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Brief paper

Gain-scheduled output-feedback controllers depending solely on scheduling parameters via parameter-dependent Lyapunov functions☆

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ABSTRACT

In this paper, we propose a new design method for Gain-Scheduled Output Feedback (GSOF) controllers for continuous-time Linear Parameter-Varying (LPV) systems via Parameter-Dependent Lyapunov Functions (PDLFs). The GSOF controllers depend solely on scheduling parameters. Although our method requires a line search to obtain suboptimal controllers, it produces practical GSOF controllers, being independent of the derivatives of scheduling parameters. Our method is proved to be no more conservative than conventional design methods via constant Lyapunov functions as well as particularly structured PDLFs.

1. Introduction

It is widely known that Gain-Scheduled (GS) controllers have better performance than robust controllers if scheduling parameters, which describe changes of plant dynamics, are available. Many researchers have therefore already tackled the design problem of GS Output-Feedback (GSOF) controllers for Linear Parameter-Varying (LPV) systems using Linear Matrix Inequalities (LMIs), e.g. Apkarian and Adams (1998), Apkarian and Gahinet (1995), Apkarian, Gahinet, and Becker (1995), Scherer (1996) and Wu, Yang, Packard, and Becker (1996). In Apkarian and Adams (1998), Scherer (1996) and Wu et al. (1996), Parameter-Dependent Lyapunov Functions (PDLFs) are adopted to reduce the conservatism due to Parameter-inDependent Lyapunov Functions (PiDLFs) used in Apkarian et al. (1995) and Apkarian and Gahinet (1995). Although these methods successfully reduce conservatism, the designed GSOF controllers require not only scheduling parameters but also their derivatives, which cannot be obtained in the real world.

To circumvent this difficulty, several remedies have been proposed such as controller design via structured PDLFs (Apkarian & Adams, 1998) and controller implementations incorporating filter systems (Masubuchi & Kurata, 2009). However, these methods have drawbacks: the former still has conservatism due to the structured parameter-dependency of PDLFs, and the filters in the latter increase the numerical complexity of the implemented GSOF controllers. From a practical perspective, simple GSOF controllers are preferable.

This paper tackles the design problem of GSOF controllers for continuous-time LPV systems via PDLFs, with the controllers incorporating no additional systems (such as filters) and being dependent only on the scheduling parameters. We successfully propose a new design method for the problem in terms of a set of Parameter-Dependent LMIs (PDLMIs) with single line search parameters. Our method is proved to be no more conservative than conventional design methods via PiDLFs or particularly structured PDLFs. Recently, Köroğlu has independently proposed a design method for GSOF controllers for regulation problems using a similar technique to ours (Köroğlu, 2010).

We use standard notation. $\mathrm{He}\{X\}$ is a shorthand for $X + X^T$, I_n, $\mathbf{I}$ and $\mathbf{0}$ respectively denote an $n \times n$ dimensional identity matrix, an identity matrix and a zero matrix of appropriate dimensions, $\mathcal{R}^{n\times m}$ and $\mathcal{S}^n$ respectively denote sets of $n \times m$ dimensional real matrices and $n\times n$ dimensional symmetric real matrices, $\otimes$ denotes the Kronecker product, $\mathrm{diag}(X_1, \ldots, X_l)$ denotes a block-diagonal matrix composed of $X_1, \ldots$ and X_l, and $*$ in symmetric matrices denotes an abbreviated off-diagonal block.

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2. Preliminaries

2.1. System definitions

Suppose that the state-space representation of an LPV system with k independent scheduling parameters is given as follows:

$$G(\theta):\begin{cases}\dot{x}=A(\theta)x+B_1(\theta)w+B_2(\theta)u\\ z=C_1(\theta)x+D_{11}(\theta)w+D_{12}(\theta)u\\ y=C_2(\theta)x+D_{21}(\theta)w\end{cases},\tag{1}$$

where $x\in\mathcal{R}^n, w\in\mathcal{R}^{n_w}, u\in\mathcal{R}^{n_u}, z\in\mathcal{R}^{n_z}$ and $y\in\mathcal{R}^{n_y}$ respectively denote the state with $x=\mathbf{0}$ at $t=0$, the disturbance input, the control input, the controlled output and the measurement output. The state-space matrices in (1) are supposed to be polynomially parameter-dependent and to have compatible dimensions. The scheduling parameters θ_i and their derivatives with respect to time $\dot{\theta}_i$ are supposed to lie in *a priori* given hyper-rectangles Ω_θ and Λ_θ, respectively; that is, $\theta(t)\in\Omega_\theta$ and $\dot{\theta}(t)\in\Lambda_\theta$ hold for $\theta=[\theta_1\cdots\theta_k]^T$ and $\dot{\theta}=\left[\dot{\theta}_1\cdots\dot{\theta}_k\right]^T$. Λ_θ is supposed to include the origin.

