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Speedup of water distribution simulation by domain decomposition

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ABSTRACT

The Schur complement domain decomposition method is used for solution of large linear systems. The algorithm is based on the subdivision of the domain into smaller ones and the solution of those subdomains independently. Regarding water distribution systems modeling, the hydraulic simulation could be formulated as a sequence of systems of linear equations. Therefore, this paper utilizes the domain decomposition method to accelerate the simulation process further. The method is evaluated using a large scale real-world system with 63,616 junctions and 64,200 pipes as case study. The case study shows that the methodology could improve the performance of hydraulic simulation app. by a factor of 8 without losing accuracy at a suitable level of domain decomposition. Although the optimal level of decomposition is case specific, considerable speedup might still be achievable by decomposing a large system into only a few subsystems.

1. Introduction

Regarding computer modeling of water distribution systems, very sophisticated methods of hydraulic analysis have been developed to deal with complex issues related to the design, operation and management of distribution systems. Nonetheless, there is still a need to put effort into improving the computational efficiency and stability of the hydraulic simulation. This is due to the opportunities given by the development in computer science and the increasing demand for computing power from more complicated model-based analyses, such as detailed realistic networks modeling, optimization problems, and real-time simulation (Alonso et al., 2000). For instance, users are becoming increasingly interested in simulating the time-varying behavior of the WDS. This new problem requires solving the network (sometimes repeatedly) at each time step, especially for large scale systems. Hence, this paper explores the possibility to reduce the computational complexity of the simulation process by utilizing the Schur complement domain decomposition method. The proposed method could be applied to a variety of cutting edge issues in WDS analysis (Rogers et al., 2007; Izquierdo et al., 2008; Laucelli et al., 2012; Zhang et al., 2013; Sitzenfrei et al., 2013; Scholten et al., 2013) as the reduction of the computational load allows for comprehensive hydraulic simulation-based analyses.

Thus far, two major groups of approaches are commonly used for steady state hydraulic analysis (Todini and Rossman, 2013). The early local approaches (Cross, 1936), which solve one equation at a time, and the more recent simultaneous equation approaches including: the simultaneous loop method (Epp and Fowler, 1970); the simultaneous node method (Martin and Peters, 1963; Shamir and Howard, 1968); the simultaneous pipe method (Wood and Charles, 1972); the hybrid approach (Hamam and Brameller, 1971; Osiadacz, 1987) and the Global Gradient Algorithm (Todini and Pilati, 1988; Salgado et al., 1988). These methods have been developed and coded over the last 56 years (Ormsbee, 2006). Most notably, the EPANET software (Rossman, 2000), which is the most widely used water network simulation package, has realized extremely speedy and stable hydraulic simulation through the application of Todini's approach and sophisticated code design. Recently, the upcoming EPANET 3.0 is expected to improve the computation efficiency even further by using modified node reordering techniques.

To speedup the simulation, the Newton–Raphson method is applied as an extension of the Hardy Cross method (Todini and Pilati, 1988). By simultaneously considering all loops or nodes in the whole system (Featherstone and Nalluri, 1988; Bhave, 2003;

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Crous, 2009; Crous et al., 2012), the method converges faster than any method that corrects either the flow or the head at a single component in the system (Featherstone and Nalluri, 1988; Bhave, 2003; Crous, 2009; Crous et al., 2012). The Newton-based linear theory method simplifies the solution scheme by converting the system of equations into a set of linear equations, i.e. linearization of the non-linear equations governing the conservation of mass (Featherstone and Nalluri, 1988). The Gradient Algorithm (GA) constructs a linear system with its coefficient matrix being symmetrical and positive definite (Stieltjes type). Subsequently, the system can be efficiently solved for both the unknown heads and unknown pipe flows simultaneously using a variety of methods, such as the Conjugate Gradient Method, the Incomplete Cholesky Factorization, or the Modified Conjugate Gradient Method (Todini and Pilati, 1988). The Two Point Linear Method (Van Zyl et al., 2008) reduces computational cost by trading off accuracy and convergence speed. Furthermore, as a flow range is assigned to each pipe, the accuracy of the solution can be checked for each pipe before conducting another full iteration. Zecchin et al. (2012) proved the efficiency of using the algebraic multigrid (AMG) method, which is a defect-correction approach that iteratively corrects an approximate solution of the original system using solutions from a sequence of constructed lower dimensional systems.

