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Brief paper

Stable $\mathcal{H}^\infty$ controller design for systems with multiple input/output time-delays[☆]

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ABSTRACT

The stable $\mathcal{H}^\infty$ controller design problem is considered for multi-input–multi-output systems with multiple input/output time-delays. An algorithm is presented to solve this problem. The algorithm makes use of the small-gain theorem and the structure of the $\mathcal{H}^\infty$ controller for the class of systems under consideration.

1. Introduction

Many systems, biological, economical, or physical, include time-delays. These delays may be ignored for controller design, when they are sufficiently small. However, when they become significant, they may have a detrimental effect on the system. In such a case, time-delays should be taken into account during controller design. Existence of time-delays, however, challenges the controller design problem, since such systems are infinite-dimensional. In the literature, numerous approaches have been proposed for controller design for time-delay systems (see Niculescu (2001) for a wide survey). Operator theoretical approaches have been used by Curtain and Zwart (1995) and by Foias, Özbay, and Tannenbaum (1996) to design controllers for general infinite-dimensional systems. Toker and Özbay (1995) used Hankel+Toeplitz operator theory to design an $\mathcal{H}^\infty$ controller for single-input–single-output (SISO) infinite-dimensional systems. A J-spectral factorization approach was used to design a controller for systems with a single time-delay by Meinsma and Zwart (2000). Later, by using the chain-scattering framework and J-spectral factorizations, an $\mathcal{H}^\infty$ controller design approach for multi-input–multi-output (MIMO) systems with multiple input/output (I/O) time-delays was presented by Meinsma and Mirkin (2005).

When an optimization approach, such as $\mathcal{H}^\infty$, is used to design a controller for any system, the resulting controller may or may not be stable. An unstable controller, although theoretically stabilizes the overall system and optimizes a certain performance/robustness measure, is in general undesirable due to two reasons:

- the closed-loop system becomes highly sensitive to sensor/actuator faults, since such a fault can make the overall system unstable (a stable controller, however, guarantees overall stability under such a fault if the plant is also stable);
- an unstable controller introduces additional right-half-plane zeros, which reduce the tracking ability and disturbance rejection of the closed-loop system and makes it more sensitive to numerical errors and nonlinear effects (Vidyasagar, 1985). Such effects, may indeed cause an unstable behaviour in a practical implementation (e.g., see Ünal (2010)).

Due to above reasons, the stable controller design problem, which is also referred to as the *strong stabilization problem*, has been considered in the literature for a long time (e.g., Vidyasagar (1985); Saif, Gu, and Postlethwaite (1997); Zeren and Özbay (2000); Lee and Soh (2002); Campos-Delgado and Zhou (2003); Gümüşsoy and Özbay (2005); Cheng, Cao, and Sun (2009)). The strong stabilization problem has also been considered for SISO time-delay systems (e.g., Abedor and Poolla (1989); Suyama (1991); Toker and Özbay (1996); Gümüşsoy and Özbay (2008)).

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To the authors' best knowledge, the first works which have considered the strong stabilization problem for MIMO systems with multiple time-delays have been Özbay (2008) and Ünal and İftar (2008). Özbay (2008) (also see Özbay (2010)) uses a coprime factorization of the given plant and then suggests to use a generalization of the approach of Zeren and Özbay (2000) to infinite-dimensional systems to find a stable controller. However, obtaining a coprime factorization of a MIMO infinite-dimensional plant is a non-trivial task and no specific approach to solve this problem has been suggested in Özbay (2008) or Özbay (2010). Furthermore, how to generalize the approach of Zeren and Özbay (2000) to infinite-dimensional MIMO systems is also not clear. In Ünal and İftar (2008), on the other hand, a specific algorithm has been proposed to solve the strong stabilization problem. The presentation of Ünal and İftar (2008), however, was restricted to a specific problem: $\mathcal{H}^\infty$ rate-based flow control in networks.

In the present paper, we consider the *strong $\mathcal{H}^\infty$ stabilization problem*, i.e., the problem of finding a stable controller which stabilizes the given plant and minimizes a given $\mathcal{H}^\infty$ performance index, in a more general set-up. In Section 2, we present the structure of the controller which solves an $\mathcal{H}^\infty$ controller design problem for MIMO systems with multiple I/O time-delays as presented by Meinsma and Mirkin (2005). Main contribution of the present work is given in Section 3, where we present an algorithm (which is the generalization of the algorithm presented by Ünal and İftar (2008) for a flow control problem) to solve the strong $\mathcal{H}^\infty$ stabilization problem for MIMO systems with multiple I/O time-delays.

1.1. Notation and preliminaries

Throughout the paper, $\mathbb{Z}_+$, $\mathbb{R}$, and $\mathbb{C}$ respectively denote the set of positive integers, the set of real numbers, and the set of complex numbers. $\mathrm{Re}(\cdot)$ denotes the real part of $(\cdot)$. j denotes the imaginary unit. For $n \in \mathbb{Z}_+$, I_n represents the $n \times n$ dimensional identity matrix. 0 and I respectively denote appropriately dimensioned zero and identity matrices. For a matrix M, M^T denotes its transpose and M^{-1} denotes its inverse. For two symmetric matrices M and N, $N \leq M$ means that $M - N$ is non-negative definite. $\mathrm{bdiag}(\cdots)$ represents a block diagonal matrix with blocks $(\cdots)$ on its main diagonal. For $m, n \in \mathbb{Z}_+$, $J_{m,n} := \begin{bmatrix} I_m & 0 \\ 0 & -I_n \end{bmatrix}$ is a *signature matrix*. For appropriately dimensioned constant matrices A, B, C, and D, $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ denotes the transfer function matrix (TFM) $G(s) = C(sI - A)^{-1}B + D$. $\mathcal{H}^\infty$ represents the Hardy space of TFMs which are bounded and analytic on $\mathbb{C}_o := \{s \in \mathbb{C} \mid \mathrm{Re}(s) > 0\}$. $\|\cdot\|_\infty$, $\|\cdot\|$, and $\|\cdot\|_2$ respectively denote the $\mathcal{H}^\infty$-norm, the spectral norm, and the 2-norm.

In this paper, we consider linear systems which are represented by TFMs. The word *system* refers to either a *plant* or a *controller*. We use the words *system*, *plant*, and *controller* also to mean their TFMs. G is said to be *proper* if $\sup_{\mathrm{Re}(s)>\rho} \|G(s)\| < \infty$ for some $\rho \in \mathbb{R}$. G is said to be *finite-dimensional* if there exists appropriately dimensioned constant matrices A, B, C, and D such that $G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$. In this case, $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ is said to be a *minimal realization* of G if (A, B) is controllable and (A, C) is observable. G is said to be *stable* if $G \in \mathcal{H}^\infty$. $G \in \mathcal{H}^\infty$ is said to be *bistable* if its inverse exists in $\mathcal{H}^\infty$. A constant square matrix is said to be *Hurwitz* if all its eigenvalues have negative real parts. $Q \in \mathcal{H}^\infty$ is said to be *contractive* if $\|Q\|_\infty < 1$. A controller C is said to *stabilize* a plant P if $(I + CP)^{-1}$, $P(I + CP)^{-1}$, $(I + PC)^{-1}$, and $C(I + PC)^{-1}$ are all stable. A plant P is said to be *strongly stabilizable* if there exists a stable controller C which stabilizes P. $F_l(\cdot, \cdot)$ represents the *lower-linear fractional transformation* (Zhou, Doyle, & Glover, 1996). For TFMs G and K, where $G =: \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$ and G_{11} is $k \times k$, G_{22} is $l \times l$, and K is $k \times l$ dimensional, $HM(G, K)$ denotes the *homographic transformation* (Kimura, 1996), which is defined as

