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Brief paper

Finite-time multi-agent deployment: A nonlinear PDE motion planning approach[☆]

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ABSTRACT

The systematic flatness-based motion planning using formal power series and suitable summability methods is considered for the finite-time deployment of multi-agent systems into planar formation profiles along predefined spatial–temporal paths. Thereby, a distributed-parameter setting is proposed, where the collective leader–follower agent dynamics is modeled by two boundary controlled nonlinear time-varying PDEs governing the motion of an agent continuum in the plane. The discretization of the PDE model directly induces a decentralized communication and interconnection structure for the multi-agent system, which is required to achieve the desired spatial–temporal paths and deployment formations.

1. Introduction

In the past decades, extensive research has been conducted on the cooperative formation control of multi-agent systems with possible applications ranging from UAVs over transportation systems to micro-satellite clusters (see, e.g., Bullo, Cortés, and Martínez (2009), Murray (2007), Ren and Beard (2008) for rather recent and comprehensive overviews). Thereby, different analysis and design approaches have emerged depending on the available communication topology and the considered formation control task. In the behavior-based approach a desired set of behaviors is assigned to the individual agents and the overall behavior of the system is achieved by defining the relative importance between the individual behaviors (Balch & Arkin, 1998). The virtual structure approach relies on the consideration of the entire formation as a single (rigid) entity and the desired motion is assigned to the rigid structure (Ren & Beard, 2004). Alternatively, constraint functions relating the positions and orientations of the individual agents can be defined (Zhang & Hu, 2008). The potential field approach is based on the introduction of structural interaction forces between neighboring agents in order to stabilize the system to the equilibrium manifold (Olfati-Saber, 2006). Moreover, optimization-based approaches are analyzed to minimize the individual and cumulative formation error (Dunbar & Murray, 2006; Murray, 2007). In general, an additional distinction arises between leaderless and leader–follower systems. In the latter either a real or a virtual agent is chosen as the leader, whose motion follows a desired trajectory. The follower agents track the movement of the leader while maintaining their overall formation. Thereby in general feedback interconnection strategies are analyzed, which either rely on global or local information corresponding to a centralized or decentralized control scheme to achieve the agent deployment into prescribed formations.

Besides the discrete analysis of the interconnected individual agents, continuous models based on partial differential equations (PDEs) have been used to represent and control traffic flow (Alvarez, Horowitz, & Li, 1999) or large vehicular platoons (Barooah, Mehta, & Hespanha, 2009). In view of the analysis of multi-agent systems, Ferrari-Trecate, Buffa, and Gati (2006) introduce a semi-discrete continuous-time partial difference equation framework over graphs, where the spatial discretization corresponds to the individual agent. It is thereby shown that the graph Laplacian control proposed in Olfati-Saber and Murray (2004) coincides with the linear heat equation. To incorporate certain parameter uncertainties for multi-agent systems modeled by partial difference equations adaptive control is considered, e.g., in Kim, Kim, Natarajan, Kelly, and Bentsman (2008). A wave-like

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PDE model in the limit as the number of vehicles in a platoon moving in a straight line tends to infinity in proposed in Barooah et al. (2009). With this, the stability margin of large vehicular platoons under bidirectional decentralized control was analyzed and improved by introducing a forward–backward asymmetry in the control gains. Results on connecting PDEs and distributed systems towards the evaluation of system theoretic properties are provided in Sarlette and Sepulchre (2009). In terms of formation control, linear diffusion–advection–reaction equations with dynamic boundary conditions were studied in Frihauf and Krstic (2009) to achieve the deployment into equilibrium profiles. For this, PDE backstepping (Krstic & Smyshlyaev, 2008) was applied in order to exponentially stabilize the equilibrium profiles.

In this paper, a systematic nonlinear PDE-based motion planning framework is proposed for the realization of finite-time transitions between desired deployment formations along predefined spatial–temporal paths. For this, nonlinear time-varying Burgers-like PDEs are used to represent the location of a continuum of mobile agents in the plane. The desired deployment formations correspond to the equilibrium profiles of the governing PDEs, which include shock-like effects as are well known for Burgers equation (see, e.g., also Krstic, Magnis, and Vazquez (2008, 2009) for results on their stabilization). Moreover, a leader–follower configuration is considered, where the positions or velocities, respectively, of the leader agent and another agent, subsequently referred to as the anchor agent, serve as boundary inputs. For their design, a flatness-based approach (see, e.g., Fliess, Lévine, Martin, and Rouchon (1995) for the general theory for finite-dimensional systems and, e.g., Lynch and Rudolph (2002), Petit and Rouchon (2002), Meurer and Zeitz (2005, 2008), Meurer and Kugi (2009) for extensions to PDEs) is considered, which is based on the differential parametrization of the system states and the boundary inputs in terms of a flat or so-called basic output by making use of formal power series and suitable summability methods.

