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Observer based output feedback control of linear systems with input and output delays☆

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ABSTRACT

This paper is concerned with observer based output feedback control of linear systems with both (multiple) input and output delays. Our recently developed truncated predictor feedback (TPF) approach for state feedback stabilization of time-delay systems is generalized to the design of observers. By imposing some restrictions on the open-loop system, two classes of observer based output feedback controllers, one being finite dimensional and the other infinite dimensional, are constructed. It is further shown that, the infinite dimensional observer based output feedback controllers can be generalized to linear systems with both time-varying input and output delays. It is also shown that the separation principle holds for the infinite dimensional observer based output feedback controllers, but does not hold for the finite dimensional ones. Numerical examples are worked out to illustrate the effectiveness of the proposed approaches.

1. Introduction

Delay differential equations, which are also known as functional differential equations, can be utilized to model many practical physical systems, especially those systems influenced by the effect of transmission, transportation and inertia phenomena. The control of time-delay systems is challenging since these systems are inherently infinite dimensional. Existing methods that have been well developed for conventional control systems modeled by ordinary differential equations are generally not directly applicable. As a result, control of time delay systems has received much attention for several decades and a large number of research results have been reported in the literature that deal with various analysis and design problems (see, for instance, Chen, Fu, Niculescu, and Guan (2010), Foias, Ozbay, and Tannenbaum (1996), Fridman, Shaked, and Liu (2009), Gu, Kharitonov, and Chen (2003), He, Wang, Lin, and Wu (2007), Karafyllis (0000), Lam, Gao, and Wang (2007) and Zhang, Zhang, and Xie (2004) and the references therein).

State feedback control is very powerful for both ordinary differential equations and functional differential equations as the full information of the state vectors is assumed to be accessible for feedback. Therefore, if the state vectors are measurable, state feedback is the best choice. It is under the assumption of accessibility of state vectors that numerous results on control of time delay systems have been reported in the literature (see Chen and Zheng (2007), He et al. (2007), Xie, Fridman, and Shaked (2001), Xu, Lam, and Yang (2001), Yakoubi and Chitour (2007) and Zhou, Gao, Lin, and Duan (2012) and the references cited there). State feedback is particularly efficient for linear systems with input delay by using the predictor feedback (Furukawa & Shimemura, 1983; Misaki, Uchida, Azuma, & Fujita, 2004). However, in many real world control systems, only the measured output information, rather than the full state information, is available for feedback. As the ability of static output feedback is generally limited (see Cao, Lam, and Sun (1998) for more detailed introduction), it is more realistic to use an observer based output feedback controller, which is a dynamic output feedback controller that estimates the system states on-line. Therefore, from the practical point of view, observer based output feedback design is important.

Several results on the observer based output feedback control of time delay systems are available in the literature (see, for example, Bhat and Koivo (1976), Krstic (2009), Leyva-Ramos and Pearson

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(2000) and Sun (2002) and the references therein). One of the most remarkable results is on the observer based output feedback control of linear systems with both multiple input and multiple output delays (Watanabe & Ito, 1981). The underlying observers are constructed by using the predictor approach (Bekiaris-Liberis & Krstic, 2011; Kojima, Uchida, Shimemura, & Ishijima, 1994; Krstic, 2010a; Mazenc, Niculescu, & Krstic, 2012; Olbrot, 1978) and have been generalized to more general cases in Klamka (1982). These observers, as designed in, Klamka (1982), Kojima et al. (1994) and Watanabe and Ito (1981), are naturally the dual results of the predictor feedback for linear systems with input delays (Olbrot, 1978). The predictor feedback can also be interpreted as model reduction based controllers (Artstein, 1982) and finite spectrum assignment (Manitius & Olbrot, 1979). However, dual to the predictor feedback, the predictor based observer designed in Klamka (1982) and Watanabe and Ito (1981) involves distributed terms that are integration of the past output signals (Artstein, 1982; Krstic, 2010a; Manitius & Olbrot, 1979). As a result, if the open-loop system is not exponentially stable, the predictor based observer designed in Klamka (1982) and Watanabe and Ito (1981) can only be implemented via approximating the integral terms with a sum of point-wise delays by numerical quadrature rule such as rectangular, trapezoidal and Simpson's rules (Manitius & Olbrot, 1979; Mondie & Michiels, 2003). However, even in the case of state feedback, the effect of such a semi-discretization on the asymptotic stability of the closed-loop system is very complicated (Van Assche, Dambrine, Lafay, & Richard, 1999). More detailed exposition of the problems encountered in implementing the distributed terms in the predictor based controllers can be found in Mirkin (2004), Mondie and Michiels (2003) and Zhou, Lin, and Duan (2011a, 2012). We mention that for linear systems with input and output delays, the standard four-block output feedback H_∞ control problem was solved in Meinsma and Mirkin (2005) by treating the multiple delay operator as a special series connection of the adobe delay operators.

