Automatica 47 (2011) 2743–2748

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Passification-based robust flight control design ☆

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ARTICLE INFO

Article history:
Received 9 July 2010
Received in revised form
7 February 2011
Accepted 30 May 2011
Available online 5 October 2011

Keywords:
Flight control
Uncertain dynamic systems
Shunt compensation
Robustness
Passification

ABSTRACT

A passification-based robust autopilot for attitude control of flexible aircraft under parametric uncertainty is designed. A high gain controller with forced sliding motions is used to secure good performance over a wide range of the aircraft model parameters. The shunting method is applied to ensure closed-loop system stability under lack of aircraft state information. The series reference model is used to assign the desired closed-loop system performance. An example illustrating a typical design procedure for aircraft attitude control in the horizontal plane for different flight conditions is given. The simulation results demonstrate the efficiency and high robustness of the suggested control system.

1. Introduction

Modern highly maneuverable aircraft, such as fighters, operate over a wide range of flight conditions, which vary with altitude, Mach number, angle of attack, and engine thrust. The mechanical characteristics of the airframe, such as the center of gravity, change as well. The aircraft autopilot has to be able to produce a response that is accurate and fast despite severe variations in speed and altitude of the airframe or, in other words, in the face of large parametric uncertainty (Belkharraz & Sobel, 2007; Gurfil, 2001; Singh, Steinberg, & Page, 2003; Tsourdos & White, 2001). A promising way to fulfill these requirements is application of the adaptive control technique. The adaptation method has to meet the conflicting requirements on the tuning rate and performance quality under conditions of lack of aircraft state measurements (Andrievsky, Fradkov, & Stotsky, 1996; Ben Yamin, Yaesh, & Shaked, 2007; Fradkov & Andrievsky, 2005, 2007; Schumacher & Kumar, 2000; Singh et al., 2003; Wise, Lavretsky, & Hovakimyan, 2006).

The term "passification-based adaptive control" was introduced in Seron, Hill, and Fradkov (1994), though the structures of passification-based adaptive controllers for linear plants with underlying theory were introduced as early as in the 1970s (Fradkov, 1974, 1976) under different names. Initially they were named adaptive systems with implicit reference models (ASIRMs). The main results are presented in a number of books and surveys (Andrievskii & Fradkov, 2006; Fomin, Fradkov, & Yakubovich, 1981; Fradkov, 2003; Fradkov & Andrievsky, 2005; Fradkov, Miroshnik, & Nikiforov, 1999). Later related structures were used in so-called simple adaptive control (SAC) systems (Barkana, 1987, 1989, 2007; Barkana & Kaufman, 1985; Belkharraz & Sobel, 2007; Ben Yamin et al., 2007; Iwai & Mizumoto, 1994; Kaufman, Barkana, & Sobel, 1994). The connection between the two approaches was studied in Andrievsky, Fradkov, and Kaufman (1994). Usage of passification-based flight control is motivated by its simplicity and its close relation to stability: if a system is passive with respect to some output y, it can be asymptotically stabilized by the output feedback $u = -ky$ for any $k > 0$. Applicability conditions of the method (necessary and sufficient conditions of passifiability) were provided in Fradkov (1974, 1976) for linear systems and in Byrnes, Isidori, and Willems (1991) and Fradkov and Hill (1998) for nonlinear systems.

In this study, the passification method is applied for robust attitude control of flexible aircraft with a high-order model. To ensure the applicability conditions of the method, a parallel feedforward compensator (*a shunt*) (Andrievsky, Churilov, & Fradkov, 1996; Andrievsky & Fradkov, 1994; Barkana, 1987, 1994;

☆ This work was partly supported by the Russian Foundation for Basic Research (Proj. Nos 09-08-00803, 11-08-01218), by the Russian Federal Program "Research and Teaching Cadres", contract No 16.740.11.0042, and the Program of basic research of OEMPPU RAS No 2 "Control and safety in energy and technical systems". The material in this paper was partially presented at the 10th IFAC Workshop on Adaptation and Learning in Control and Signal Processing (ALCOSP 2010), August 26–28, 2010, Antalya, Turkey. This paper was recommended for publication in revised form by Associate Editor Yong-Yan Cao under the direction of Editor Toshiharu Sugie.