We design a full-order GSOF controller $C(\theta)$ for $G(\theta)$,

$$C(\theta):\begin{cases}\dot{x}_c=A_c(\theta)x_c+B_c(\theta)y\\ u=C_c(\theta)x_c+D_c(\theta)y,\end{cases}\tag{2}$$

where $x_c\in\mathcal{R}^n$ denotes the state with $x_c=\mathbf{0}$ at $t=0$. The state-space matrices in (2) are supposed to have compatible dimensions and to be rationally parameter-dependent. In contrast to existing design methods via PDLFs, e.g. Apkarian and Adams (1998), Scherer (1996) and Wu et al. (1996), the GSOF controller is supposed to depend only on the scheduling parameters θ_i.

Then, the closed-loop system is given as follows:

$$G_{\mathrm{cl}}(\theta):\begin{cases}\dot{x}_{\mathrm{cl}}=A_{\mathrm{cl}}(\theta)x_{\mathrm{cl}}+B_{\mathrm{cl}}(\theta)w\\ z=C_{\mathrm{cl}}(\theta)x_{\mathrm{cl}}+D_{\mathrm{cl}}(\theta)w,\end{cases}\tag{3}$$

where $x_{\mathrm{cl}}=\left[x^T\ x_c^T\right]^T$ and

$$A_{\mathrm{cl}}(\theta)=\begin{bmatrix}A(\theta)+B_2(\theta)D_c(\theta)C_2(\theta) & B_2(\theta)C_c(\theta)\\ B_c(\theta)C_2(\theta) & A_c(\theta)\end{bmatrix},$$

$$B_{\mathrm{cl}}(\theta)=\begin{bmatrix}B_1(\theta)+B_2(\theta)D_c(\theta)D_{21}(\theta)\\ B_c(\theta)D_{21}(\theta)\end{bmatrix},$$

$$C_{\mathrm{cl}}(\theta)=\left[C_1(\theta)+D_{12}(\theta)D_c(\theta)C_2(\theta)\quad D_{12}(\theta)C_c(\theta)\right],$$

$$D_{\mathrm{cl}}(\theta)=D_{11}(\theta)+D_{12}(\theta)D_c(\theta)D_{21}(\theta).$$

2.2. Problem definitions

We tackle the following two problems.

Problem 1 (H_∞-*Type Problem*)**.** For a given positive number γ_∞, design a GSOF controller (2) depending solely on θ_i which stabilizes the closed-loop system (3) and satisfies (4) for all admissible pairs $(\theta,\dot{\theta})\in\Omega_\theta\times\Lambda_\theta$.

$$\sup_{w\in\mathcal{L}_2,w\neq\mathbf{0}}\|z\|_2/\|w\|_2<\gamma_\infty.\tag{4}$$

Problem 2 (H_2-*Type Problem*)**.** Suppose that $D_{11}(\theta)=\mathbf{0}$ holds for all $\theta\in\Omega_\theta$. For a given positive number γ_2, design a GSOF controller (2) depending solely on θ_i with $D_c(\theta)=\mathbf{0}$ which stabilizes the closed-loop system (3) and satisfies (5) for all admissible pairs $(\theta,\dot{\theta})\in\Omega_\theta\times\Lambda_\theta$.

$$\mathcal{E}\left(\int_0^\infty z^Tzdt\right)<\gamma_2^2\quad\text{for } w=\begin{cases}w_0(t=0)\\ \mathbf{0}(t\neq0)\end{cases}\tag{5}$$

with a random variable w_0 satisfying $\mathcal{E}\left(w_0w_0^T\right)=\mathbf{I}$.

2.3. Basic lemmas

Hereafter, $\dot{X}(\theta)$ denotes $\frac{d}{dt}X(\theta):=\sum_{i=1}^k\frac{d\theta_i}{dt}\frac{\partial X(\theta)}{\partial\theta_i}$.

For given controller $C(\theta)$, the following are well known.