To avoid the implementation problem for the predictor based controllers, we recently proposed in Zhou, Lin, Duan (2012) (see also Zhou et al. (2011a)) a new approach named truncated predictor feedback (TPF). The idea of the TPF is that, provided the open-loop system satisfies some assumptions, and with the parameterization of the nominal feedback gain in the predictor based feedback controllers by a positive scalar γ, the distributed term in the predictor based controller is a high order infinitesimal with respect to γ and can thus be safely neglected when the value of γ is small enough (Lin & Fang, 2007; Zhou et al., 2011a; Zhou, Lin, Duan, 2012). This TPF approach has been proven to be very effective even the delay in the system are time-varying (Zhou et al., 2011a; Zhou, Lin, Duan, 2012) and distributed (Zhou, Gao et al., 2012).

The aim of the present paper is to generalize the idea of the TPF approach in Zhou et al. (2011a); Zhou, Lin, Duan (2012) to the design of observer based output feedback controllers for linear systems with both input and output delays. As a result, the implementation difficulty inherent in the predictor based output feedback controllers proposed in Klamka (1982) and Watanabe and Ito (1981) is completely avoided. In particular, we will propose two classes of observer based output feedback controllers by using the TPF approach. The first type of observer based output feedback controllers can be regarded as the generalization of the TPF designed in Zhou et al. (2011a); Zhou, Lin, Duan (2012) to the design of observers with the help of the separation principle. However, this class of observer based output feedback controllers are infinite dimensional and may still be hard to implement if the open-loop system contains distributed delays. To avoid this problem, another class of *finite dimensional* observer based output feedback controllers are proposed. This class of finite dimensional controllers are very easy to implement as only the input and output vectors themselves are required to be used. The derivation of this class of finite dimensional observer based output feedback controllers is highly nontrivial as the separation principle no longer holds. Indeed, an involved stability analysis should be carried out. A detailed numerical example shows that a finite dimensional observer based output feedback controller outperforms the infinite dimensional one.

The remainder of this paper is organized as follows. The problem formulation and some preliminary results are given in Section 2. The infinite dimensional and finite dimensional observer based output feedback controllers by the TPF approach are then respectively studied in Sections 3 and 4. A numerical example is given in Section 5 to show the effectiveness of the proposed design. Finally, Section 6 concludes the paper.

Notation. The notation used in this paper is fairly standard. For a vector $u \in \mathbf{R}^m$, we use $\|u\|_\infty$ to denote the ∞-norm of u. For a matrix $A \in \mathbf{R}^{n\times n}$, A^{T} and $\|A\|$ are respectively its transpose and 2-norm. For a positive scalar τ, let $\mathscr{C}_{n,\tau} = \mathscr{C}([-\tau, 0], \mathbf{R}^n)$ denote the Banach space of continuous vector functions mapping the interval $[-\tau, 0]$ into $\mathbf{R}^n$ with the topology of uniform convergence, and let $x_t \in \mathscr{C}_{n,\tau}$ denote the restriction of $x(t)$ to the interval $[t-\tau, t]$ translated to $[-\tau, 0]$, that is, $x_t(\theta) = x(t+\theta), \theta \in [-\tau, 0]$. Finally, for a linear system characterized by the matrix pair (A, B), we say that it is asymptotically null controllable with bounded controls (ANCBC) if (A, B) is stabilizable in the ordinary linear systems theory sense and all the eigenvalues of A are located on the closed left-half plane.

2. Problem formulation and preliminary results

2.1. Problem formulation

Consider the following linear system with multiple delays in both the inputs and the outputs

$$\begin{cases} \dot{x}(t) = Ax(t) + \displaystyle\sum_{i=1}^{p} B_i u(t-h_i), \\ y(t) = \displaystyle\sum_{j=1}^{q} C_j x\left(t-l_j\right), \end{cases} \tag{1}$$

where $A \in \mathbf{R}^{n\times n}$, $B_i \in \mathbf{R}^{n\times m}$, $i \in \mathbf{I}[1, p]$, and $C_j \in \mathbf{R}^{r\times n}$, $j \in \mathbf{I}[1, q]$, are constant matrices, and $h_i, i \in \mathbf{I}[1, p]$, and $l_j, j \in \mathbf{I}[1, q]$, are known nonnegative constant scalars representing respectively the input delays and the output delays. Without loss of generality, we assume that

$$0 \le h_1 < h_2 < \cdots < h_p = h, \tag{2}$$

$$0 \le l_1 < l_2 < \cdots < l_q = l. \tag{3}$$

The initial conditions for system (1) are assumed to be $x_0(\theta), \forall \theta \in [-l, 0]$, and $u_0(\theta), \forall \theta \in [-h-l, 0)$ (Watanabe & Ito, 1981).

We suppose that u and y are measurable, but x and the initial functions $x_0(\theta)$ and $u_0(\theta)$ are unknown and unmeasurable, which are the standard assumptions on observer based output feedback scheme (Watanabe & Ito, 1981). In this paper, we are interested in the design of observer based output feedback stabilizing controllers that are easy to implement for the time-delay system in (1).

Associated with the time-delay system in (1), we define two constant matrices

$$\begin{cases} B = B\left(\{h_i\}_{i=1}^{p}\right) = \displaystyle\sum_{i=1}^{p} \mathrm{e}^{-Ah_i} B_i, \\ C = C\left(\left\{l_j\right\}_{j=1}^{q}\right) = \displaystyle\sum_{j=1}^{q} C_j \mathrm{e}^{-Al_j}. \end{cases} \tag{4}$$