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Optimal filtering for networked systems with stochastic sensor gain degradation ☆

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ABSTRACT

In this paper, the optimal filtering problem is investigated for a class of networked systems in the presence of stochastic sensor gain degradations. The degradations are described by sequences of random variables with known statistics. A new measurement model is put forward to account for sensor gain degradations, network-induced time delays as well as network-induced data dropouts. Based on the proposed new model, an optimal unbiased filter is designed that minimizes the filtering error variance at each time-step. The developed filtering algorithm is recursive and therefore suitable for online application. Moreover, both currently and previously received signals are utilized to estimate the current state in order to achieve a better accuracy. A numerical simulation is exploited to illustrate the effectiveness of the proposed algorithm.

1. Introduction

In the past decade, the research on networked systems has been gaining momentum due to the rapid advances in communication technology and the increasing interest in systems connected via wireless and shared links (Gao, Lam, Wang, & Wang, 2004; Richard, 2003). In networked systems, it is often the case that the system outputs are measured by various sensors spatially located in a wide area, where the measurements are transmitted to a remote central estimator parallelly (Zhang, Yu, & Feng, 2011). Measurements in a networked environment might be seriously impaired by imperfect data transmissions, for example, communication delay and packet dropout, which may deteriorate the performance or cause instability. Thus, many traditional control/filtering techniques, which focus on interconnected dynamical systems linked through ideal channels, should be re-designed before being applied in networked systems (Hespanha, Naghshtabrizi, & Xu, 2007; Yang, 2006). Much research attention has been devoted to control/filtering in networked systems to deal with the phenomena that shared communication network induces (Garcia-Ligero, Hermoso-Carazo, & Linares-Perez, 2011; Karimi, 2009; Sun, Xie, Xiao, & Xiao, 2008; Tian, Yue, & Peng, 2010).

However, the sensor gain degradation issue in a networked environment has not been investigated extensively. Sensor gain variation occurs frequently in engineering practice. This is particularly true for real-world systems under changeable working conditions. Examples include thermal sensors for vehicles (Yalcin, Collins, & Hebert, 2007), radiation detectors for nuclear/radiological threat (Manor et al., 2009), and the platform mounted sonar for the acoustic signals from the ocean (Solomon & Knight, 2002). It is quite common in a networked system that the sensor gains are degraded in a random fashion, which is due probably to sensor aging, intermittent sensor outages, or network-induced saturations/congestions. As such, the so-called stochastic sensor gain degradation has received some initial research attention (He, Wang, & Zhou, 2009). To date, the corresponding results on *time-varying* systems using *recursive algorithms* under *stochastic sensor gain degradations* have been very few in the literature. Moreover, the network-induced time-delay and data dropout during transmissions over shared networks should be taken into consideration.

In this paper, we are motivated to study the optimal filtering problem for networked time-varying systems with stochastic sensor gain degradations by a recursive matrix equation

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approach. In the developed filtering algorithm, both current and previous measurements are used to estimate the system state so as to achieve a better estimation performance. The filter parameters are calculated recursively and the developed algorithm is thus suitable for online computation. The main contributions of the paper are outlined as follows: (1) a new model is proposed that is comprehensive to cater for sensor gain degradations, network-induced time delays as well as data dropouts within a unified framework; (2) an optimal unbiased filter is designed that minimizes the filtering error variance under sensor gain degradations whose occurrence probability is allowed to vary over the interval [0, 1]; and (3) the developed recursive algorithm is shown to be both effective and efficient as compared with the standard Kalman filtering method.

2. Problem formulation and preliminaries

Consider the following class of time-varying discrete-time linear systems:

$$x_{k+1} = \left(A_k + g_k\hat{A}_k\right) x_k + w_k, \tag{1}$$

where the matrices A_k and $\hat{A}_k$ are known; $x_k \in \mathbb{R}^n$ is the state; $w_k \in \mathbb{R}^n$ is the additive white noise with $\mathbb{E}\{w_k\} = 0$ and $\mathbb{E}\left\{w_k w_k^T\right\} = W_k$; g_k is the multiplicative white noise with $\mathbb{E}\{g_k\} = \bar{g}_k$ and $\mathbb{E}\left\{g_k^2\right\} = \tilde{g}_k$.

Assuming that there are μ different classes of sensors and the sensors belonging to the same class receive/send signals in one go, the measurements before transmitting are described by

$$y_k^{(i)} = f_k^{(i)} C_k^{(i)} x_k + v_k^{(i)}, \quad i = 1, 2, \ldots, \mu, \tag{2}$$

where $C_k^{(i)}$ is a known matrix; $y_k^{(i)} \in \mathbb{R}^{m^{(i)}}$ is the measurement of the sensors of class i; $v_k^{(i)} \in \mathbb{R}^{m^{(i)}}$ is the measurement noise of the sensors of class i with $\mathbb{E}\left\{v_k^{(i)}\right\} = 0$ and $\mathbb{E}\left\{v_k^{(i)}\left(v_k^{(i)}\right)^T\right\} = V_k^{(i)}$. $v_k^{(i)}$ is assumed to be independent of w_k and g_k. $f_k^{(i)}$ is a random variable distributed over the interval $\left[a^{(i)}, b^{(i)}\right]$ $\left(0 \le a^{(i)} \le b^{(i)} \le 1\right)$ with $\mathbb{E}\left\{f_k^{(i)}\right\} = \bar{f}_k^{(i)}$ and $\mathbb{E}\left\{\left(f_k^{(i)}\right)^2\right\} = \tilde{f}_k^{(i)}$, where $\bar{f}_k^{(i)}$ and $\tilde{f}_k^{(i)}$ are known scalars.

