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Brief paper

Broadcast stochastic receding horizon control of multi-agent systems☆

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ABSTRACT

Optimal regulation of stochastically behaving agents is essential to achieve a robust aggregate behavior in a swarm of agents. How optimally these behaviors are controlled leads to the problem of designing optimal control architectures. In this paper, we propose a novel broadcast stochastic receding horizon control architecture as an optimal strategy for stabilizing a swarm of stochastically behaving agents. The goal is to design, at each time step, an optimal control law in the receding horizon control framework using collective system behavior as the only available feedback information and broadcast it to all agents to achieve the desired system behavior. Using probabilistic tools, a conditional expectation based predictive model is derived to represent the ensemble behavior of a swarm of independently behaving agents with multi-state transitions. A stochastic finite receding horizon control problem is formulated to stabilize the aggregate behavior of agents. Analytical and simulation results are presented for a two-state multi-agent system. Stability of the closed-loop system is guaranteed using the supermartingale theory. Almost sure (with probability 1) convergence of the closed-loop system to the desired target is ensured. Finally, conclusions are presented.

1. Introduction

Regulating a swarm of stochastic entities simultaneously with the goal of achieving a desired aggregate behavior has long been of interest in several disciplines. Since the last decade, this interest has grown with a burst of emerging applications in submicroscopic level systems. Examples include the design of smart robots and multi-agent intelligent systems for applications in automation (see Chattopadhyay and Ray (2009), Ueda, Odhner, and Asada (2007) and references therein), regulation of submicroscopic particles in microfluidic systems for fascinating applications in drug delivery and laboratory-on-a-chip technology (Ashkin, 2000; Edwards, Engheta, & Evoy, 2005; Grier, 2003) suppression of the Brownian motion of subcellular biological structures suspended in a solution for revealing their intrinsic behaviors (Cohen & Moerner, 2006; Gosse & Croquette, 2002) etc. In all these novel applications, the primary goal is to achieve a desired system behavior optimally by regulating behaviors of individual entities within the system.

Historically, the theory of optimal control policies in stabilizing stochastic dynamical systems traces back to the seminal results of Kalman (1960) and Kushner (1964). These results provide a theoretical framework which allows the incorporation of probabilistic tools, such as conditional expectations and the theory of martingales (Durrett, 2005), in designing closed-loop control policies for such systems (Kushner, 1967). Incorporation of receding horizon based predictive control policies (Kwon & Han, 2005) within this framework can potentially result in a unique optimal control strategy for stabilizing stochastic dynamical systems. A recent application on stabilization of linear stochastic systems with multiplicative noise and linear constraints (see Primbs (2009) and references therein) shows the capability of this framework for designing a model based receding horizon control (RHC) policy. However, a direct application of this framework in regulating the stochastic behavior of a swarm of agents is limited by the complexity in assigning individual controllers to each of the agents in the system with very few available actuators.

Asada and his group (Ueda et al., 2007) have recently introduced the concept of broadcast feedback control, a centralized control strategy, for stabilizing the aggregate behavior of a vast number of identical agents when limited feedback information is available from the system. The authors have applied this control framework to the problems of endothelial cell migration (Wood, Das, Kamm, & Asada, 2009) and artificial muscle actuators (Odhner, Ueda, & Asada, 2007; Ueda et al., 2007). Similar control architecture has later been applied to achieve the desired aggregate behavior in a colony of *E. coli* bacteria (Julius et al., 2008) and in supervising a swarm of simple agents (Chattopadhyay & Ray, 2009). In all these

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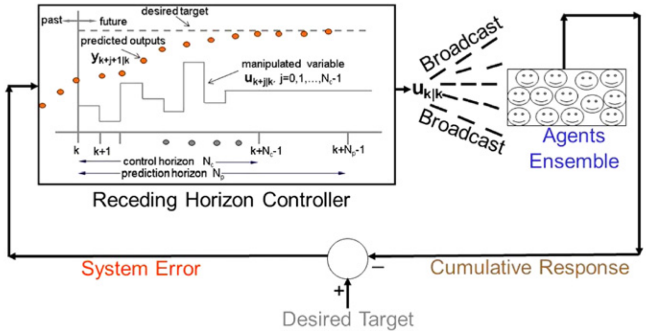

Fig. 1. Broadcast Stochastic Receding Horizon Control (BSRHC): here at the current time step k, the "Receding Horizon Controller" designs control inputs, "$u_{k|k}$" and "Broadcast" them to the system ("Agents Ensemble"). Each agent in the system makes an independent stochastic decision and contributes to the "Cumulative Response" of the system. The "Cumulative Response" of the system is then compared with the "Desired Target" and the "System Error" is fed back to the controller for designing new control inputs at time step $k+1$.

works, the dynamical behavior of individual agents in the system has been represented by the finite state Markov chain model and their state transition probabilities have been used as manipulated variables to achieve the desired system behavior. The computed transition probabilities in most of these works are non-optimal (except Odhner et al. (2007), Odhner and Asada (2010)) which may lead to an overall poor performance of the system.

