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# Scenario-based robust optimization of a water supply system under risk of facility failure

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ABSTRACT

In this paper, we propose a scenario-based robust optimization model for the design of a water supply system considering the risk of facility failure, which is represented as an uncertainty set generated by a finite set of scenarios. New facilities are planned to be built to hedge against the possible failure of existing system facilities that would potentially damage the capacity of the system to meet given user demands. The goal is to build facilities that are both cost-effective and make the system robust. The system robustness is defined as the ability to satisfy user demands for every data realization in the uncertainty set. The proposed model is shown to be equivalent to a large-scale mixed-integer linear program that is solved by a Benders decomposition algorithm. Computational results demonstrate the efficiency of the proposed algorithm, and show that substantial improvement in system robustness can be achieved with minimal increase in system cost.



## 1. Introduction

Population growth, economic development and diminishing freshwater supplies pose serious challenges for water supply planning and management in many urban areas. An integrative approach is thus needed for efficiently managing water resources. Mathematical optimization models have proven their usefulness for this purpose. Early models for water supply planning and management were generally deterministic in that the problem parameters are assumed to be perfectly known. When implemented in practical situations, however, such models may lead to solutions that are suboptimal, or even infeasible, when the problem parameters are revealed and differ from their assumed values.

The need to incorporate uncertainty in water supply system planning and management has been increasingly acknowledged, and research in this area has led to a variety of optimization models. Several models handle uncertainty in problem parameters by applying stochastic programming methods, including recourse-based stochastic programming (Housh et al., 2013b) and scenarios analysis (Escudero, 2000; Pallottino et al., 2005; Kang and Lansey, 2014).

Stochastic programming models in general minimize the expected value of a cost function assuming a known probability distribution, or a finite set of known scenarios. They do not account for the risk associated with the variability of the cost. Robust optimization (RO), however, attempts to incorporate risk aversion in the modeling process. One RO method to capture this risk-averse behavior is to add a risk term to the objective in an otherwise stochastic programming model. While the use of variance (Watkins and McKinney, 1997; Housh et al., 2013a; Kang and Lansey, 2013; Ray et al., 2014) is common, other types of risk terms, such as the conditional value-at-risk (*CVaR*) (Yamout et al., 2007), have also been reported. An alternative RO method that circumvents the need to specify a probability distribution for uncertain problem parameters is the so-called robust counterpart approach (Ben-Tal and Nemirovski, 1999). This approach seeks a solution that is optimal for every possible data realization in a user-defined

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uncertainty set for the parameters (Chung et al., 2009; Housh et al., 2011).

Almost all models for water supply system planning under uncertainty in existing literature consider uncertainty in water supply and/or demand only. They implicitly assume that, once constructed, the system facilities, such as water conveyance structures and water treatment plants, will always function as desired. In reality, facilities will malfunction, or "fail", due to events ranging from mechanical failures to natural catastrophes, leading to partial or complete loss of their normal capacities. Facility failures can potentially make the system incapable of meeting given user demands.

In this paper, we present a scenario-based robust optimization model for designing a water supply system. Given a finite set of assumed *failure scenarios* with each scenario specifying which facilities will fail and when, as well as the capacity loss for these failed facilities, an uncertainty set is generated as a convex hull of the capacity realizations under all the failure scenarios and a *no-failure scenario* where all facilities are operational for the entire planning horizon. The model attempts to find the most cost-effective way to construct and operate system facilities while making the system *robust*. The system robustness is defined as the ability to meet given user demands for every data realization in the uncertainty set. The RO model is shown to be equivalent to a large-scale mixed-integer linear program (MILP). A Benders decomposition method (Nemhauser and Wolsey, 1999) is then proposed to solve the MILP. Computational experience on a practical water supply planning problem demonstrates the efficiency of the proposed algorithm and suggests that a large improvement to system robustness is possible without a large increase of system cost.

The remainder of this paper is organized as follows. Section 2 gives the problem statement, and Section 3 formulates a deterministic model for the problem assuming known facility capacities. Section 4 introduces the RO model which extends the deterministic model by considering the risk of facility failure, and shows that it is equivalent to a MILP. The Benders decomposition method is presented in Section 5 for solving the MILP. Section 6 presents computational results, and Section 7 concludes the paper.

