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Brief paper

Feasibility of piecewise-constant control sequences for timed continuous Petri nets[☆]

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ABSTRACT

In this study, the determination of control actions for timed continuous Petri nets is investigated by the characterisation of attractive regions in marking space. In particular, attraction in finite time, which is important for practical issues, is considered. Based on the characterisation of attractive regions, the domain of admissible piecewise constant control actions is computed, and sufficient conditions to verify the feasibility of the control objectives are proposed. As a consequence, an iterative procedure is presented to compute piecewise constant control actions that correspond to local minimum time control for timed continuous Petri nets.

1. Introduction

Timed continuous Petri nets (contPNs) are used to eliminate the combinatory explosion that occurs by applying enumeration techniques with discrete event systems (DESs) and to benefit from the main advances in continuous systems control theory (David & Alla, 1992; Júlvez, Recalde, & Silva, 2005; Silva & Recalde, 2002; Vazquez & Silva, 2011). Conditions for controllability have been investigated (Jimenez, Júlvez, Recalde, & Silva, 2005; Vazquez, Ramirez, Recalde, & Silva, 2008). Stationary markings and flows have also been characterised (Mahulea, Ramírez-Treviño, Recalde, & Silva, 2008b). Then, linear control designs (Apaydin-Ozkan, Júlvez, Mahulea, & Silva, 2011; Lefebvre, 1999; Lefebvre, Delherm, Leclercq, & Druaux, 2007) have been considered for contPNs. Optimal controls have also been investigated. In particular, affine control laws (Vazquez & Silva, 2009), model predictive control (Giua et al., 2006; Mahulea, Giua, Recalde, Seatzu, & Silva, 2008a), constrained feedback control (Kara, Ahmane, Loiseau, & Djennoune, 2009) and piecewise-linear marking trajectories in minimum time (Apaydin-Ozkan et al., 2011) have been designed.

In this study, a geometric approach is proposed to verify the feasibility of piecewise constant control actions (PCCAs) for the discrete time approximation of contPNs with some controllable transitions and some uncontrollable ones. Control actions are maintained constant in specific regions of the marking space, namely r-regions that depend on the contPN structure (Lefebvre & Leclercq, 2012). Feasibility concerns the determination of the domain of suitable control actions for a given control objective. Feasibility is required to compute admissible trajectories, attracted near a reference marking. The reachability of the neighbourhood of a desired marking in the marking space and the stability within this neighbourhood are both concerned. These properties are studied in discrete time with a single formulation, namely (τ, τ')-attractivity. The (τ, τ')-attractive regions are computed based on linear matrix inequalities (LMIs). As long as PCCAs are considered, manufacturing systems (Cassandras, 1993) are concerned at first. The motivation to use constant or piecewise constant actions is to prevent the stress of actuators. An algorithm that computes local minimum time trajectories with PCCAs is then proposed. It drives the marking from an initial value to the neighbourhood of a target value according to temporal specifications: feasible trajectories with respect to (wrt) a given sequence of r-regions are computed. The proposed approach is related to the computation of invariant sets for nonlinear systems by means of LMIs (Benlaoukli, Hovd, Olaru, & Boucher, 2009; Benlaoukli & Olaru, 2007; Blanchini, 1999). In this paper, the method is extended to characterise attractive regions and conditions are relaxed by taking advantages of the piecewise linear structure of contPNs. Our contribution is compared with model predictive control (MPC) for contPNs (Bemporad, Morari, Dua, & Pistikopoulos, 2002; Giua et al., 2006).

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This paper is organised into five sections. Section 2 introduces contPNs with PCCAs. Section 3 discusses the approximation of attractive regions for contPNs. Section 4 concerns control design in finite time with PCCAs. Section 5 presents some conclusions.

2. ConPNs with constant control actions

2.1. Controlled timed continuous Petri nets

ContPNs under infinite server semantics have been developed to provide continuous approximations of DESs (David & Alla, 1992; Silva & Recalde, 2002). The marking of each place is a continuous non-negative real-valued function of time, and $M(t, M_I) \in (\mathbf{R}^+)^n$, $t \geq 0$ ($\mathbf{R}^+$ is the set of non-negative real numbers) is the continuous marking trajectory whose initial marking is M_I at instant $t = 0$. $\boldsymbol{P} = \{P_i\}$ is a set of n places and $\boldsymbol{T} = \{T_j\}$ is a set of q transitions, $W_{PO} \in (\mathbf{Z}^+)^{n\times q}$ is the post-incidence matrix ($\mathbf{Z}^+$ is the set of non-negative integer numbers), $W_{PR} \in (\mathbf{Z}^+)^{n\times q}$ is the pre-incidence one and $W = W_{PO} - W_{PR}$ is the incidence matrix. The marking variation is given by $dM(t, M_I)/dt = W.X(t, M_I)$. The vector $X(t, M_I) \in (\mathbf{R}^+)^q$ with $X(t, M_I) = (x_j(t, M_I))$ is the firing speed (i.e. flow) vector at time t in the free regime, which continuously depends on the marking of the places. The flow through the transition T_j is defined by (1):