Motivated by this, in this paper, we develop an optimal control strategy called "Broadcast Stochastic Receding Horizon Control (BSRHC)" for regulating the ensemble behavior of stochastically behaving agents. The central idea of the proposed strategy is to design and broadcast the optimal control input in a predictive framework to all the agents in a swarm using the aggregate behavior of agents as the only available feedback information. Our novelty here is in integrating the broadcast concept with the existing probabilistic tools and the theory of finite receding horizon based optimal control policy. Fig. 1 illustrates the overall framework of "BSRHC".

We theoretically demonstrate the stabilization of a swarm of stochastically behaving agents to the desired state in the framework of "BSRHC". The need to regulate a swarm of agents simultaneously using a centralized controller as well as the presence of non-linear constraints on control inputs make the framework suitable for the particular problem. The dynamical behavior of individual agents is represented by the discrete time finite state Markov chain model. The controller uses the measured aggregate system error as the only available feedback information and designs the optimal control inputs in a predictive framework. The computed control inputs are the state transition probabilities of the agents which are then broadcast to all agents in the swarm to achieve the desired system behavior. Probabilistic tools such as the supermartingale theory and the bounded convergence theorem are applied to guarantee the almost sure convergence of the closed-loop system behavior to the desired one. The derived stability and convergence results establish key principles applicable to stabilize general stochastic dynamical systems.

The paper starts with the definition of a two-state multi-agent system which is followed by the formulation of a constrained finite horizon "BSRHC" problem to achieve desired system behavior of a swarm of agents and the development of predictive models of the system. Next, we establish convergence and stability conditions for the closed-loop system. Analytical and numerical solutions of the system of two-state agents are provided next to support the concept of "Broadcast RHC" and establish the underlying stability and convergence theorem. Finally, we extend the capability of the "BSRHC" design to multi-state multi-agent systems.

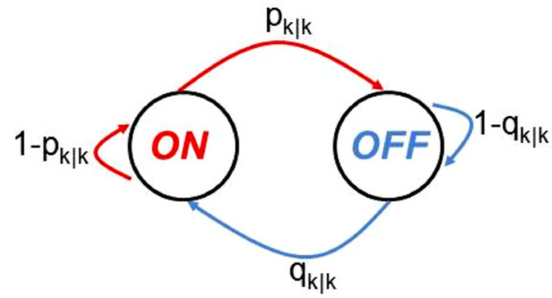

Fig. 2. State transition behavior of agents in a two-state multi-agent system. $p_{k|k}$ and $q_{k|k}$ are the transition probabilities at time k.

2. Stochastic two-state multi-agent system

We define a stochastic two-state multi-agent system as an ensemble of agents, where each agent assumes a state value stochastically out of two possible states. Each state has an associated value which is defined as

$$X_{i,k} = \begin{cases} 1 & \text{when "ON"}, \\ 0 & \text{when "OFF"}. \end{cases} \quad (1)$$

$X_{i,k}$ is the state of the ith agent at time k. We assume that all agents in this system behave independently. At time k, an agent at the state 'ON' can make transition to 'OFF' with a transition probability $p_{k|k}$. Similarly, an agent at the state 'OFF' can make transition to 'ON' with a transition probability $q_{k|k}$. Fig. 2 illustrates the state transition behavior of agents in a two-state multi-agent system.

We define the ensemble behavior of the system at time k as

$$N_k = \sum_{i=1}^{N} X_{i,k}. \quad (2)$$

N_k is the number of agents at the state 'ON' at time k. N is the predefined number of agents present in the system. We define the system error at time k as $e_k = N_r - N_k$, where N_r is the time invariant desired number of agents possessing 'ON' state.

2.1. Broadcast finite stochastic RHC problem

We formulate a constrained non-linear finite stochastic RHC problem at time k as follows:

$$\min_{\substack{p_{k|k},\ldots,p_{k+N_c-1|k},\\ q_{k|k},\ldots,q_{k+N_c-1|k}}} \mathbb{J}_k \quad (3)$$

s.t.

$$(p_{k+m|k}, q_{k+m|k}) \in [0,1] \times [0,1] \quad \text{for } 0 \le m \le N_c - 1, \quad (4a)$$

$$p_{k+m|k} = q_{k+m|k} = 0 \quad \text{for } m \ge N_c, \quad (4b)$$

$$\mathbb{E}[e^2_{k+m+1} \mid \mathcal{F}_k] < e^2_k \quad \text{for } e_k \neq 0,\ 0 \le m \le N_c - 1, \quad (4c)$$

$$\mathbb{E}[e^2_{k+m+1} \mid \mathcal{F}_k] = e^2_k \quad \text{for } e_k = 0,\ 0 \le m \le N_c - 1. \quad (4d)$$

The cost function $\mathbb{J}_k$ at time k is defined as

$$\mathbb{J}_k = \sum_{m=0}^{N_p-1} \mathbb{E}[e^2_{k+m+1} \mid \mathcal{F}_k] + \sum_{n=0}^{N_c-1} (p^2_{k+n|k} + q^2_{k+n|k}). \quad (5)$$

N_p and N_c are time invariant prediction and control horizon respectively.

2.2. System model

To compute the conditional expectations appeared in (4c), (4d) and (5), we write the conditional expectation of the number of