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Brief paper

A design algorithm using external perturbation to improve Iterative Feedback Tuning convergence[☆]

Jakob K. Huusom[a,1], Håkan Hjalmarsson[b], Niels K. Poulsen[c], Sten B. Jørgensen[a]

[a] Department of Chemical and Biochemical Engineering, Technical University of Denmark, DK - 2800 Lyngby, Denmark
[b] Department of Signals, Sensors and Systems, Royal Institute of Technology, SE - 100 44 Stockholm, Sweden
[c] Department of Informatics and Mathematical Modelling, Technical University of Denmark, DK - 2800 Lyngby, Denmark

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ABSTRACT

Iterative Feedback Tuning constitutes an attractive control loop tuning method for processes in the absence of process insight. It is a purely data driven approach for optimization of the loop performance. The standard formulation ensures an unbiased estimate of the loop performance cost function gradient, which is used in a search algorithm for minimizing the performance cost. A slow rate of convergence of the tuning method is often experienced when tuning for disturbance rejection. This is due to a poor signal to noise ratio in the process data. A method is proposed for increasing the data information content by introducing an optimal perturbation signal in the tuning algorithm. The theoretical analysis is supported by a simulation example where the proposed method is compared to an existing method for acceleration of the convergence by use of optimal prefilters.

1. Introduction

Control design and tuning for disturbance rejection is one of the classical disciplines in control theory and control engineering science. Design of compensators for disturbance rejection is well documented (Åström, 1970; Åström & Hägglund, 1995; Box & Jenkins, 1970). Given a particular control design, the tuning of the control parameters can be conducted based on tuning rules or by minimization of some loop performance criterion. Given a model of the system, the set of optimal control parameters which minimize the performance cost can be evaluated. In the absence of a sufficiently reliable model, the tuning can be performed based on data obtained from the loop, by a data driven optimization. Iterative Feedback Tuning is a method for optimizing control parameters using closed loop data, which forms the basis for the modifications presented here. The basic algorithm was first presented in Hjalmarsson, Gunnarsson, and Gevers (1994) and has since then been analyzed, extended and tested in a number of papers. For an extensive overview of the development of the method and references to applications, see Gevers (2002), Hjalmarsson (2002) and Huusom (2008). Alternative data driven tuning algorithms are Correlation based Tuning (Karimi, Mišković, & Bonvin, 2003, 2004) and Virtual Reference Feedback Tuning (Campi, Lecchini & Savaresi, 2002; Lecchini, Campi, & Savaresi, 2002).

The performance criterion, $F_N(y_t, u_t)$, used in the controller tuning is a function of the output and the control action. Hence it is a function of the true system, the controller and external signals acting on the loop. We will use the set-up in Fig. 1, where G is a causal scalar linear time-invariant system, C is the controller, which also is assumed to be causal scalar linear time-invariant, and where r_t is the reference signal and v_t is the disturbance, respectively. Assuming, as we will, that the disturbance is stochastic implies that the performance cost is itself a random variable. However, as in, e.g., LQG-control, it is natural to minimize the expected cost

$$F(\cdot) \triangleq \mathrm{E}\left[F_N(\cdot)\right] \tag{1}$$

where $\mathrm{E}[\cdot]$ is the mathematical expectation over the random disturbances acting on the closed loop system. Notice that in the following, when expectation of $F(\cdot)$ is taken, the expectation does not refer to the random disturbances acting on the system when assessing the closed loop performance. Instead it refers to the random variables that have affected the experimental data that has been used to design the controller for which the performance of $F(\cdot)$ is to be assessed.

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E-mail addresses: jkh@kt.dtu.dk (J.K. Huusom), hakan.hjalmarsson@ee.kth.se (H. Hjalmarsson), nkp@imm.dtu.dk (N.K. Poulsen), sbj@kt.dtu.dk (S.B. Jørgensen).

[1] Tel.: +45 4525 2801; fax: +45 4593 2906.

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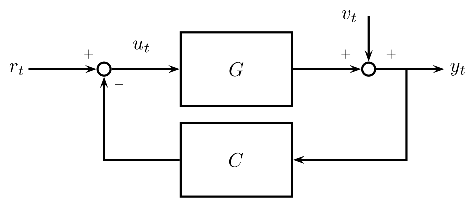

Fig. 1. A general feedback loop designed for disturbance rejection. The process, G, and the compensator in the feedback loop, C, is given as scalar linear transfer functions.