Alonso et al. (2000) implemented parallel computing in hydraulic simulations based on the multifrontal Cholesky method. Crous (2009), Crous et al. (2012) examined the feasibility of using graphical processing units (GPU) with the Conjugate Gradient Method. Thus far, however, these studies do not indicate that application of parallel processing to hydraulic network solvers guarantees more efficient computation. This is due to the overhead, of which the extra cost cannot be compensated unless for exceptionally large water distribution models (Crous et al. 2012).

This study aims at making a substantial modification on the computation scheme of the hydraulic simulation in order to reduce the computational complexity, and consequently to increase simulation efficiency. Accordingly, the Schur complement domain decomposition method (Toselli and Widlund, 2005) is applied. The algorithm is based on subdivision of the domain into smaller sub-domains and reordering of the nodes within the sub-domains (Aleksandrov and Samuel, 2010). Thus, the solution of a large system of linear equations (the domain) is converted to the solution of a series of smaller systems (the sub-domains). Consequently, a nearly linear computation complexity can be reached. In terms of the water distribution systems, the domain is the whole system and the sub-domains are subsystems specified after decomposition. As it is not uncommon that a distribution system is divided into smaller metered subsystems to improve water audit (Farley, 2001; Thornton et al., 2008), the computation scheme could therefore be formed based on the subsystems' layout. Hence, in the context of water distribution systems, this method has a close tie to practice instead of being an abstract mathematical concept.

2. The Schur complement domain decomposition method

The hydraulic simulation could be formulated to the iterative solution of a sequence of systems of linear equations ($\mathbf{A}x = b$). For different solution methods (e.g. the simultaneous loop method; the Global Gradient Algorithm, etc.), different systems ($\mathbf{A}x = b$) are specified (Boulos et al., 2006; Ormsbee, 2006). In this study, the Global Gradient Algorithm (GGA) is selected to formulate the equations (for details refer to the appendix), since it is currently the most efficient solution method. However, the proposed decomposition method is general applicable to all solution methods of water distribution simulation.

To implement the Schur complement domain decomposition method, the whole distribution system is first divided into a number of subsystems. Correspondingly, the system of linear equations ($\mathbf{A}x = b$) is decomposed into a set of matrix equations with reduced dimensionalities. Thus, the problem is converted from solving a huge dimensional system into solving sequentially a series of low dimensional subsystems (Kron, 1963; Tselishcheva and Shishkin, 2008).

2.1. System decomposition: nested dissection partitioning

System decomposition is the prerequisite for using the domain decomposition method. In this regard, graph partitioning algorithms can be used for distribution system decomposition based on mapping the system into an undirected graph $G = (V, E)$ in which the vertices V represent consumers, sources, and tanks - the edges E the connecting pipes, pumps, and valves (Perelman and Ostfeld, 2011). Corresponding to the hydraulic simulation scheme, the vertices represent rows and columns of the system of linear equations, and an edge represents a nonzero entry in the coefficient matrix ($\mathbf{A}$) representing the system.

In this study, the graph partitioning package METIS is invoked to nested dissection partitioning (Karypis and Kumar, 1998a,b). Operated with the reduced-size graph, the partitioning algorithms in METIS are extremely fast compared to traditional partitioning algorithms that compute a partition directly on the original graph. Extensive testing has also shown that the partitions provided by METIS are consistently better than those produced by spectral partitioning algorithms (Karypis and Kumar, 1998a,b; Miettinen et al., 2006). Nested dissection (George, 1973) is an approach that recursively splits a graph into almost equally-sized (balanced numbers of components) subgraphs using separators. The removal of small subsets of components in the graph (e.g. vertices and/or edges) allows the graph to be partitioned into subgraphs with at most a constant fraction of the number of components. At each level of recursion, the components of the graph are numbered in such a way that the separator components are ordered after the components in the partitions (Karypis and Kumar, 1998a,b). Based on the reordering strategy above, the original system of linear equations is transformed into a linear, bordered, block-diagonal model (Eq. (1)) (Kron, 1963).

