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Formation control and coordinated tracking via asymptotic decoupling for Lagrangian multi-agent systems[☆]

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ABSTRACT

We study the problem of formation control and trajectory tracking for a group of robotic systems modeled by Lagrangian dynamics. The objective is to achieve and maintain a stable formation for a group of multi-agent systems, while guaranteeing tracking of a specified trajectory. In order to do so, we partition the state space for the collective system into coordinates of the geometric center of mass of the group and coordinates that describe the relative positions of the robots with respect to the center of mass, thus defining the formation shape. The relative positions can be further partitioned in coordinates which describe the absolute distances and orientation of each robot to the center of mass. We can rewrite the total system as dynamics of the center of mass of the formation, and dynamics of the shape, where the systems are, in general, coupled. By imposing holonomic constraints between the subsystems (i.e., imposing a configuration constraint) and hence reducing the system's dimension, we guarantee that the group can be driven to follow a desired trajectory as a unique rigid body. Using high gain feedback, we achieve asymptotic decoupling between the center of mass and the shape dynamics and the analysis is performed using a singular perturbation method. In fact, the resulting system is a singularly perturbed system where the shape dynamics describe the boundary layer while the center of mass dynamics describes the reduced system. After an initial fast transient in which the robots lock to the desired shape, a slower tracking phase follows in which the center of mass converges to a desired trajectory while maintaining a stable formation.

1. Introduction

The technological revolution of the last century with the advent of wireless communications brought a breadth of innovation and provided ways to efficiently share information between systems. Interacting systems are no longer constrained to be physically connected. Thus, in several applications, single complex systems have been replaced by interacting multi-agent systems with interconnected structure. In the automotive and aerospace areas, examples range from assembling structures, carrying large objects, exploring unknown environment and many others. In fact, a group of robots with simple structure can achieve more complex tasks at less cost than a single complex robot due to its modularity and flexibility. In this framework, a new set of problems needs to be addressed such as formation control and coordinated tracking. The aim is to have the robots converge and maintain a stable formation while following a desired trajectory as a group. The steering commands for the group are provided by a supervisor, that is, the group should appear to the supervisor as a rigid body.

The problem of coordination of multiple agents has been addressed through different approaches, various stability criteria and control techniques. The recent literature on the subject shows a rich collection of results. Some of the existing approaches, as highlighted in Tanner (2004), include the behavior based approach as in Balch and Arkin (1998) in which an interaction law between subsystems is defined that leads to the emergence of collective behavior. The leader–follower approach as in Tanner, Pappas, and Kumar (2004) defines a hierarchy between the agents where one or more leaders drive the configuration scheme generating commands while the followers follow the commands generated

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by the leaders. Another approach focuses on maintaining a certain group configuration and forces each agent to behave as a particle in a rigid virtual structure (Desai, Kumar, & Ostrowski, 2001; Egerstedt & Hu, 2001). As we are considering groups of interacting systems, classical stability needs to be redefined to include the interconnection aspects between the systems. To this end, the classical concept of stability has been extended as, for example, in Swaroop and Hedrick (1996), where string stability is adopted to analyze the system's behaviors. In Šiljak (1991), the concept of connective stability for multiple interacting systems has been introduced. In a recent survey (Murray, 2007), two main methods were identified to solve formation control problems: optimization based method as in Dunbar and Murray (2006), Parker (1993) and a potential fields method (Atkar, Choset, & Rizzi, 2002; Leonard & Fiorelli, 2001).

In the first part of this work, we recall the approach in Desai et al. (2001) and model the total system of robots as a group element that describes the gross motion of the team and a set of shape variables that describe the relative positions of the robots with respect to the center of mass. We then consider another decomposition in which we model the system as a group element that describes the position and orientation of the total system considered as a rigid body and a set of shape variables that describe the absolute distances between the robots, and therefore define the formation shape. We proceed by defining a desired configuration in terms of holonomic constraints between subsystems; hence the group behaves as a rigid body. Our solution involves designing a controller to direct the robots to the desired configuration, thus reducing the system's dimension, and then driving the team as a unique rigid body according to a desired trajectory. Using high gain feedback, we obtain a system in a singular perturbation form and perform singular perturbation stability analysis. Then we can approximate the dynamics of the group with the dynamics of the group's center of mass.