Lemma 3 (Wu et al., 1996)**.** *For a positive number γ_∞, if there exists a continuously differentiable parameter-dependent matrix $X_{\mathrm{cl}}(\theta)\in\mathcal{S}^{2n}$ such that (6) and (7) hold, then the controller $C(\theta)$ stabilizes the closed-loop system (3) and satisfies (4) for all admissible pairs $(\theta,\dot{\theta})\in\Omega_\theta\times\Lambda_\theta$.*

$$X_{\mathrm{cl}}(\theta)>0,\quad\forall\theta\in\Omega_\theta\tag{6}$$

$$\begin{bmatrix}\mathrm{He}\{A_{\mathrm{cl}}(\theta)X_{\mathrm{cl}}(\theta)\}-\dot{X}_{\mathrm{cl}}(\theta) & * & B_{\mathrm{cl}}(\theta)\\ C_{\mathrm{cl}}(\theta)X_{\mathrm{cl}}(\theta) & -\gamma_\infty I_{n_z} & D_{\mathrm{cl}}(\theta)\\ * & * & -\gamma_\infty I_{n_w}\end{bmatrix}<0,$$
$$\forall\left(\theta,\dot{\theta}\right)\in\Omega_\theta\times\Lambda_\theta.\tag{7}$$

Lemma 4 (Sznaier, 1999)**.** *Suppose that $D_{\mathrm{cl}}(\theta)=\mathbf{0}$ holds for all $\theta\in\Omega_\theta$. For a positive number γ_2, if there exists a continuously differentiable parameter-dependent matrix $X_{\mathrm{cl}}(\theta)\in\mathcal{S}^{2n}$ such that (6), (8) and (9) hold, then the controller $C(\theta)$ stabilizes the closed-loop system (3) and satisfies (5) for all admissible pairs $(\theta,\dot{\theta})\in\Omega_\theta\times\Lambda_\theta$.*

$$\begin{bmatrix}\mathrm{He}\{A_{\mathrm{cl}}(\theta)X_{\mathrm{cl}}(\theta)\}-\dot{X}_{\mathrm{cl}}(\theta) & *\\ C_{\mathrm{cl}}(\theta)X_{\mathrm{cl}}(\theta) & -I_{n_z}\end{bmatrix}<0,$$
$$\forall\left(\theta,\dot{\theta}\right)\in\Omega_\theta\times\Lambda_\theta\tag{8}$$

$$\gamma_2^2-\mathrm{Tr}\left(B_{\mathrm{cl}}(\theta)^TX_{\mathrm{cl}}(\theta)^{-1}B_{\mathrm{cl}}(\theta)\right)>0,\quad\forall\theta\in\Omega_\theta.\tag{9}$$

3. Main results

In Lemmas 3 and 4, PDLFs are set as $x_{\mathrm{cl}}^TX_{\mathrm{cl}}(\theta)^{-1}x_{\mathrm{cl}}$ with a continuously differentiable parameter-dependent matrix $X_{\mathrm{cl}}(\theta)$. According to (Apkarian & Adams, 1998; Chilali & Gahinet, 1996), matrix $X_{\mathrm{cl}}(\theta)$ is now set as $\Pi_1(\theta)\Pi_2(\theta)^{-1}$ with $\Pi_1(\theta)=\begin{bmatrix}R(\theta) & I_n\\ M(\theta)^T & \mathbf{0}\end{bmatrix}$, $\Pi_2(\theta)=\begin{bmatrix}I_n & S(\theta)\\ \mathbf{0} & N(\theta)^T\end{bmatrix}$, where $S(\theta),R(\theta)\in\mathcal{S}^n$ are continuously differentiable parameter-dependent matrices to be determined and matrices $N(\theta)$ and $M(\theta)$ are arbitrary nonsingular matrices satisfying the factorization problem $I_n-S(\theta)R(\theta)=N(\theta)M(\theta)^T$.

Note that the solvability of the method in Apkarian and Adams (1998) does not depend on the factorization problem. Bearing this in mind, let us suppose that $N(\theta)$ and $M(\theta)$ are set as $-S(\theta)$ and $R(\theta)-S(\theta)^{-1}$ respectively. Obviously, these matrices satisfy the above factorization problem and do not change the solvability of the method in Apkarian and Adams (1998). Then, matrix $X_{\mathrm{cl}}(\theta)$ is derived as

$$X_{\mathrm{cl}}(\theta)=\begin{bmatrix}X(\theta) & Y(\theta)\\ Y(\theta) & Y(\theta)\end{bmatrix},\tag{10}$$

where $X(\theta)=R(\theta)$ and $Y(\theta)=R(\theta)-S(\theta)^{-1}$. Therefore, PDLFs can be set as (10) without loss of generality as long as $X_{\mathrm{cl}}(\theta)$ is set as $\Pi_1(\theta)\Pi_2(\theta)^{-1}$. (Details of the above can be also found in Sato, 2008.)

3.1. Proposed methods

We propose the following theorem for Problem 1.

Theorem 5. *For a given positive number γ_∞, suppose that there exist a positive number ε, continuously differentiable parameter-dependent matrices $X(\theta),Z(\theta)\in\mathcal{S}^n$, and parameter-dependent matrices*