The paper is organized as follows: Section 2 introduces the considered PDE-based leader-enabled deployment problem which is solved in Section 3 following a flatness-based approach. For the determination of the feedforward formation control the assignment of suitable desired trajectories is analyzed in Section 4 in view of the realization of the leader-enabled deployment into planar curves. Simulation results are presented in Section 5 and some final remarks conclude the paper.

2. PDE-based leader-enabled deployment

The leader-enabled deployment of mobile agents is considered under the assumptions that the agents are fully actuated and operate in a common reference frame. Motivated by the correspondence of graph Laplacian control and the linear heat equation in the limit as the number of interconnected agents approaches infinity (Ferrari-Trecate et al., 2006), subsequently, the planar motion of the agents in the (x^1, x^2)-domain is introduced in terms of two decoupled nonlinear heat equations in the form of modified viscous Burgers equations with time-varying parameters.

2.1. Burgers equations and continuous agent topology

The motivation to use Burgers-type equations in the deployment of a continuum of interconnected agents is twofold. To generate complex profiles involving corners (Fig. 1, left) and "switchback" shapes (Fig. 1, right), one option is to use a linear PDE model that is of high order in the spatial variable α, where α denotes the continuous index of the agents. This option creates both considerable challenges for stabilization and for actuation.

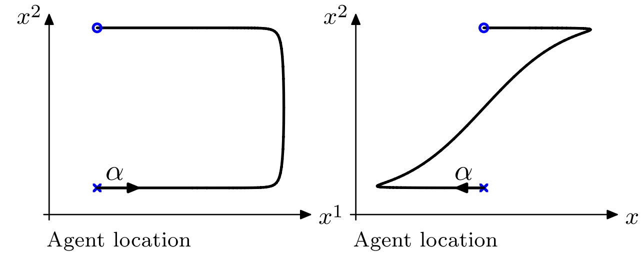

Fig. 1. Examples of deployment profiles for multi-agent continuum in the (x^1, x^2)-plane with the α-coordinate representing the continuous communication path. Anchor and leader agents are marked by $\times$ and $\circ$.

With regards to stabilization, linear Korteweg–de Vries (third order), Kuramoto–Sivashinsky (fourth order), and higher-order PDEs are much harder to stabilize than parabolic PDEs. With regards to actuation, higher numbers of derivatives in α in the PDE require to employ a higher number of boundary conditions, meaning, a higher number of leaders and anchors.

The second option is to stick with PDEs that are second-order in α, namely, parabolic, but allow nonlinearities, such as in the Burgers equation. The quadratic nonlinearity in the Burgers equation, which generates shock-like equilibrium profiles, allows for corner-like shapes in deployment profiles. At the same time, the motion planning and stabilization problems for Burgers equation are tractable, with a number of boundary conditions/inputs that is no higher than for linear parabolic PDEs.

Thus, the Burgers equation is a natural choice for considerably expanding the catalog of achievable deployment profiles, without dramatically expanding the complexity of the problem of deploying the agents to the desired profile.

We consider an agent continuum in the (x^1, x^2)-plane (cf. Fig. 1) with the communication path represented by the continuous independent coordinate $\alpha \in [0, 1]$ referring to the agent index in the continuum. The locations of the anchor and leader agents at $\alpha = 0\,(\times)$ and $\alpha = 1\,(\circ)$ serve as inputs, whose temporal paths are determined by flatness-based motion planning and feedforward control. Each $x^j = x^j(\alpha, t)$, $j = 1, 2$, is thereby governed by a boundary controlled PDE, which determines the individual motion of the agent continuum in the x^j-direction. By superimposing the respective x^1- and x^2-contributions, the desired planar deployment is achieved along prescribed spatial–temporal motion paths. The PDE formulation thereby in particular enables a design, which is independent of the actual communication topology. The latter is induced by means of a finite difference discretization scheme to transfer the results from an agent continuum to a discrete set of agents, where any follower agent processes only local information.

2.2. Distributed-parameter agent dynamics

As pointed out above, in the following a modified viscous Burgers equation is considered to model the motion of the mobile agent continuum with respect to the $x^j(\alpha, t)$-coordinate, $j \in \{1, 2\}$, i.e.

$$\begin{aligned}\partial_t x^j(\alpha, t) &= a^j \partial_\alpha^2 x^j(\alpha, t) - b^j x^j(\alpha, t)\partial_\alpha x^j(\alpha, t)\\ &\quad + c^j(t) x^j(\alpha, t), \quad \alpha \in (0, 1), t \in \mathbb{R}_{t_0} \qquad (1a)\end{aligned}$$

with $a^j, b^j > 0$, the time-varying parameter $c^j(t) \in \mathbb{R}$, and $\mathbb{R}_{t_0} = \{t \in \mathbb{R} \mid t > t_0\}$. The independent coordinate α corresponds to an agent index in a large group (continuum) of agents. The positions of the anchor agent ($\alpha = 0$) and the leader agent ($\alpha = 1$) are governed by the inhomogeneous Dirichlet boundary conditions

$$x^j(0, t) = u_a^j(t), \qquad x^j(1, t) = u_l^j(t), \quad t > t_0. \qquad (1b)$$