$$x_j(t, M_I) = x_{\max j}.enab_j(M(t, M_I)) \tag{1}$$

with $enab_j(M) = \min\{m_k/w^{PR}{}_{kj}$ for all $P_k \in {}^\circ\boldsymbol{T_j}\}$, ${}^\circ\boldsymbol{T_j}$ stands for the set of T_j upstream places, $x_{\max\ j}, j = 1, \ldots, q$ are the transition firing rates, which are constant parameters and $X_{\max} = \mathrm{diag}(x_{\max\ j})$.

Control actions may be introduced according to a reduction in the flow through the transitions (Jimenez et al., 2005). Such control actions can be interpreted as slowing down the server activities in the considered systems. Transitions in which a control action can be applied are called controllable, and $\boldsymbol{T_c} = \{T_1, \ldots, T_{qc}\}$ is the set of q_c controllable transitions. Similarly, $\boldsymbol{T_{nc}} = \{T_{qc+1}, \ldots, T_q\}$ is the set of $q - q_c$ uncontrollable transitions. Let us define $Q_c \in (\mathbf{Z}^+)^{qc\times q}$ and $Q_{nc} \in (\mathbf{Z}^+)^{(q-qc)\times q}$ according to (2):

$$Q_c = (I_{qc}|0_{qc\times(q-qc)})$$
$$Q_{nc} = (0_{(q-qc)\times qc}|I_{q-qc}). \tag{2}$$

I_{qc} is the identity matrix of size $q_c \times q_c$, and $0_{qc\times(q-qc)}$ is the null matrix of size $q_c \times (q - q_c)$. The control actions are represented by the control vector $U(t) \in (\boldsymbol{R}^+)^q$ with $U(t) = (u_j(t))$. The flow through transition T_j is given by (3):

$$x_{c\,j}(t, M_I) = x_j(t, M_I) - u_j(t) = x_{\max j}.enab_j(M(t, M_I)) - u_j(t) \tag{3}$$

with $0 \leq u_j(t) \leq x_{\max j}.enab_j(M(t, M_I))$ if $T_j \in \boldsymbol{T_c}$ and $u_j(t) = 0$ if $T_j \in \boldsymbol{T_{nc}}$. In other words, $X_c(t, M_I) = X(t, M_I) - U(t)$. The marking variation of a controlled contPN is given by $dM(t, M_I)/dt = W.X_c(t, M_I)$ or by (4):

$$dM(t, M_I)/dt = W.(X(t, M_I) - U(t)) \tag{4}$$

with $0 \leq Q_c.U(t) \leq Q_c.X(t, M_I)$ and $Q_{nc}.U(t) = 0$. Control actions $\{U(t), t \geq 0\}$ that satisfy the preceding conditions for a given marking trajectory are named admissible Bounded Input Controls. The set $\mathrm{BIC}(\boldsymbol{T_c}, M_I)$ of admissible control actions $\{U(t), t \geq 0\}$ for initial marking M_I is defined as follows:

$$BIC(\boldsymbol{T_c}, M_I) = \{\{U(t), t \geq 0\} \text{ such that } 0 \leq Q_c.U(t) \leq Q_c.X(t, M_I) \text{ and } Q_{nc}.U(t) = 0, \text{ for all } t \geq 0\}. \tag{5}$$

2.2. Regions for contPNs

Below, a "region" denotes any polyhedral set in marking or control space. Such a set is characterised by the LMI: $H.Z \leq h$, where Z stands either for M or U and the couple (H, h) defines the perimeter of the polytope.

Switches occur in contPNs according to the function "min(.)" in the expression of the enabling degree (1). Let us denote the critical place(s) for transition T_j at time t as the place(s) P_i such that i correspond(s) to the value(s) of the index k for which the quantity of tokens $m_k(t, M_I)/w^{PR}{}_{kj}$ is minimal for all $P_k \in {}^\circ T_j$.