Our objective is to design a controller such that F is minimized when $r_t \equiv 0$, i.e. we are interested in disturbance rejection. Adding a reference signal during the experimentation phase may however improve the quality of the obtained controller C. In Iterative Feedback Tuning, one tries to minimize F with respect to the controller using noisy closed loop experiments. The accuracy of this design very much depends on the shape of the cost function F one tries to minimize. Any change in the spectrum Φ_r of the reference signal, will affect the output spectrum Φ_y and the input spectrum Φ_u. Hence the reference signal spectrum affects the minimum and the shape of the performance cost surface. By designing the spectrum of an external reference it is consequently possible to shape the performance cost function in order to improve the convergence properties of the search algorithm. However, one has to bear in mind that shaping the cost function will also influence the location of the minimum in the controller parameter space. Despite this unfortunate consequence, successful simulation studies are reported with respect to convergence using Iterative Feedback Tuning with external perturbation, when tuning for disturbance rejection (Huusom, Poulsen, & Jørgensen, 2009).

1.1. Formulating a design criterion

Let $F(\boldsymbol{\rho}, \boldsymbol{\vartheta})$ denote the cost function that we are interested in minimizing, where $\boldsymbol{\rho}$ and $\boldsymbol{\vartheta}$ represent the free control parameters which are to be tuned and a set of parameters which characterize the reference signal spectrum, respectively. The objective is to find the optimal $\boldsymbol{\rho}$ for a given $\boldsymbol{\vartheta} = \boldsymbol{\vartheta}^0$, where $\boldsymbol{\vartheta}^0$ corresponds to $r_t \equiv 0$. We denote the optimum $\boldsymbol{\rho}$ by $\bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta})$, indicating its dependence on $\boldsymbol{\vartheta}$. Since the system will be affected by noise it is only possible to obtain a minimizer, $\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta})$, with a certain accuracy; we use subscript n to denote that n iterations are performed in the tuning method. Hence Iterative Feedback Tuning will produce a solution with the following error

$$\boldsymbol{\Sigma}_n(\boldsymbol{\vartheta}) \triangleq \mathrm{E}\left[\left(\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta}) - \bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta})\right)\left(\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta}) - \bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta})\right)^T\right]. \quad (2)$$

Using a continuity argument it may therefore be advantageous to optimize $\boldsymbol{\rho}$ for a $\boldsymbol{\vartheta} \neq \boldsymbol{\vartheta}^0$, i.e. it may be that the controller corresponding to $\boldsymbol{\vartheta}$ may result in a smaller expected cost for the desired excitation conditions (which correspond to $\boldsymbol{\vartheta}^0$) than the controller tuned with the desired operating conditions $\boldsymbol{\vartheta}^0$ i.e. $\mathrm{E}\left[F(\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta}), \boldsymbol{\vartheta}^0)\right] < \mathrm{E}\left[F(\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta}^0), \boldsymbol{\vartheta}^0)\right]$ Our objective is to determine operating conditions $\boldsymbol{\vartheta}$ such that $\mathrm{E}\left[F(\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta}), \boldsymbol{\vartheta}^0)\right]$ is minimized. This is a difficult problem since $F(\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta}), \boldsymbol{\vartheta}^0)$ is a complicated and non-linear function of the random disturbances originating from the experiments on which $\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta})$ is based. This in turn means that the expectation with respect to these random variables is difficult to compute. Our approach to cope with this is to perform a local analysis, assuming $\boldsymbol{\vartheta}$ to be close to $\boldsymbol{\vartheta}^0$. Using Taylor expansion to second order near the optimum gives

$$F(\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta}), \boldsymbol{\vartheta}^0) \approx F(\bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta}^0), \boldsymbol{\vartheta}^0) + \frac{1}{2}\mathrm{Tr}\left\{\frac{\partial^2 F(\bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta}^0), \boldsymbol{\vartheta}^0)}{\partial \boldsymbol{\rho}^2}\left(\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta}) - \bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta}^0)\right)\left(\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta}) - \bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta}^0)\right)^T\right\}.$$