## 2. Problem statement

Consider the *centralized* water supply system (WSS) for a metropolitan area in Fig. 1 (plotted in solid lines) Surface water and groundwater aquifer (GW) provide potable (drinking-quality) water supply. The surface water must be treated at a water treatment plant (WTP) before use. This treated water and pumped groundwater are distributed to users for potable or non-potable purposes through pipelines and pump stations. Wastewater is collected from the users to a centralized wastewater treatment plant (WWTP). Most non-potable uses are outdoor watering, and this water is not returned to the WWTP. The reclaimed (i.e. treated) water can be directly provided to users for non-potable purposes, or recharged and then recovered from an aquifer and distributed for potable uses.

Given that certain facilities in the WSS are subject to failure, leading to partial or complete loss of their normal capacities, the system may not be able to satisfy the user demands when failure occurs. With this in mind, the goal of the problem is to develop optimal plans for constructing satellite wastewater treatment plants (SP) and indirect potable reuse (IPR) facilities. These *decentralized* facilities (i.e. SPs and IPR facilities) generally have lower capacities than their centralized counterparts, and are built closer to water consumers. Fig. 1 plots a pair of SP and IPR facilities (in dashed line) for illustration. Instead of conveying all wastewater after potable use to the centralized WWTP, a portion of it can now be diverted to the SP for treatment. The treated water can be filtered into the IPR facility before being extracted for potable and nonpotable uses.

## 3. A deterministic model

In this section, a deterministic model is presented for water supply system planning *without* considering the risk of facility failure. The complete notation used in the deterministic model and the rest of the paper is listed in Table 1. Note that for each subscripted/superscripted parameter or decision variable defined therein, the same symbol in bold face without the subscript/superscript will be used to denote a vector of the corresponding entity.

Consider a water supply system with a network representation $\boldsymbol{G} = (\boldsymbol{N}, \boldsymbol{A})$, where $\boldsymbol{N}$ is the set of nodes and $\boldsymbol{A}$ is the set of arcs. The nodes denote water sources, users, treatment and recharge facilities, and the arcs represent pipelines. The set of nodes $\boldsymbol{N}$ is categorized as the set of storage nodes $\boldsymbol{N^S}$ that can store water over time, such as recharge facilities, and the set of nonstorage nodes $\boldsymbol{N^{NS}}$. Given a planning horizon represented by a set of time periods $\boldsymbol{T}=\{1, 2, \ldots, T\}$ with projected water supply and demand for each time period $t \in \boldsymbol{T}$, we need to decide when, where and what capacities of the SPs and IPR facilities to build, and flow allocation over the system network for each time period, with the objective to minimize system cost while satisfying user demands. The subset $\boldsymbol{T^1}$ of time periods and subsets $\boldsymbol{N^{SP}}$ and $\boldsymbol{N^{IPR}}$ of nodes designate the time and locations where an SP and IPR facility may be built,

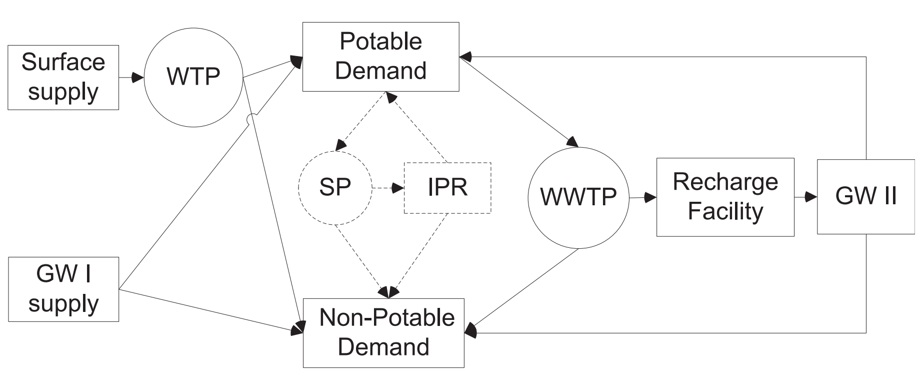


**Fig. 1.** An illustrative water supply system.