Let us define $R(contPN, M_I)$ as the untimed reachable set of a marked contPN. $R(contPN, M_I)$ is partitioned into K reachable regions $\mathbf{A_k}$ (r-regions) with $K \leq \Pi\{|{}^\circ T_j|, j = 1, \ldots, q\}$: $R(contPN, M_I) = \mathbf{A_1} \cup \cdots \cup \mathbf{A_K}$. The r-regions depend on the critical places of the transitions, and each r-region $\mathbf{A_k}$ is characterised by a constraint matrix $A_k \in (\mathbf{R}^+)^{q\times n}$ with $A_k = (a^k{}_{ij})$, where $a^k{}_{ji} = 1/w^{PR}{}_{ij}$ if P_i is the critical place of transition T_j anywhere in r-region $\mathbf{A_k}$ and $a^k{}_{ji} = 0$ otherwise. In the interior of any r-region $\mathbf{A_k}$ i.e. $\mathrm{int}(\mathbf{A_k})$) each transition has a unique critical place. At the borders (i.e. $\mathbf{A_k} \setminus \mathrm{int}(\mathbf{A_k})$), a transition may have several critical places. As a consequence, each r-region $\mathbf{A_k}$ is characterised by an LMI: $H_k.M \leq h_k$ (Lefebvre, 2011).

2.3. Piecewise constant control actions for discrete time approximation of contPNs

In each r-region $\mathbf{A_k}$, the firing speed vector can be written as $X_c(t, M_I) = X_{\max}.A_k.M(t, M_I) - U(t)$, and the marking variation satisfies $dM(t, M_I)/dt = W.(X_{\max}.A_k.M(t, M_I) - U(t))$ with $\{U(t), t \geq 0\} \in \mathrm{BIC}(\boldsymbol{T_c}, M_I)$. If the control actions are constant in r-region $\mathbf{A_k}$ (i.e., $U(t) = U_k$ if $M(t, M_I) \in \mathrm{int}(\mathbf{A_k})$), PCCAs are considered and contPNs are piecewise-affine hybrid systems given by (6):

$$\forall M(t, M_I) \in \mathrm{int}(\mathbf{A_k}),$$
$$dM(t, M_I)/dt = W.X_{\max}.A_k.M(t, M_I) - W.U_k \tag{6}$$

where $0 \leq Q_c.U_k \leq Q_c.X_{\max}.A_k.M(t, M_I)$ and $Q_{nc}.U_k = 0$, for all $t \geq 0$.

Let us notice that several values of $U(t)$ may be selected if $M(t, M_I) \in \mathbf{A_k} \setminus \mathrm{int}(\mathbf{A_k})$ because the r-region borders belong to two or more r-regions. In order to avoid ambiguity at the borders, the definition of PCCAs is extended as: $U(t) = U_k$ where $k = \min\{h$ such that $M(t, M_I) \in \mathbf{A_h}\}$.

For numerical issues, the first-order discrete time approximation (7) of the continuous trajectories of controlled contPNs will be used:

$$\forall M_D(t, M_I) \in \mathbf{A_k},$$
$$M_D(t+1, M_I) = A_{Dk}.M_D(t, M_I) - W.\Delta t.U(t) \tag{7}$$

t stands for discrete time in (7), Δt is the sampling period and $A_{Dk} = W.X_{\max}.A_k.\Delta t + I_n$. A constant control vector U_k that satisfies conditions (6) for a given discrete time marking trajectory included in $\mathbf{A_k}$ is named an admissible Constant Bounded Input Control, and the sets $\mathrm{CBIC}(\mathbf{A_k}, \boldsymbol{T_c}, M_I)$ of admissible constant control vectors for initial marking $M_I \in \mathbf{A_k}$ are defined as:

$$\mathrm{CBIC}(\mathbf{A_k}, \boldsymbol{T_c}, M_I) = \{U_k \in (\mathbf{R}^+)^q \text{ such that for all } t \geq 0 \text{ and } M_D(t, M_I) \in \mathbf{A_k} : \text{(i) } 0 \leq Q_c.U_k \leq Q_c.X_{\max}.A_k.M_D(t, M_I); \text{ (ii) } Q_{nc}.U_k = 0\}. \tag{8}$$

If $M_D(\tau, M_I) \in \mathbf{A_k}$, for $\tau = 0, \ldots, t-1$, and $U_k \in \mathrm{CBIC}(\mathbf{A_k}, \boldsymbol{T_c}, M_I)$ then (7) leads to (9) from an iterative calculation of $M_D(t, M_I)$ starting from initial marking M_I:

$$M_D(t, M_I) = (A_{Dk})^t.M_I - \Sigma_k(t).U_k \tag{9}$$