By taking the expectation and rearranging using Eq. (2) it is seen that

$$\begin{aligned}\mathrm{E}\left[F(\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta}), \boldsymbol{\vartheta}^0)\right] - F(\bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta}^0), \boldsymbol{\vartheta}^0) &\approx \frac{1}{2}\mathrm{Tr}\left\{\frac{\partial^2 F(\bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta}^0), \boldsymbol{\vartheta}^0)}{\partial \boldsymbol{\rho}^2}\left(\bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta}) - \bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta}^0)\right)\left(\bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta}) - \bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta}^0)\right)^T\right\} \\ &+ \frac{1}{2}\mathrm{Tr}\left\{\frac{\partial^2 F(\bar{\boldsymbol{\rho}}(\boldsymbol{\vartheta}^0), \boldsymbol{\vartheta}^0)}{\partial \boldsymbol{\rho}^2}\boldsymbol{\Sigma}_n(\boldsymbol{\vartheta})\right\} \triangleq \Delta F_n(\boldsymbol{\vartheta}).\end{aligned} \quad (3)$$

Now, if the covariance, $\boldsymbol{\Sigma}_n(\boldsymbol{\vartheta})$, can be evaluated then $\Delta F_n(\boldsymbol{\vartheta})$ is a quantity that can be minimized with respect to $\boldsymbol{\vartheta}$ in order to find the (approximately) optimal (reference) perturbation signal spectrum to be used in the experiments when tuning the controller parameters $\boldsymbol{\rho}$ using Iterative Feedback Tuning. The two terms in $\Delta F_n(\boldsymbol{\vartheta})$ can be interpreted as follows: The first term is the bias error due to that $\boldsymbol{\vartheta} \neq \boldsymbol{\vartheta}^0$ is used in the optimization whereas the second term is the variance error incurred on $F(\hat{\boldsymbol{\rho}}_n(\boldsymbol{\vartheta}), \boldsymbol{\vartheta}^0)$. The bias error will typically increase as $\boldsymbol{\vartheta}$ moves away from $\boldsymbol{\vartheta}^0$. As noted above, it may be possible to decrease the variance error if $\boldsymbol{\vartheta}$ is suitably chosen. The optimal perturbation choice $\boldsymbol{\vartheta} = \bar{\boldsymbol{\vartheta}}$ will balance these two terms. The aim of this study is to construct a systematic and formal algorithm for designing an optimal external perturbation signal for Iterative Feedback Tuning for the disturbance rejection problem based on (3).

The paper is organized as follows: Section 2 presents the basic Iterative Feedback Tuning algorithm for disturbance rejection and the error $\boldsymbol{\Sigma}_n(\boldsymbol{\vartheta})$ of the method derived in Hildebrand, Lecchini, Solari, and Gevers (2005b). In Section 3 the effect of adding an external perturbation signal to the loop in the tuning method is analyzed. In Section 4, a formal design criterion for the perturbation spectrum in Perturbed Iterative Feedback Tuning is derived. Finally a simulation example serves to illustrate the advantages of introducing optimal external perturbation when tuning the loop for disturbance rejection.

2. Iterative feedback tuning for disturbance rejection

The Iterative Feedback Tuning algorithm for disturbance rejection is illustrated in the following Hjalmarsson, Gevers, Gunnarsson, and Lequin (1998). The feedback loop in Fig. 1 depicts the signals and transfer functions which are used in the algorithm for tuning the parameters $\boldsymbol{\rho}$ in C. The objective is minimization of the cost function:

$$F(\rho_i) = \frac{1}{2N}\mathrm{E}\left[\sum_{t=1}^{N}(y_t(\rho_i) - y_t^d)^2 + \lambda(u_t(\rho_i))^2\right] \quad (4)$$

where N is the number of discrete time data points and $y^d = 0$ is the desired output response for disturbance rejection. The sensitivity of the cost function with respect to the control parameters is

$$\boldsymbol{J}(\rho_i) = \frac{1}{N}\mathrm{E}\left[\sum_{t=1}^{N} y_t(\rho_i)\frac{\partial y_t(\rho_i)}{\partial \boldsymbol{\rho}} + \lambda u_t(\rho_i)\frac{\partial u_t(\rho_i)}{\partial \boldsymbol{\rho}}\right]. \quad (5)$$

The minimization is realized by iterating in the scheme

$$\boldsymbol{\rho}_{i+1} = \boldsymbol{\rho}_i - \gamma_i \mathbf{R}_i^{-1}\boldsymbol{J}(\rho_i) \quad (6)$$

where $\mathbf{R}_i$ is a positive definite matrix typically chosen as the Hessian of the cost function with respect to the control parameters. If a model for the system is unknown, the gradients of the input and output and hence the cost function gradient cannot be evaluated analytically. In the traditional Iterative Feedback Tuning framework the minimization of the cost function, (4), is based on data from two successive experiments: