Environmental Modelling & Software 65 (2015) 50–57

Contents lists available at ScienceDirect

Environmental Modelling & Software

journal homepage: www.elsevier.com/locate/envsoft

Impulse controls and uncertainty in economics: Method and application

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ARTICLE INFO

Article history:
Received 13 June 2014
Received in revised form
25 November 2014
Accepted 26 November 2014
Available online 22 December 2014

JEL Classification:
C61
C63
Q22
Q27

Keywords:
Dynamic programming
Consistent approximation
Discontinuous jumps
Linear programming
Marine reserves
Principle of optimality
Unexpected disturbances

ABSTRACT

We develop a stochastic optimal control framework to address an important class of economic problems where there are discontinuities and a decision maker is able to undertake impulse controls in response to unexpected disturbances. Our contribution is two fold: (1) to develop a linear programming algorithm that produces a consistent approximation of the maximum value and optimal policy functions in the context of stochastic impulse controls; and (2) to illustrate the economic benefits of impulse controls optimized, using our framework, and calibrated to the population dynamics of a marine fishery. We contend that the framework has wide applicability and offers the possibility of higher economic pay-off for a wide-range of policy problems in the presence of discontinuities and adverse shocks.

1. Introduction

Optimal control methods have been widely used to address many economic phenomena that include the dynamics of saving and investment behaviours or the optimal extraction of natural resources. Such methods, however, are rarely used for an important class of problems where a decision maker is able to undertake impulse controls and there are discontinuities in the system. The importance of impulse controls in economics has long been recognized, at least in renewable exploitation (e.g., Clark, 1976; Hannesson, 1975), but economic applications are scant. One possible explanation, as observed by Erdlenbruch et al. (2010), is that deterministic impulse control can only rarely be applied at an aggregate level in terms of optimal policy responses because of the possibility that anticipated state jumps may give rise to arbitrage opportunities by economic agents. By contrast, in the mathematics literature there are multiple descriptions (Getz and Martin, 1980; Perthame, 1984; Dar'in et al., 2005), and impulse controls are included in the hybrid control literature (Branicky et al., 1998; Attia et al., 2007).

There are at least two broad categories of economic problems that can be analysed by impulse controls. In the first category, decision makers select to implement cyclical policies where the controls are activated at the beginning of a cycle. This class of problems has been recently analysed by Erdlenbruch et al. (2013) who propose a condition for the existence of such a cyclical policy, based on a deterministic unidimensional impulse-control framework introduced by Vind (1967) and Leonard and Long (1992).

Our paper focuses on the second category of impulse controls which are designed to deal with uncertainties and when and how to apply impulse controls in response to an unexpected disturbance to the state variable(s). Such decisions are of practical relevance and include the policy responses that may arise following a catastrophic event, such as an earthquake or cyclone, or what might be the

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http://dx.doi.org/10.1016/j.envsoft.2014.11.027
1364-8152/

optimal investment or spending decision following an adverse financial event. The approach we use to solve this class of economic problems, and that requires impulse controls, incorporates: (1) Bellman's (1957) state-dependent decision-making rule and (2) uncertainty with a well-defined probabilistic distribution that allows for shocks consistent with real-world phenomena. Our model uses state-jump uncertainties based on Poisson diffusions (Walde, 2010) that can cause a sudden change in state variables and also allows for the activation of unanticipated impulse controls.

Our impulse control is a combination of the standard optimal control and real options (Dixit and Pindyck, 1994) with discontinuities in response to unanticipated disturbances. Managing environmental risks could greatly reduce losses from negative shocks (Balica et al., 2013) and real options are not limited to finance but can be used in environmental and economic modelling (see Marques et al., 2015 for a list of examples). In our view, risk management is the beginning of any policy modelling process (Refsgaard et al., 2007) where both routine performance and the ability to quickly respond to unexpected disturbances are important (e.g., Zagonari and Rossi, 2013). Our impulse control provide a modelling framework to specify not only the optimal actions under normal circumstances, but also the optimal recovery option from a disturbance by making relocation of resources.

Another important contribution of our paper is to develop a linear programming algorithm that, under the regular assumption of twice differentiability, produces a consistent approximation of the maximum value and optimal policy functions for impulse control problems, including deterministic ones. While linear programming has been used in dynamic programming for more than half a century (Manne, 1960; Ross, 1970), it has been limited to discrete-state problems because of the requirement that there be a finite number of possible states. Recent developments by Farias and Roy (2003, 2004) have used a contraction mapping Bellman operator and introduced a consistent parametric approximation via linear programming that can be applied to continuous-state discrete-time problems. Han and Roy (2011), and also Kompas and Chu (2012), have shown how this parametric approximation can be applied to standard Hamilton–Jacobi–Bellman (HJB) operators. The consistency of such an approach is, as yet, not confirmed because HJB operators do not possess a contraction mapping as does the discrete-time analogue. By contrast, in our solution algorithm, the parametric linear programming approach is extended to impulse controls, but without using contraction mapping. Unlike the random approximation and probabilistic error bound employed by Farias and Roy (2004), we focus on a non-random consistent approximation for the impulse control framework.