For the application to water distribution systems, a simple example is given in Figs. 1 and 2. As shown in Fig. 1, the network is divided into two subsystems using the specified separators. Selecting both a pipe and a node as separators is a result of using the GGA that solves the pipe flows and nodal heads simultaneously. By such a division, each sub matrix equation will have the same form as the whole matrix equation of the GGA (Appendix). Consequently, the accuracy and stability of the simulation is guaranteed. After the decomposition, the two subsystems are nearly equally-sized with 6 components (in subsystem 1) and 5 components (in subsystem 2), respectively. Based on the decomposition, the coefficient matrix ($\mathbf{A}$) is reordered by numbering the components in subsystems first and then the components in separators (Fig. 2).

2.2. The Schur complement domain decomposition algorithm

This section introduces how the Schur complement domain decomposition method is applied to solve the linear, bordered, block-diagonal model (Eq. (1)).

$$\begin{bmatrix} \mathbf{A}_{(1,1)} & 0 & \cdots & 0 & \mathbf{A}_{(1,B)} \\ 0 & \mathbf{A}_{(2,2)} & \cdots & 0 & \mathbf{A}_{(2,B)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \mathbf{A}_{(N,N)} & \mathbf{A}_{(3,B)} \\ \mathbf{A}_{(B,1)} & \mathbf{A}_{(B,2)} & \cdots & \mathbf{A}_{(B,N)} & \mathbf{A}_{(B,B)} \end{bmatrix} \begin{bmatrix} \mathbf{X}_{(1)} \\ \mathbf{X}_{(2)} \\ \vdots \\ \mathbf{X}_{(N)} \\ \mathbf{X}_B \end{bmatrix} = \begin{bmatrix} \mathbf{b}_{(1)} \\ \mathbf{b}_{(2)} \\ \vdots \\ \mathbf{b}_{(N)} \\ \mathbf{b}_B \end{bmatrix} \tag{1}$$

In Eq. (1), $\mathbf{A}_{(i,i)}$ is the coefficient matrix of subsystem i, and N is the total number of subsystems; $\mathbf{A}_{(B,B)}$ is the coefficient matrix of the separators. $\mathbf{A}_{(i,B)}$ represents the coupling between subsystem i and the separators, in which the nonzero entries correspond to edges connecting them. $\mathbf{A}_{(B,i)} = \mathbf{A}^T_{(i,B)}$; $\mathbf{X}_{(i)}$ and $\mathbf{X}_B$ are unknowns for subsystem i and the separators respectively. $\mathbf{b}_{(i)}$ and $\mathbf{b}_B$ are right hand side terms for subsystem i and the separators respectively. In each matrix or vector introduced above, all the entries are hydraulic method-specific since different hydraulic methods would formulate different systems of linear equations ($\mathbf{A}x = b$) (Boulos et al., 2006; Ormsbee, 2006). For this study, the Global Gradient Algorithm (GGA) is applied to build the equations (See as the Appendix).

From Eq. (1), it yields,

$$\mathbf{A}_{(i,i)}\mathbf{X}_{(i)} + \mathbf{A}_{(i,B)}\mathbf{X}_B = \mathbf{b}_{(i)} \tag{2}$$

$$\sum_{i=1}^{N} \mathbf{A}_{(B,i)}\mathbf{X}_{(i)} + \mathbf{A}_{(B,B)}\mathbf{X}_B = \mathbf{b}_B \tag{3}$$

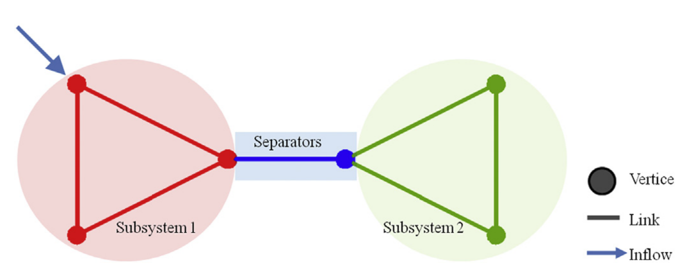

Fig. 1. A water distribution system graph partitioned using nested dissection.