Eqs. (8) and (9)).

### 2.2. Description of a smooth switching LPV controller

In this paper, we are interested in designing a special type of LPV controller, which is a *smooth switching LPV controller* described by

$$K(\theta) = \begin{cases} K^{(j)}(\theta), & \text{if } \theta \in \Theta^{(j)},\ j \in \mathbb{N}_J, \\ K^{(j,j+1)}(\theta), & \text{if } \theta \in \Theta^{(j,j+1)},\ j \in \mathbb{N}_{J-1}. \end{cases} \tag{8}$$