Automatica 46 (2010) 1703–1711

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Adaptive dynamic surface control for linear multivariable systems ☆

Wang Chenliang, Lin Yan *

School of Automation, Beijing University of Aeronautics and Astronautics, Beijing 100191, China

ARTICLE INFO

Article history:
Received 31 May 2009
Received in revised form
29 April 2010
Accepted 29 May 2010
Available online 14 July 2010

Keywords:
Dynamic surface control
Backstepping control
Adaptive control
Multivariable systems
Output-feedback

ABSTRACT

In this paper, an output-feedback adaptive control is presented for linear time-invariant multivariable plants. By using the dynamic surface control technique, it is shown that the explosion of complexity problem in multivariable backstepping design can be eliminated. The proposed scheme has the following features: (1) The $\mathcal{L}_\infty$ performance of the system's tracking error can be guaranteed, (2) it has least number of updated parameters in comparison with other multivariable adaptive schemes, and (3) the adaptive law is necessary only at the first design step, which significantly reduces the design procedure. Simulation results are presented to demonstrate the effectiveness of the proposed scheme.

1. Introduction

Adaptive control for linear time-invariant (LTI) multi-input multi-output (MIMO) plants has long been a challenging issue in the control community and can be traced back to the work by Monopoli, Wolovich et al. in 1975 and 1976, respectively (Monopoli & Hsing, 1975; Wolovich & Falb, 1976). Since then, considerable achievements have been made by many authors; see, for instance, Ioannou and Sun (1996), Narendra and Annaswamy (1989), Tao (2003) and the references contained therein for more details. The research has also led to the emergence of some important problems attributed to linear multivariable systems, say, the high-frequency gain matrix (HFGM), the computational burden and the possibly poor transient performance.

The main interests in earlier research for linear multivariable adaptive control are to extend the results obtained in single-input single-output (SISO) adaptive control schemes to the multivariable case. In Das and Loh (1986), Dion, Dugard, and Carrillo (1988) and Singh and Narendra (1984), model reference adaptive control (MRAC) for multivariable plants was considered for the purpose of using less a *priori* knowledge of the interactor matrix or deciding the interactor matrix from information of plant relative degree. In Mathelin and Bodson (1995), using indirect adaptation, a hysteresis switching control strategy was introduced to weaken the assumptions for HFGM and a more general case was investigated by Weller and Goodwin (1994). In Tao and Ioannou (1989), a robust MRAC design for multivariable plants with unmodeled dynamics and bounded disturbance was considered.

Since the mid-1990s, the research for linear multivariable adaptive control has been divided into two branches: matrix factorization (LDU, SDU, LDS) based MRAC and backstepping control. In Costa, Hsu, Imai, and Kokotovic (2003) and Imai, Costa, Hsu, Tao, and Kokotovic (2004), a matrix factorization based MRAC was proposed with the assumption that only the signs of the leading principal minors of HFGM are known, which significantly weakens the assumption for HFGM. However, since all MRAC schemes are based on the certainty equivalence principle, their transient performance may be poor because the derivation of the so-called *normalization signal* basically slows down the adaptation (Ortega, 1993). Besides, the computational burden may be large due to the weakening of the assumption for HFGM. To improve the transient performance, another line of research such as that by Costa, Hsu, Imai, and Tao (2002) and Ling and Tao (1997) is focused on the extension of the backstepping design to the multivariable case, whose SISO counterpart has been proved an effective way to improve the transient performance (Krstic, Kokotovic, & Kanellakopoulos, 1993; Krstic, Kanellakopoulos, & Kokotovic, 1994). However, this kind of design suffers from the problem of "explosion of complexity" (Swaroop, Gerdes, Yip, &

☆ This work was supported by NSF of China under Grant 60874044 and Research Foundation for Key Disciplines of Beijing Municipal Commission of Education under Grant XK100060422. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Raul Ordóñez under the direction of Editor Miroslav Krstic.

