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Bivariate extreme value modeling for road safety estimation

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ABSTRACT

Surrogate safety measures have been advocated as a complementary approach to study safety from a broader perspective than relying on crash data alone. This study proposes an approach to incorporate different surrogate safety measures in a unified framework for road safety estimation within the bivariate extreme value theory framework. The model structure, model specification, threshold selection method, and parameter estimation method of the bivariate threshold excess model are introduced. Two surrogate safety measures, post encroachment time (PET) and length proportion of merging (LPM), are chosen to characterize the severity of merging events on freeway entrance merging areas. Based on the field data collected along Highway 417 in the City of Ottawa, Ontario, Canada, the bivariate modelling methods with seven distribution functions are applied and compared, and the model with logistic distribution function is selected as the best model. The best bivariate models' estimation results are then evaluated by comparing them to their two marginal (univariate Generalized Pareto distribution) models. The results show that the bivariate models tend to generate crash estimates that are much closer to observed crashes than univariate models. A more important finding is that incorporating two surrogate safety measures into the bivariate models can significantly reduce the uncertainty of crash estimates. The efficiency of a bivariate model is not evidently better than either of its marginal models, but it is expected to be improved with data of a prolonged observation period. This study is also a step forward in the direction of developing multivariate safety hierarchy models, since models of the safety hierarchy have been predominantly univariate.

1. Introduction

Utilizing surrogate safety measures for road safety analysis has been gaining popularity in recent years (Zheng et al., 2014a). A surrogate safety measure should be based on observable non-crash events that are physically related to crashes and can be converted into crash frequency and/or severity using practical methods (Tarko et al., 2009). Several surrogate safety measures that satisfy previous conditions have been proposed in the literature, and examples include time to collision (TTC), post encroachment time (PET) and proportion of stopping distance (Gettman and Head, 2003; Laureshyn et al., 2016). Although previous studies assumed that one single measure is sufficient to classify all traffic events in a meaningful way, different measures inherently represent partial severity aspects of traffic events, and integrating all these sources of information is a promising way to gain more comprehensive understanding on the underlying level of safety (Ismail et al., 2011; Zheng et al., 2014a). This study makes an explorative attempt in this regard. Different from previous studies, e.g., Ismail et al. (2011), in which several conflict indicators were integrated by a mapping function to generate an index of severity, this study develops a model to utilize different surrogate safety measures and the same model can be utilized to generate crash counts. Specifically, crashes are modeled as extremes of traffic events that are characterized by the joint behavior of two surrogate safety measures within the multivariate extreme value theory framework.

2. Background

Extreme Value Theory (EVT) provides models and techniques to enable extrapolation from observed levels to unobserved levels of a stochastic phenomenon. For example, it is vital to ensure that a bridge is protected against flood events that it is likely to experience within its projected life span, e.g., 100 years. Local data could record daily water levels including minor flood events for a short period, e.g., 10 years. The EVT enables the estimation of the occurrence of the extreme flood event over the next 100 years (100-year flood) given the 10-year

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history. This idea can readily meet a practical need of surrogate safety measures that are sought to utilize observable more frequent non-crash events to predict rarely occurred crashes.

With the development of surrogate safety measures, the EVT approach is introduced to connect non-crash events to crashes in early seminal studies (Songchitruksa, 2004; Songchitruksa and Tarko, 2006; Tarko, 2012). The authors concluded the advantages of the EVT approach as abandoning the assumption of fixed crash-to-surrogate ratio and no need of crash data in the model estimation process. Other work also made contributions in this area. Gordon et al. (2013) used the EVT approach to estimate the road departure crash frequency based on the surrogate safety measure of time to road edge crossing, and they found reasonable crash estimates compared to the observed crash data. Zheng et al. (2014b) used the EVT approach to estimate safety related to lane changing maneuver on freeways. They also suggested that the EVT fitted well within the safety continuum and thus developed a parametric safety continuum model using this theory (Zheng et al., 2014c; Zheng and Ismail, 2017). Zheng et al. (2016) also employed the EVT approach to determine the PET thresholds that distinguish traffic conflicts from non-conflict events. Åsljung et al. (2016, 2017) used the EVT approach to validate the safety level of vehicles and they implied that the EVT approach was promising in safety analysis related to autonomous vehicles. In a recent study, Farah and Azevdo (2017) employed the EVT approach to analyze the safety of passing maneuvers on two-lane rural highways.

Using the EVT approach for road safety analysis has been so far mainly limited to univariate models, by which crashes are estimated as extremes of a single surrogate safety measure. Jonasson and Rootzén (2014) introduced the bivariate block maxima modeling methods to investigate whether the near-crashes are representative of the crashes in naturalistic driving study. They used surrogate safety measure (TTC) and an explanatory variable (nine variables as max speed, min distance left lane marking, etc. were tested separately) combined in the crash estimation process. In contrast to the rare application in the road safety area, the bivariate/multivariate extreme value models have been used in multidisciplinary areas, as drought or flood prediction in hydrology (Yue, 2001; Hamdi et al., 2016), financial crisis prediction in finance (Cumperayot and Kouwenberg, 2013), and failure risk prediction in structure engineering (Valamanesh et al., 2015).