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0005-1098/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2011.09.004

Barkana & Kaufman, 1985; Fradkov, 1994; Iwai & Mizumoto, 1992; Kaufman et al., 1994; Mareels, 1984) is introduced into the controller.

The paper is organized as follows. Some essentials of the passification method are outlined in Section 2. Section 3 is devoted to the application of the passification method for robust control of flexible aircraft. Concluding remarks are given in Section 4.

2. Preliminaries. Passification and shunting methods

2.1. Passification theorem

Consider a linear time invariant (LTI) single-input multiple-output (SIMO) system

$$\dot{x} = Ax + Bu, \qquad z = Cx, \tag{1}$$

where $x = x(t) \in \mathbb{R}^n$ is a state vector, $u = u(t) \in \mathbb{R}^1$ is a scalar control variable, $z = z(t) \in \mathbb{R}^l$ is a measured output vector, and A, B, C are constant real matrices of size $n \times n, n \times 1, l \times n$, respectively. Let G be a $(1 \times l)$ matrix.

The *passification problem* for system (1) is understood here as finding an $(l \times 1)$ matrix K such that the closed-loop system with feedback $u = -K^{\mathrm{T}}z + v$ is strictly passive with respect to an auxiliary output $\sigma = Gz$: inequality $\int_0^T (\sigma v - \rho|x|^2)\,\mathrm{d}t \geq 0$ for some $\rho > 0$ and all $T > 0$ holds for all trajectories of (1) starting from $x(0) = 0$. This is equivalent (as follows from the Kalman–Yakubovich–Popov lemma) to finding a matrix K satisfying the *strict positive realness* (SPR) condition: transfer function $W(\lambda) = GC(\lambda I_n - A + BK^{\mathrm{T}}C)^{-1}B$ of the closed-loop system[2] from the input, v, to the output, $\sigma = Gz$, satisfies the relations

$$\mathrm{Re}\, W(i\omega) > 0 \quad \text{for all } \omega \in \mathbb{R}^1,\ i^2 = -1$$
$$\text{and} \lim_{\omega \to +\infty} \omega^2\, \mathrm{Re}\, W(i\omega) > 0. \tag{2}$$

Definition 1. System (1) is called *minimum phase* with respect to the output $\sigma = Gz$, if the polynomial

$$\varphi_0(s) = \det \begin{bmatrix} sI_n - A & -B \\ GC & 0 \end{bmatrix} \tag{3}$$

is Hurwitz; it is called *hyper minimum phase* (HMP), if it is minimum phase and $GCB > 0$.

Theorem 2 (*Passification Theorem, or Feedback Kalman–Yakubovich –Popov Lemma (Fradkov, 2003; Fradkov et al., 1999)*). *The following statements are equivalent.*

(A1) *There exist a positive definite $(n \times n)$ matrix H and an $(l \times 1)$ matrix K such that the relations*

$$H(A + BK^{\mathrm{T}}C) + (A + BK^{\mathrm{T}}C)^{\mathrm{T}}H < 0, \qquad HB = C^{\mathrm{T}}G^{\mathrm{T}} \tag{4}$$

hold.

(B1) *System (1) is hyper minimum phase with respect to the output $\sigma = Gz$.*

(C1) *There exists a feedback*

$$u = K^{\mathrm{T}}z + v \tag{5}$$

rendering the closed-loop system (1), (5) strictly passive with respect to the output $\sigma = Gz$.

Note that, if condition (B1) is satisfied, then matrix K in (4) can be found in the form $K = -\kappa G^{\mathrm{T}}$, where κ is a sufficiently large positive real number. Extension of Theorem 2 to the multiple-input multiple-output (MIMO) case can be found in Fradkov (2003) and Fradkov et al. (1999).

Remark 3. For the MIMO case, an additional requirement of symmetry $(GCB)^{\mathrm{T}} = GCB$ is included in the HMP definition. It follows from the recent results by Barkana, Teixeira, and Hsu (2006) that, for $l = m$, if the nonsymmetric positive definite matrix $WGCB$ is diagonizable, an unknown positive definite symmetric matrix R exists that makes the product $RGCB$ positive definite symmetric. Therefore, for $l = m$, the original adaptive controllers can be used without additional G (or R).