Remark 1. The random variable $f_k^{(i)}$ accounts for the stochastic sensor gain degradation. Compared with existing literature, (2) provides a more precise means for quantifying the sensor gain degradations, since $f_k^{(i)}$ is not restricted to take values at 0 or 1 only, and its statistical properties could be time-varying. When $f_k^{(i)}$ specializes to the traditional Bernoulli distributed one, our model covers the binary one in Wang, Yang, Ho, and Liu (2006).

The received signal impaired by communication delays and data dropouts can be described as:

$$\begin{cases} r_k^{(i)} = \sum_{j=0}^{L} \delta\left(\tau_k^{(i)}, j\right) y_{k-j}^{(i)} + n_k^{(i)}, \\ y_d^{(i)} = 0, \quad d = -L, -L+1, \ldots, -1, \end{cases} \tag{3}$$

where L is the maximum time-delay; $r_k^{(i)} \in \mathbb{R}^{m^{(i)}}$ is the received signal of the ith channel; $n_k^{(i)} \in \mathbb{R}^{m^{(i)}}$ is the transmission white noise of the ith channel with $\mathbb{E}\left\{n_k^{(i)}\right\} = 0$ and $\mathbb{E}\left\{n_k^{(i)}\left(n_k^{(i)}\right)^T\right\} = N_k^{(i)}$, and is independent of w_k and $v_k^{(i)}$. $\delta(\cdot)$ is the standard Dirac function with $\mathbb{E}\left\{\delta\left(\tau_k^{(i)}, j\right)\right\} = \text{Prob}\left\{\tau_k^{(i)} = j\right\} = p_{j,k}^{(i)}$ and $\sum_{j=0}^{L} p_{j,k}^{(i)} \le 1$. When $\tau_k^{(i)} = j$, the time delay is j at time k at the sensors of class i. When $\tau_k^{(i)} = 0$, the transmission is perfect, and no time delay or data dropout occurs. During the transmission at time k, the measurements get lost with the probability $1 - \sum_{j=0}^{L} p_{j,k}^{(i)}$. With knowledge on the order of the received signal, the receiver would discard the previously transmitted packets when several measurements arrive at the receiver in the same interval.

The following assumptions are needed in the derivation of the main results.

Assumption 1. For any i and k, $f_k^{(i)}$, g_k and $\tau_k^{(i)}$ are independent of each other and all the noises.

Assumption 2. The initial state x_0 is stochastic with $\mathbb{E}\{x_0 x_0^T\} = X_{0,0}$, and x_0 is independent of the noises, transmissions and sensor gain degradations.

For notational convenience, we denote

$$\hat{x}_0 = \mathbb{E}\{x_0\},$$
$$r_k = \left[\left(r_k^{(1)}\right)^T, \ldots, \left(r_k^{(\mu)}\right)^T\right]^T,$$
$$\bar{F}_k = \text{diag}\left\{\bar{f}_k^{(1)} I_{m^{(1)}}, \ldots, \bar{f}_k^{(\mu)} I_{m^{(\mu)}}\right\},$$
$$\bar{P}_{j,k} = \text{diag}\left\{p_{j,k}^{(1)} I_{m^{(1)}}, \ldots, p_{j,k}^{(\mu)} I_{m^{(\mu)}}\right\},$$
$$\tilde{C}_k = \left[\left(C_k^{(1)}\right)^T, \ldots, \left(C_k^{(\mu)}\right)^T\right]^T.$$

In this paper, we are interested in designing a full-order filter of the following form

$$\hat{x}_{k+1} = L_k\left(r_k - \sum_{j=0}^{L} \bar{P}_{j,k}\bar{F}_{k-j}\tilde{C}_{k-j}\hat{x}_{k-j}\right) + \left(A_k + \bar{g}_k\hat{A}_k\right)\hat{x}_k, \tag{4}$$

where L_k are filter parameters to be determined.

The filter structure given in (4) is set so as to achieve the unbiased estimation. The unbiasedness can be proven by mathematical induction as follows.

Denote $\tilde{x}_k = x_k - \hat{x}_k$. Firstly, one can verify that the assertion is true for $k = 0$ according to $\hat{x}_0 = \mathbb{E}\{x_0\}$. Secondly we assume that it is true for the integers from 0 to k, then we have

$$\begin{aligned} \mathbb{E}\{\tilde{x}_{k+1}\} &= \mathbb{E}\left\{\left(A_k + g_k\hat{A}_k\right)x_k + w_k\right\} \\ &\quad - \left(A_k + \bar{g}_k\hat{A}_k\right)\hat{x}_k - L_k\left(\mathbb{E}\{r_k\} - \sum_{j=0}^{L}\bar{P}_{j,k}\bar{F}_{k-j}\tilde{C}_{k-j}\hat{x}_{k-j}\right) \\ &= \left(A_k + \bar{g}_k\hat{A}_k\right)\mathbb{E}\{\tilde{x}_k\} - L_k\sum_{j=0}^{L}\bar{P}_{j,k}\bar{F}_{k-j}\tilde{C}_{k-j}\mathbb{E}\{\tilde{x}_{k-j}\} \\ &= 0. \end{aligned}$$

This concludes the proof.

Remark 2. A particular feature of the structure in (4) is that both currently and previously received signals are utilized to estimate the state of the system. It is worth mentioning that the state augmentation method has been largely used in the literature to tackle the measurement delays, which would inevitably cause exponentially increasing computational burden due to the increased dimension. In contrast, the recursive matrix equation method to be developed in this paper possesses the advantage of efficient computation for the addressed filter design problem. On the other hand, note that the terms reflecting the statistics, $\bar{F}_k$, $\bar{P}_{j,k}$ and $\bar{g}_k$, are fixed values which make the filter structure easy-to-implement.