* Corresponding author. Tel.: +86 13683678258; fax: +86 010 82338435.
E-mail addresses: sin316liang@sohu.com (W. Chenliang), linyanee2@yahoo.com.cn (L. Yan).

0005-1098/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2010.06.020

Hedrick, 1997; Swaroop, Hedrick, Yip, & Gerdes, 2000). In other words, the control law becomes highly nonlinear and complicated as plant relative degree is high due to the repeated derivatives of certain nonlinear functions and may become much more severe for multivariable adaptive control, where more parameters are to be estimated. Moreover, HFGM remains a puzzle: the positive definiteness assumption concerning HFGM in Ling and Tao (1997) is quite restrictive, and the SDU factorization in Costa et al. (2002) would cause excessive over-parameterization.

Recently, to overcome the drawback of explosion of complexity in backstepping design, a new technique named *dynamic surface control* (DSC) was proposed by Swaroop et al. for a class of SISO nonlinear systems (Swaroop et al., 1997, 2000), which introduces, at each design step, a low pass filter to prevent the derivative of nonlinear functions and therefore, eliminates the phenomenon of explosion of complexity. The recent research on DSC for different nonlinear systems and the applications to various engineering fields can be referred to Wang and Huang (2005), Yoo, Park, and Choi (2009) and Zhang and Ge (2008). The main disadvantage of the current DSC research is that it requires full state to be available for measurement and therefore, it is difficult to be applied to the case where only the plant input and output signals can be obtained. We point out that the utilization of a low pass filter to prevent the direct derivative of a control signal is not new. Variable structure model reference adaptive control (VS-MRAC) proposed by Hsu et al. in 1989 (Hsu & Costa, 1989; Hsu, Lizarralde, & Araujo, 1997), which is similar to backstepping design but with an ad hoc controller structure, may be the earliest effort to prevent the derivative of nonlinear signals for LTI plants with relative degree greater than one. Some latest developments of the VS-MRAC, including the extension to the multivariable case, can be found in Hsu, Cunha, Costa, and Lizarralde (2002), Hsu, Peixoto, Cunha, Costa, and Lizarralde (2006), Oliveira, Peixoto, Nunes, and Hsu (2007) and Yan, Hsu, Costa, and Lizarralde (2008).

In this paper, based on SDU factorization, an output-feedback adaptive dynamic surface control for LTI multivariable plants with relative degree greater than one is proposed. By introducing a first-order low pass filter at each design step, it is shown that the explosion of complexity problem in multivariable backstepping design can be eliminated, which is an extension of the original work Swaroop et al. (1997, 2000). Moreover, the proposed scheme possesses the following features:

- The transient performance of the tracking error can be improved. We show that by choosing the design parameters and initializing the filters and parameter estimators properly, the $\mathcal{L}_\infty$ performance of the system's tracking error can be guaranteed. Compared with the SISO backstepping design (Krstic et al., 1993), where the $\mathcal{L}_\infty$ performance is only available when the high-frequency gain is known, the $\mathcal{L}_\infty$ performance of our scheme can be obtained under a weaker assumption that only the signs of the leading principal minors of HFGM are known.
- Compared with those multivariable adaptive schemes (Costa et al., 2002; Imai et al., 2004; Ling & Tao, 1997; Mathelin & Bodson, 1995; Weller & Goodwin, 1994), the proposed scheme has least number of updated parameters.
- Compared with the current adaptive DSC schemes and linear multivariable adaptive backstepping schemes, our adaptive law is necessary only at the first design step, which significantly reduces the design procedure.

This paper is organized as follows. In Section 2, the controlled plant is introduced and the control purpose is formulated. In Section 3, the multivariable adaptive DSC design procedure is presented. Section 4 gives the stability analysis for the closed-loop system. In Section 5, we show that the $\mathcal{L}_\infty$ performance can be guaranteed. Finally, simulation results are given to demonstrate the effectiveness of the proposed design scheme.

