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Motion/force tracking control of nonholonomic mechanical systems via combining cascaded design and backstepping[☆]

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ABSTRACT

This paper considers motion/force tracking control of a class of Lagrange mechanical systems with classical nonholonomic constraints. A tracking control method is proposed by combining cascaded methods and backstepping techniques. The main results of this paper include three parts: (1) error dynamics between the kinematic system and the desired paths are transformed into a cascaded system consisting of two subsystems and an interconnection function; (2) under the framework of cascaded methods, virtual controllers for the subsystems are designed to stabilize the error dynamics; (3) the tracking controller is designed for the overall mechanical systems using backstepping techniques.

1. Introduction

Nonholonomic systems, as an important class of mechanical systems, abound in robotics (De Luca, Oriolo, & Samson, 1998; Murray & Shankara, 1993; Samson, 1995). In the last three decades, studies on nonholonomic systems have attracted much attention from the control community (Fierro & Lewis, 1997; McClamroch & Wang, 1988; Panagou & Kyriakopoulos, 2013; Zhang, Liu, Luo, & Wang, 2013), because no time-invariant smooth state feedback control law can stabilize nonholonomic systems with restricted mobility to a desired configuration (Kolmanovsky & McClamroch, 1995; Li, Ge, Adams, & Wijesoma, 2008a). Thus many publications have focused on the stabilization problem (Ge & Lewis, 2006; Li et al., 2008a).

In practice, tracking a reference trajectory, as a more interesting issue, has received relatively less attention in the literature. According to the nonholonomic systems presenting at a kinematic or dynamic level (Ge, Wang, Lee, & Zhou, 2001; Li et al., 2008a), the tracking problem is usually classified into kinematic tracking or dynamic tracking problems. Thus the kinematic and dynamic tracking problems yield kinematic control such as driving speed and physical controls such as driving torques, respectively (Fukao, Nakagawa, & Adachi, 2000). Like the stabilization case, most of the work on the tracking problem in the literature focuses on the kinematic level (Oya, Su, & Katoh, 2003). Recognizing the importance of addressing the tracking control problem at the dynamic level, several works dealing with this problem have been reported (Dong, 2002; Jin & Fu, 2012; Oya et al., 2003). The author of Dong (2002) proposed an adaptive controller for nonholonomic dynamic systems with unknown inertia parameters. Oya et al. (2003) and Jin and Fu (2012) develop robust adaptive controllers for nonholonomic dynamic systems with some uncertainties in their dynamics, but need the assumption that the Lagrangian dynamics are linearly parameterizable. Mauder (2008) presents a sliding-mode-based robust controller for nonholonomic dynamic systems with disturbances. As pointed out in Michino and Mizumoto (2010), the above-mentioned results suffer from an overly complicated structure stemming from the number of adaptive adjusting terms needed to construct the controllers against considered uncertainties. Although the high gain adaptive feedback controller is rather simple, in practice it may lead to instability or high

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noise or both (Maples & Becker, 1986). On the other hand, control of the forces of the contact interactions is at least as important as the position control. Since physically the constraints provide the necessary reactions, no matter what the proposed control algorithms may be, they are not practical unless the position and the force of interaction are controlled in a simultaneous way. And yet, the literature on controlling the two objectives simultaneously is relatively rare. For this goal several fundamental results were proposed for mechanical systems with nonholonomic constraints in Li et al. (2008a); Li and Zhang (2010); Oya et al. (2003), electrically driven mobile manipulators in Li, Ge, Adams, and Wijesoma (2008b), wheeled mobile manipulators with hybrid joints in Li, Tao, Ge, Adams, and Wijesoma (2009), and multiple mobile manipulators in Li and Kang (2010); Li, Li, and Kang (2010).

At a dynamic level, although each of the above-mentioned methods has different characteristics, in practice these control algorithms may be insufficient to solve the problems in terms of computational burden, once they are designed and are ready to be implemented. Realizing the advantages and disadvantages of the methods in the literature on motion and force tracking control of nonholonomic systems and the computational insufficiencies in practical implementation, this paper proposes a new control strategy for motion and force tracking control of nonholonomic dynamic systems by combining cascaded methods and backstepping techniques. The method is designed in three steps. Step one is that the kinematic dynamics is feedback-transformed into a chained form and the tracking error dynamics is converted into a cascaded system. Step two is that under the framework of cascaded methods the stability of the cascaded system coinciding with that in Tian and Cao (2007) without dilation is guaranteed by designing two controllers for the two linear subsystems with the interconnection term satisfying a certain growth condition. Step three is that, with the two controllers of the kinematic dynamics viewed as virtual controllers, an overall controller is designed at the dynamic level to obtain motion/force tracking control of the overall nonholonomic mechanical system by using backstepping techniques.

