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Reversing hydrology: Estimation of sub-hourly rainfall time-series from streamflow

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ABSTRACT

A novel solution to the estimation of catchment rainfall at a sub-hourly resolution from measured streamflow is introduced and evaluated for two basins with markedly different flow pathways and rainfall regimes. It combines a continuous-time transfer function model with regularised derivative estimates obtained using a recursive method with capacity for handling missing data. The method has general implications for off-line estimation of unknown inputs as well as robust estimation of derivatives. It is compared with an existing approach using a range of model metrics, including residuals analysis and visuals; and is shown to recover the salient features of the observed, sub-hourly rainfall, sufficient to produce a precise estimate of streamflow, indistinguishable from the output of the catchment model in response to the observed rainfall data. Results indicate potential for use of this method in environment-related applications for periods lacking sub-hourly rainfall observations.

1. Introduction

Accurate simulation of stream hydrographs is strongly dependent on the availability of rainfall data at a sufficiently high, sub-daily sampling intensity (Hjelmfelt, 1981; Littlewood and Croke, 2013). Additionally, hydrograph simulation may be sensitive to the spatial intensity of rainfall sampling (Ogden and Julien, 1994; Bardossy and Das, 2008) or to the uncertainties arising from local calibrations of rainfall radar (Cunha et al., 2012) or individual raingauges (Yu et al., 1997). Despite this importance, most gauged basins lack the necessary long-term, sub-hourly rainfall records (and adequate spatial rainfall sampling) to combine with the streamflow records that are, by contrast, typically monitored at sub-hourly intervals for several decades. If those short-term rainfall characteristics responsible for producing stream hydrographs (see Eagleson, 1967; Obled et al., 1994) can be estimated from streamflow, the resultant synthetic rainfall series may be useful in many applications. For example, synthetic rainfall records could be derived for basins with long-term streamflow, but only short-term rainfall, to: (1) evaluate long-term, rainfall estimates from Global Circulation Models for specific catchments (see Fujihara et al., 2008), (2) provide long-term rainfall records for long-term aquatic ecology studies (e.g., Ormerod and Durance, 2009), and (3) identify localised rainfall cells or snowfall events that affect the streamflow but are poorly represented in raingauge records (Kirchner, 2009).

This study uses a Data-Based Mechanistic (DBM) modelling approach to identify linear Continuous-Time Transfer Function (CT-TF) models (Young and Garnier, 2006) between sub-hourly rainfall and streamflow. These forward CT-TF models are then inverted to derive rainfall time-series using a novel method that utilises regularisation techniques. Algorithms within the CAPTAIN Toolbox (Taylor et al., 2007) are used for this modelling and the methodology evaluated by application to two micro- or headwater-catchments with contrasting rainfall and response characteristics, namely the humid tropical Baru catchment and the humid temperate Blind Beck catchment. Classical rainfall–runoff non-linearity utilises a power law relationship between measured and effective rainfall (Beven, 2011) implemented as a Hammerstein type non-linearity (Wang and Henriksen, 1994) separated from the linear dynamics of the transfer function. As the power function is monotonic, it is easily inverted, making it trivial to apply in

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combination with the effective rainfall estimate generated by the proposed method as illustrated in Fig. 1.

The graphical expression of the forward CT-TF model of a rainfall–streamflow response in discrete time is the impulse response function and this is directly equivalent to the unit hydrograph or UH developed by Sherman (1932). Inversion of the UH or its CT-TF equivalent to derive rainfall from streamflow has been attempted by Hino (1986), Croke (2006), Kirchner (2009), Andrews et al. (2010) and Young and Sumisławska (2012). These studies have used a range of different approaches. For example, Hino (1986) applied a standard regularised Least Squares (LS) solution to the inversion of a catchment model of ARX form (i.e., autoregressive with exogenous variables: see Box et al., 2008). This approach differs from the CT-TF based approach proposed here, in that potentially huge matrix inversions are needed. Kirchner (2009) used a very different method that involved the construction of a first-order, non-linear differential equation linking rainfall, evaporation and streamflow through the sensitivity function, resulting in a compound measure of precipitation and evaporation, which is then reduced to rainfall through making assumptions about the relationship between the rainfall and residual rainfall (i.e., rainfall minus evaporation). Kirchner's method has been applied to the Rietholzbach catchment in Switzerland (Teuling et al., 2010) and to 24 diverse catchments in Luxembourg (Krier et al., 2012) where it reproduces the streamflow and storage dynamics for catchments characterised by a single storage–discharge relationship but cannot explain more complex travel times. Andrews et al. (2010) used inverse filtering, applying similar CAPTAIN modelling methods to the ones proposed here, but using a direct inverse transfer function in discrete time. As this is methodologically the nearest approach to the proposed one and, at the same time, highlights the practical problems with direct inversion of transfer function models, it was chosen as a comparison in this study. Young and Sumisławska (2012) applied non-minimal state-space feedback control methods to inversion of discrete time transfer function models, based on the work of Antsaklis (1978).