Singular perturbations have been extensively used to obtain model reduction, for example, in Ghorbel and Spong (2000), the problem of multibody systems with rigid links and flexible joints is addressed using singular perturbation-based model reduction. Moreover, as it is shown in Young, Kokotović, and Utkin (1977), high-gain feedback systems can be analyzed as singularly perturbed systems. In this paper, we focus on second order dynamics, and consider a general setting which includes systems operating in a three dimensional space with multiple degrees of freedom, for example, robots equipped with manipulators.

An important feature of this approach is that, due to a Lyapunov based analysis, it guarantees robustness with respect to uncertainties and unmodeled dynamics in multi-agent systems. This is an important aspect as it accounts for uncertainties in the interactions among agents. The paper is organized as follows: in Section 2, we state the problem and briefly recall the main result in singular perturbation stability analysis to be used later. In Section 3, we study the formation control problem for two systems and then we address the general case of n-dimensional systems. We show, in simulation, the resulting system's behaviors when the multiple objectives are imposed. Finally, in Section 4, we study the decoupling of the absolute distances within the group elements from the position and orientation of the total group.

2. Problem statement and background

Consider a group of n dynamical systems, each one with m dimensional configuration space $S_i = \mathrm{SE}(3) \times \mathrm{TSE}(3) \times L$, $i = 1, \ldots, n$, where the special Euclidean group SE(3) describes the set of rigid body configurations (position and orientation) in three dimensional space, TSE(3) is the tangent space of velocities, and L (of dimension $m - 12$) is the space of configurations for the remaining degrees of freedom (DOF) that each agent is equipped with. Then the collective configuration space $Q = S_1 \times \cdots \times S_n$ has dimension $n \times m$. In addition, we can partition the configuration space $Q = G \times B$, into two subspaces where B is the *Shape or Base Space* and describes the internal configuration of the system, or internal shape and G is the *Structure Group* which describes the system with respect to the environment, (position, orientation, etc.). It is possible to achieve the desirable shape for the system by imposing holonomic constraints between the systems. We formalize two problems as follows:

Problem 1. Formation Control and Coordinated Tracking: Given a set of n robots, design a controller so that they converge to a desired formation and then drive the geometric center of mass of the formation along a desired trajectory.

Problem 2. Formation Control, Coordinated Tracking and Steering: Given a set of n robots, design a controller so that they converge to a desired formation and then drive the geometric center of mass of the formation along a desired trajectory and steer the group according to a desired orientation.

Note 1. In both problem formulations, we assume that there is a communication link between any two robots.

2.1. Stability of singularly perturbed systems

We recall the main result in the singular perturbation literature which will be used in our analysis. For extensive reviews of results on singular perturbation, see Khalil (2002) and Kokotović, Khalil, and O'Reilly (1987). The following theorem which is needed in proving our results, is from (Khalil, 2002):

Theorem 1. *Consider the singularly perturbed system*

$$\begin{aligned}\dot{x} &= f(t, x, z, \epsilon)\\ \epsilon\dot{z} &= g(t, x, z, \epsilon).\end{aligned} \tag{1}$$

Assume that the following assumptions are satisfied for all $(t, x, \epsilon) \in [0, \infty) \times B_r \times [0, \epsilon_0]$:

1. $f(t, 0, 0, \epsilon) = 0, g(t, 0, 0, \epsilon) = 0.$
2. *The equation* $g(t, x, z, 0) = 0$ *has an isolated solution* $z = h(t, x)$ *such that* $h(t, 0) = 0$.
3. *The functions* f, g, h *and their partial derivatives up to the second order are bounded for* $y = z - h(t, x) \in B_\rho$*, for some* ρ.
4. *The origin of the reduced system* $\dot{x} = f(t, x, h(t, x), 0)$ *is exponentially stable.*
5. *The origin of the boundary-layer system* $\frac{dy}{d\tau} = g(t, x, y + h(t, x), 0)$ *is exponentially stable, uniformly in* (t, x).

Then, there exists ϵ^* *such that for all* ϵ, $0 < \epsilon < \epsilon^*$*, the origin of* (1) *is exponentially stable.*

3. Decoupling position from shape

We now address Problem 1 on formation control and coordinated tracking for a group of robots/agents with Lagrangian dynamics. First, we consider the case of two agents, and then extend the result to the case of N agents.

3.1. Two robots with m degrees of freedom

We consider the m-dimensional systems S_1 and S_2, each one defined on a manifold $\mathcal{M}$ and described by the following dynamics:

$$S_i : M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i = \tau_i \tag{2}$$

where $i = 1, 2$, $x_i \in M$ are m-dimensional configurations, τ_i are control inputs, M_i's are $m \times m$ inertia matrices, and C_i's are $m \times m$