2. Problem statement

Consider the following mechanical systems:

$$D(q)\ddot{q} + C(q,\dot{q}) + G(q) = J^T(q)\lambda + B(q)\tau \tag{1}$$

$$J^T(q)\dot{q} = 0 \tag{2}$$

where q and τ denote the n vector of generalized coordinates and the r vector of generalized control input force, respectively; $\lambda \in R^m$ is the associated Lagrangian multiplier expressing the contract force; $D(q), C(q,\dot{q})\dot{q}$ and $G(q)$ are the $n\times n$ symmetric, bounded and positive definite inertia matrix, the n vector of centripetal and Coriolis torques, and the n vector of gravitational torques, respectively; $B(q)$ and $J(q)$ are the $n \times r$ assumed known input transformation and $m \times n$ constraint matrix, respectively. Here assumptions are needed that Eq. (2) is completely nonholonomic for all q and t, $B(q)$ is a full-rank matrix and r is not less than $n - m$.

Given a desired contact force λ_d and desired trajectories q_d and $\dot{q}_d$, the control objective is to determine a control law for τ such that λ, q and $\dot{q}$ asymptotically converge to λ_d, q_d and $\dot{q}_d$, respectively.

Let v be a vector of independent generalized velocities and define $R(q)$ such that it maps vector v into a vector of feasible generalized velocities $\dot{q}$ that satisfies constraint (2), namely

$$\dot{q} = R(q)v. \tag{3}$$

Combining (2) and (3) yields

$$R^T(q)J^T(q) = 0. \tag{4}$$

Differentiating (3) gives

$$\ddot{q} = R(q)\dot{v} + \dot{R}(q)v. \tag{5}$$

Thus, the dynamic system (1) satisfying (2) can be transformed into

$$\dot{q} = R(q)v \tag{6}$$

$$D(q)R(q)\dot{v} + C_1(q,\dot{q}) + G(q) = B(q)\tau + J^T(q)\lambda \tag{7}$$

where $C_1(q,\dot{q}) = D(q)\dot{R}(q) + C(q,\dot{q})r(q)$.

Remark 1. Please note that (6) is a purely kinematic subsystem which will be transformed into chained form.

3. Main results

3.1. Derivation of the cascaded system

We focus on the following class of two-input chained-form nonholonomic systems, which are converted from (6) with a coordinate transformation $x = \Psi(q)$, and a state feedback $v = \Omega_1(q)u$ under certain explicit conditions satisfied by only systems with two degrees of freedom (see e.g. Martin & Rouchon, 1994)

$$\begin{cases} \dot{x}_1 = u_1, \\ \dot{x}_2 = u_2, \\ \dot{x}_3 = x_2u_1 \\ \vdots \\ \dot{x}_n = x_{n-1}u_1 \end{cases} \tag{8}$$

where $u = (u_1, u_2)^T$ is the input and x is the state.

Remark 2. Under the coordinate transformation, the dynamic model (7) is accordingly converted into

$$D_2(x)R_2(x)\dot{u} + C_2(x,\dot{x})u + G_2(x) = B_2(x)\tau + {J_2}^T(x)\lambda \tag{9}$$

where

$$\begin{aligned} D_2(x) &= D(q)\,|_{q=\Psi^{-1}(x)} \\ R_2(x) &= R(q)\Omega_1(q)\,|_{q=\Psi^{-1}(x)} \\ C_2(x,\dot{x}) &= [D(q)R(q)\dot{\Omega}_1(q) + C(q,\dot{q})\Omega_1(q)]\,|_{q=\Psi^{-1}(x)} \\ G_2(x) &= G(q)\,|_{q=\Psi^{-1}(x)} \\ J_2(x) &= J(q)\,|_{q=\Psi^{-1}(x)} \\ B_2(x) &= B(q)\,|_{q=\Psi^{-1}(x)}. \end{aligned}$$

The tracking problem for (8) is to design an appropriate controller so that the state trajectory of system (8) follows a vector-valued reference signal $x_d(t) = (x_{1d}(t), \ldots, x_{nd}(t))^T$, which is generated by a system of the same form as system (9) with $u_d = (u_{1d}, u_{2d})^T$.

Assumption 1. Assume $u_{1d} = d(t)$ and there exist nonzero T and D such that $|D| > T > 0$ and the bounded continuous $d(t)$ satisfies $|d(t) - D| < T$.

Assumption 2. The trajectories x_d, their first time derivative and up to $(n-1)$th time derivative inclusive are bounded.

Define the tracking errors as $y_i = x_i - x_{id}$, $i = 1, \ldots, n$, and it is easy to obtain the tracking error dynamics

$$\begin{cases} \dot{y}_1 = u_1 - u_{1d} \\ \dot{y}_2 = u_2 - u_{2d} \\ \dot{y}_3 = u_{1d}y_2 + x_2(u_1 - u_{1d}) \\ \vdots \\ \dot{y}_n = u_{1d}y_{n-1} + x_{n-1}(u_1 - u_{1d}). \end{cases} \tag{10}$$