Jakeman and Young (1984) were the first to indicate that recursive regularisation might be a useful approach to derive rainfall time-series from the UH, but without offering an implementation of the algorithm or examples. The novel method proposed here has been developed by combining these ideas with developments in the identification of CT-TF models (e.g., Young and Garnier, 2006) and improvements in the CAPTAIN routines (Taylor et al., 2007). The inverse process is based on differentiation (Young, 2006), and so may be expected to be ill-posed and sensitive to noise in the streamflow data (O'Sullivan, 1986; Neumaier, 1998; Tarantola, 2005). The direct inverse of the discrete transfer function method involves differencing, the key issue addressed in the proposed method by using regularised derivatives, potentially its major advantage.

The generality of our approach indicates that it could be used within any modelling framework involving DBM or top-down catchment modelling. Integrating it within other frameworks, for instance to assess the information content of hydrological data (Beven and Smith, 2014) is already a part of an existing project which partly funded this study (NERC CREDIBLE project – see Acknowledgements for details). Another good example of the use for this approach would be within the **hydromad** framework (Andrews et al., 2011) where it could be a part of either model or data evaluation process. Such application could be based on the reasoning that a model and data combo (the principle of DBM approach), which invert well should be more reliable (this assertion will be the subject of future work). Within the same **hydromad** framework a similar reasoning could be used to verify the placement of raingauges within a catchment. If the inversion generates poorly fitting inferred rainfall with many negative periods it could indicate that the present raingauges do not provide full information about the catchment rainfall due to their placement. Andrews et al. (2011) also indicate the use of such inversion routines in calibration of full hydrological models.

Reaching further out, beyond the discipline of hydrology, there are many other situations where either input estimation of a dynamic system (e.g., Maquin, 1994; Yang and Wilde, 1988 and many others), or more generally, robust derivative estimation problems (De Brabanter et al., 2011) could benefit from the solution provided here. The off-line character of the method, characteristic for regularisation-based methods, excludes on-line applications, such as input observers in control engineering, but provides more flexibility, for instance by easy compensation of pure time delays in the transfer functions.

2. Novel parsimonious method for input estimation using reduced order output derivatives

To obtain a well-defined and effective inverse of any transformation (e.g., UH or equivalently a TF), the transformation itself must be well defined. It must capture the character of the system without any unnecessary complexity that would result in the transformation itself being ill-defined. This is the essence of the philosophy of the Data-Based Mechanistic (DBM) approach of Young (1998, 1999) that aims to produce models that fit the data well with as few parameters as are necessary to capture the dominant dynamic modes of the system. CAPTAIN tools are used to identify models using this underlying philosophy.

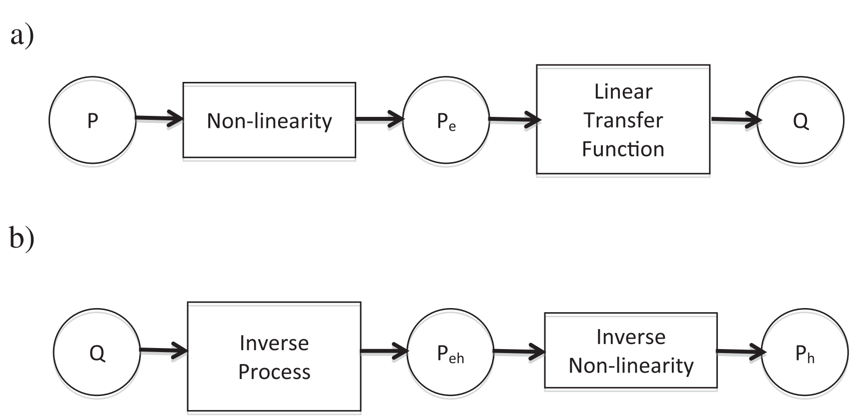

Fig. 1. The use of Hammerstein-type non-linearity in the model identification (a) and inversion (b) processes where P is the observed rainfall, P_e is the effective rainfall, Q is the observed streamflow, P_{eh} is the inferred effective rainfall and P_h is the inferred rainfall with the non-linearity reapplied.