$$HM(G, K) := (G_{11}K + G_{12})(G_{21}K + G_{22})^{-1}.$$

Two important properties of the homographic transformation are the *chain property*:

$$HM(\Psi, HM(G, K)) = HM(\Psi G, K), \tag{1}$$

where Ψ is a TFM which has the same dimensions as G, and the *inverse property*: if $Q = HM(G, K)$, where G is invertible, then

$$K = HM(G^{-1}, Q). \tag{2}$$

2. Structure of the $\mathcal{H}^\infty$ controller

Since our approach, to be presented in the next section, is based on the structure of the $\mathcal{H}^\infty$ controller for a MIMO plant with multiple I/O time-delays, in this section we present this structure, which was derived by Meinsma and Mirkin (2005). The problem is to find a stabilizing proper controller K for the generalized structure shown in Fig. 1, such that the $\mathcal{H}^\infty$ norm of the closed-loop TFM from w to z is as small as possible. In Fig. 1, $\Lambda_y(s) := \mathrm{bdiag}\,(I_{m_0}, e^{-h_1^y s} I_{m_1}, \ldots, e^{-h_q^y s} I_{m_q})$, where m_0 is a non-negative integer (denotes the number of delay-free output channels), m_i is a positive integer (denotes the number of output channels subject to time-delay $h_i^y > 0$), for $i = 1, \ldots, q$, where q is the number of distinct output time-delays, and $0 < h_1^y < \cdots < h_q^y$ are the output time-delays, $\Lambda_u(s) := \mathrm{bdiag}\,(e^{-h_p^u s} I_{r_p}, \ldots, e^{-h_1^u s} I_{r_1}, I_{r_0})$, where r_0 is a non-negative integer (denotes the number of delay-free input channels), r_i is a positive integer (denotes the number of input channels subject to time-delay $h_i^u > 0$), for $i = 1, \ldots, p$, where p is the number of distinct input time-delays, and $0 < h_1^u < \cdots < h_p^u$ are the input time-delays[2] and $\widehat{P}$ is a finite-dimensional TFM given as

$$\widehat{P} = \begin{bmatrix} \widehat{P}_{11} & \widehat{P}_{12} \\ \widehat{P}_{21} & \widehat{P}_{22} \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right], \tag{3}$$

where the following conditions are satisfied.

(i) (A, B_2) is stabilizable and (A, C_2) is detectable.
(ii) $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank $\forall \omega \in \mathbb{R} \cup \{\infty\}$.
(iii) $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank $\forall \omega \in \mathbb{R} \cup \{\infty\}$.

The problem of finding a stabilizing proper controller K such that the $\mathcal{H}^\infty$ norm of the closed-loop TFM from w to z in Fig. 1, $\|F_l(\widehat{P}, \Lambda_u K \Lambda_y)\|_\infty$, is less than a given positive γ is called the *standard four-block problem*. It is shown by Meinsma and Mirkin (2005) that, when (i)–(iii) above are satisfied, there exists a $\gamma^{\mathrm{opt}} > 0$, such that there exists a solution to this problem for any $\gamma \geq \gamma^{\mathrm{opt}}$. Furthermore, the solution, when it exists, is of the form

$$K = \hat{V}_{11} F_u H (I + FH)^{-1} F_y \hat{V}_{22}^{-1} + D_K, \tag{4}$$

where $\hat{V}_{11}$, $\hat{V}_{22}$, and D_K are constant matrices ($\hat{V}_{11}$ and $\hat{V}_{22}$ being invertible),

$$F := F_y F_f + F_y \hat{V}_{22}^{-1} \hat{V}_{21} F_u, \tag{5}$$

[2] Note that, the assumption on the respectively descending and ascending arrangement of the input and output time-delays is not restrictive, since a channel permutation can be used to obtain this representation.