A basic starting point for those studies using multivariate modeling methods is that the extremal events they investigated are often characterized by several features or processes. For instance, in hydrology a storm could be characterized by its density and duration, and a flood event could be characterized by its peak, volume, and duration. Similarly, a pre-crash event leading to a crash or a traffic conflict can also be characterized from different aspects by different surrogate safety measures, such as temporal proximity, spatial proximity, likelihood of evasive actions, and consequence of a potential collision (Davis et al., 2011; Laureshyn et al., 2017; Tageldin and Sayed, 2016). Modeling these aspects jointly will certainly bring complications as defining extremal events in a multidimensional space and deducing safety implications from the joint behavior of surrogate safety measures. This study attempts to investigate these issues using the multivariate extreme value modeling techniques.

The remainder of the study is organized as follows. Section 3 describes the bivariate threshold excess models and the model estimation methods. The data including the development of surrogate safety measures is presented in Section 4. The modeling results based on the practical data is discussed in Section 5. The conclusions are presented in Section 6.

3. Bivariate extreme value modelling methods

Bivariate extreme value theory models the joint distribution of two extreme variables and it is an extension of the univariate extreme value theory. The reader can refer to Coles (2001) and Beirlant et al., (2004) for detailed theoretical foundations on bivariate extreme value models as well as univariate extreme value models. This study mainly describes the models for exceedances over high thresholds, i.e., bivariate threshold excess models.

3.1. Bivariate threshold excess models

Suppose $\{(x_1, y_1), (x_2, y_2), \ldots\}$ are independent realizations of a random vector (X, Y) with joint distribution function $F(x, y)$. The bivariate threshold excess model approximates the joint distribution $F(x, y)$ on regions of the form $x > u_x$, $y > u_y$, for large enough u_x and u_y. For suitable thresholds u_x and u_y, each of the two marginal distributions of F can be approximated in the form of a univariate generalized Pareto (GP) distribution, with respective parameter sets (σ_x, ξ_x) and (σ_y, ξ_y). The functional form of GP approximation is as follows:

$$G\left(z; u, \sigma, \xi\right) = \begin{cases} 1-\left(1+\frac{\xi}{\sigma}(z-u)\right)^{-1/\xi} & \xi \neq 0 \\ 1\text{-}\exp\left(-\frac{z-u}{\sigma}\right) & \xi = 0 \end{cases} \tag{1}$$

Defining $\varsigma_x = \Pr\{x > u_x\}$ and $\varsigma_y = \Pr\{y > u_y\}$, the following transformations:

$$\tilde{x} = -\left(\log\left\{1-\varsigma_x\left[1+\frac{\xi_x(x-u_x)}{\sigma_x}\right]^{-1/\xi_x}\right\}\right)^{-1} \quad \text{and} \quad \tilde{y} = -\left(\log\left\{1-\varsigma_y\left[1+\frac{\xi_y(y-u_y)}{\sigma_y}\right]^{-1/\xi_y}\right\}\right)^{-1}$$

induce a pair of variables $(\tilde{x}, \tilde{y})$ whose marginal distributions are approximately standard Fréchet distributions for $x > u_x$ and $y > u_y$. As detailed in Coles (2001), the joint distribution $F(x, y)$ can be expressed as

$$F(x, y) = \tilde{F}(\tilde{x}, \tilde{y}) \approx \exp\{-V(\tilde{x}, \tilde{y})\},\ x > \mu_x,\ y > \mu_y \tag{2}$$

where

$$V(\tilde{x}, \tilde{y}) = 2\int_0^1 \max\left(\frac{w}{\tilde{x}}, \frac{1-w}{\tilde{y}}\right) dH(w) \tag{3}$$

and H (also called spectral measure) is a distribution function on [0, 1] satisfying the constraint:

$$\int_0^1 w dH(w) = 1/2 \tag{4}$$

Although Equations (2) to (4) provide a complete characterization of bivariate limit distributions, the class of possible limits constrained only by Equation (3) is wide. Particularly, any distribution function H on [0, 1] in Equation (3), satisfying the mean constraint of Equation (4), gives rise to a valid limit in Equation (2). This leads to difficulties in the use of bivariate limit distributions, as they have virtually infinite parametric forms. One approach to address this problem is to use specific parametric distributions for H, leading to sub-families of distributions for G. For instance, letting H have the density function of $h(w) = \frac{1}{2}(\alpha^{-1}-1)\{w(1-w)\}^{-1-1/\alpha}\{w^{-1/\alpha}+(1-w)^{-1/\alpha}\}^{\alpha-2}$ on $0 < w < 1$. The mean constraint of Equation (4) is satisfied for this H because of symmetry at $w = 0.5$. Substitution into (3) and (2) and then generates the logistic family of bivariate extreme value distributions, as shown in Table 1. There are also other parametric families available in the literature, including asymmetry logistic, negative logistic, asymmetry negative logistic, bilogistic, negative bilogistic, and Husler-Reiss, as listed in Table 1.

3.2. Threshold selection

To ensure that the approximation in Equation (2) is valid, a pair of optimal thresholds need to be selected. The approximation entails that the marginal distributions of excess over corresponding thresholds are modeled by a GP distribution while the dependence structure between two margins by that of a bivariate extreme value distribution. Marginal