The passification theorem (Theorem 2) provides conditions for solvability of matrix inequalities related to the feedback version of the classical Kalman–Yakubovich–Popov (KYP) lemma (Andrievsky et al., 1996; Fradkov, 1974, 1976). It also provides solvability conditions for the system passification problem by means of static output feedback. It has had various applications in control design since the 1970s, for example, the design of adaptive controllers with the implicit reference model (Andrievskii, Stotskii, & Fradkov, 1988; Andrievsky & Fradkov, 1994).

2.2. Passification-based design of variable-structure systems and signal-parametric adaptive controllers

In this section, an application of the passification theorem to design of variable-structure systems (VSSs) (Utkin, 1992) and signal-parametric adaptive controllers (Andrievskii & Fradkov, 2006; Andrievskii et al., 1988; Stotsky, 1994) is briefly described.

Consider the LTI plant (1) for $m = 1$ and the control objective $\lim_{t\to\infty} x(t) = 0$. Let the auxiliary objective be chosen as maintaining the *sliding mode* on the plane $\sigma = 0$, where $\sigma = Gz$ is the auxiliary variable and G is a $(1 \times m)$ matrix. Using the speed-gradient method (Andrievskii et al., 1988; Fradkov, 1979) with the goal function σ^2, we arrive at the following control law:

$$u(t) = -\gamma\, \mathrm{sign}\, \sigma_t, \quad \sigma_t = Gz(t), \tag{6}$$

where $\gamma > 0$ is the gain parameter. As is shown in Andrievskii et al. (1988) and Fradkov et al. (1999), the goal $x(t) \to 0$ may be achieved in system (1), (6) if there exist matrix $H = H^{\mathrm{T}} > 0$ and vector K_* such that $HA_* + {A_*}^{\mathrm{T}}H < 0, HB = C^{\mathrm{T}}G^{\mathrm{T}}, A_* = A + BK_*{}^{\mathrm{T}}C$. As is clear from Theorem 2, the mentioned condition is fulfilled if and only if the function $W(s)$ is HMP, where $W(s) = GC\,(sI_n - A)^{-1}B$, and the sign of the high-frequency gain GCB is known. In that case, for sufficiently large γ, the relation $\lim_{t\to\infty} x(t) = 0$ holds. To eliminate the dependence of system stability on initial conditions and plant parameters, the following combined (so called "signal-parametric") adaptive control law may be used instead of (6) (Andrievskii & Fradkov, 2006; Andrievskii et al., 1988):

$$u(t) = -K(t)^{\mathrm{T}}z(t) - \gamma\, \mathrm{sign}\, \sigma_t, \quad \sigma_t = Gz(t) \tag{7}$$
$$\dot{K}(t) = \sigma_t \Gamma z(t), \tag{8}$$

where $\Gamma = \Gamma^{\mathrm{T}} > 0, \gamma > 0$ are design parameters.

It should be noticed that convergence of σ_t to zero in a finite time is essential for VSS-like systems. It can be shown (see, e.g., Fradkov, 1990 and Fradkov et al., 1999) that this property is valid for any bounded region of initial conditions for system (1), (7), (8). To ensure boundedness of the gain $K(t)$ in practice, parametric feedback may be added to the algorithm. Such a robustification of the adaptation algorithm (8) leads to the following adaptation law:

$$\dot{K}(t) = \sigma_t \Gamma z(t) - \alpha\,(K(t) - K_0)\,, \qquad K(0) = K_0, \tag{9}$$

where $\alpha > 0$ is the parametric feedback gain and K_0 is some initial "guessed" value of the gain matrix K.

Application of the signal-parametric algorithm (7)–(9) to flight control design is demonstrated in Andrievskii and Fradkov (2006) and Fradkov, Andrievsky, and Peaucelle (2008). In the present study we focus our attention on application of control law (6).

[2] I_n denotes the $n \times n$ identity matrix.