To show the practical relevance of impulse controls, we illustrate the framework in the context of the establishment and relocation or switching of a marine reserve or no take area that also requires the optimization of the level of harvest for a target species. Our application is calibrated to the Pacific Halibut fishery (Grafton et al., 2006) and encompasses the key characteristic of the problems requiring impulse controls, namely, uncertainty that involves a sudden change in state variables, in this case the fish stock.

In Section 2, we provide the general problem formulation that bounds the state and control variables, defines the uncertainty components and delineates the relevant economic assumptions. Section 3 presents an easy-to-implement algorithm for a numerical solution, with requisite proofs. It shows that our method has a solution and also that the approximation errors converge to zero as more computation resources are devoted to the optimization. Our method is illustrated in Section 4 where, following a negative environmental disturbance that reduces the size of a fish stock, the decision maker must determine the optimal size of the harvest and the proportion of the stock located in a reserve or no-take area. Section 5 concludes while formal proofs are provided in the appendices.

2. Optimization with impulse controls

2.1. State, control variables and state transition

We specify a dynamic system with an n-dimensional vector of state variables that has an initial value defined in equation (1):

$$k(0) = s \text{ where } s \in \mathscr{R}^n \tag{1}$$

At each point in time, the states can be controlled via two types of control variables, namely continuous and impulse controls. The continuous control, denoted as c, can take values in correspondence $\phi^c(k)$ and the impulse control, denoted as ω, can take value in correspondence $\phi^\omega(k)$. When the state is controlled via the continuous control $c \in \phi^c(k)$, we assume, without the loss of generality, that the impulse control takes value zero $\omega = 0$ (thus $\forall k$: $0 \in \phi^\omega(k)$), and then the continuous transition is governed by:

$$\mathrm{d}k = g(k,c)\mathrm{d}t + \mathrm{d}b(k) + \mathrm{d}q(k) \tag{2}$$

where $db(k)$ is the differential of a Brownian diffusion with state-dependent standard error $b(k)$ to capture neutral uncertainty and $dq(k)$ is the differential of a state-dependent Poisson diffusion to capture unexpected disturbances. Following Walde (2010), the Poisson diffusion we use in this model is a continuous-time random process q such that

$$q(k(t+\mathrm{d}t)) - q(k(t)) = \begin{cases} \mathrm{d}q(k) \neq 0 \text{ with probability } \lambda \mathrm{d}t \\ 0 \text{ with probability } 1 - \lambda \mathrm{d}t \end{cases}$$

where $dq(k)$ is an n-dimensional vector referred to as the size and scalar λ is the rate of arrival (possibly state-independent) of the diffusion.

If controlled via the impulse control ($\omega \in \phi^\omega(k)\backslash\{0\}$), the states will jump discontinuously and the continuous control takes value zero ($0 \in \phi^c(k)\ \forall k$). To model this discontinuous jump, we define a jump function, $\varphi := (\mathscr{R}^n, \mathscr{R}) \to \mathscr{R}^n$, mapping pre-jump state and the impulse control to post-jump state:

$$k\left(\tau^+\right) = \varphi(k(\tau), \omega(\tau)) \text{ where } \forall k : \varphi(k, 0) = k \tag{3}$$

2.2. Objective function and optimization

We consider a policy plan over an infinite time horizon $\{c(t)_0^\infty, \omega(t)_0^\infty\}$ which specifies that the impulse control ω is activated at time $t = \tau_i | i = 1, 2 \ldots M$ (M can be finite or infinite). The (current value) cost to activate the impulse control is $C(k(\tau_i), \omega(\tau_i))$, where $C := (\mathscr{R}^n, \mathscr{R}) \to \mathscr{R}$. Other than the jumps, the states are controlled continuously and generate an instantaneous return $u(k,c)$. Following Kemp and Long (1977), the return associated with the policy plan $\{c(t)_0^\infty, \omega_0^\infty\}$, aggregating from time zero, with a discounting rate $\rho > 0$, will be:

$$U^{\{c,\omega\}}(0) = \sum_{i=0}^{M} \int_{\tau_i^+}^{\tau_{i+1}} e^{-\rho t} u(k(t), c(t)) \mathrm{d}t - \sum_{i=1}^{M} e^{-\rho \tau_i} C(k(\tau_i), \omega(\tau_i)) \tag{4}$$

where for the sake of shortening the notation: $\tau_0^+ \equiv 0$ and $\tau_{M+1} \equiv \infty$.

The optimization problem is to choose a feasible policy plan $\{c(t)_0^\infty, \omega_0^\infty\}$ that maximizes the expected value of the return