2. Problem formulation

We consider an r-input r-output linear time-invariant plant described by

$$y(t) = G(s)[u](t), \tag{1}$$

where $u(t), y(t) \in \mathbb{R}^r$, $r > 1$, and $G(s)$ is an $r \times r$ strictly proper rational transfer function matrix with unknown parameters. Using the left polynomial matrix decomposition, $G(s)$ can be expressed as (Tao, 2003)

$$G(s) = D^{-1}(s)N(s) = C_g(sI - A_g)^{-1}B_g, \tag{2}$$

where

$$D(s) = s^\nu I_r + A_{\nu-1}s^{\nu-1} + \cdots + A_1 s + A_0, \tag{3}$$

$$N(s) = B_m s^m + \cdots + B_1 s + B_0, \tag{4}$$

$$A_g = \begin{bmatrix} -A_{\nu-1} & I_r & 0 & \cdots & 0 \\ -A_{\nu-2} & 0 & I_r & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -A_1 & 0 & 0 & \cdots & I_r \\ -A_0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{r\nu \times r\nu}, \tag{5}$$

$$B_g = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ B_p \end{bmatrix} \in \mathbb{R}^{r\nu \times r}, \qquad B_p = \begin{bmatrix} B_m \\ \vdots \\ B_1 \\ B_0 \end{bmatrix} \in \mathbb{R}^{r(m+1)\times r}, \tag{6}$$

$$C_g = \begin{bmatrix} I_r & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathbb{R}^{r \times r\nu}, \tag{7}$$

where $A_i \in \mathbb{R}^{r\times r}$, $i = 0, 1, \ldots, \nu - 1$, and $B_j \in \mathbb{R}^{r\times r}$, $j = 0, 1, \ldots, m$, are unknown constant parameter matrices, I_r is the $r \times r$ identity matrix, and ν is the observability index of $G(s)$. In view of (2) and (5)–(7), the plant (1) can be expressed in state-space form as

$$\begin{aligned} \dot{x} &= Ax + A_p y + B_g u, \\ y &= C_g x = x_1, \end{aligned} \tag{8}$$

where

$$x = [x_1^{\mathrm{T}}, \ldots, x_\nu^{\mathrm{T}}]^{\mathrm{T}}, \qquad x_i \in \mathbb{R}^r, \tag{9}$$

$$A = \begin{bmatrix} 0 & I_r & 0 & \cdots & 0 \\ 0 & 0 & I_r & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I_r \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \qquad A_p = \begin{bmatrix} -A_{\nu-1} \\ -A_{\nu-2} \\ \vdots \\ -A_1 \\ -A_0 \end{bmatrix}. \tag{10}$$

For the controlled plant, we make the following assumptions (Imai et al., 2004; Ling & Tao, 1997).

A.1: *$G(s)$ has full rank;*

A.2: All zeros of *$\det(N(s))$ have negative real parts;*

A.3: The observability index *ν of $G(s)$ and the order m of $N(s)$ are known and satisfy $\rho = \nu - m > 1$;*

A.4: The leading principal minors of *B_m are nonzero and their signs are known.*

Remark 1. By assumption A.4, B_m is nonsingular (called high-frequency gain matrix), which, together with assumption A.3, implies that $G(s)$ has uniform vector relative degree ρ. If B_m is singular, as shown in Ling and Tao (1997), one can introduce an input compensator to make B_m nonsingular. In this paper, we only consider the nonsingular case just for the sake of simplicity.

Based on the above assumptions, the control objective is to design an adaptive dynamic surface control so that the plant output $y(t)$ tracks a given reference output $y_r(t)$ and all closed-loop signals are bounded, where it is assumed that $y_r(t)$, $\dot{y}_r(t)$, $\ddot{y}_r(t) \in \mathcal{L}_\infty$.