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Brief paper

Lyapunov–Krasovskii functionals and application to input delay compensation for linear time-invariant systems☆

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ABSTRACT

For linear systems with pointwise or distributed delay in the inputs which are stabilized through the reduction approach, we propose a new technique of construction of Lyapunov–Krasovskii functionals. These functionals allow us to establish the ISS property of the closed-loop systems relative to additive disturbances.

1. Introduction

Controlling dynamical systems including delays in the inputs was a problem of recurring interest in the past fifty years since it frequently arises in control applications, due to the transport and measurement delays that naturally occur (for more details, see, e.g., Michiels and Niculescu (2007)).

A number of approaches to deal with input delays have been proposed in both frequency- and time-domains. Among them, for linear systems, two of the most celebrated are the *Smith predictor* and the *reduction* technique, also known as finite spectrum assignment (FSA). To the best of the authors' knowledge, the reduction approach originates in Mayne (1968), with the well known contributions that have followed in Kwon and Pearson (1980), Manitius and Olbrot (1979) and Olbrot (1978), which have been systematized and generalized in Artstein (1982), to which we refer the reader for a pedagogical exposition. This technique is popular and frequently used in practice for stabilizing linear systems with delay in the input, due to the fact that, under an appropriate transformation, the system reduces to a finite-dimensional one. However, the control applied to the original dynamics is complicated. Alternatives to the popular reduction approach include the general observer–predictor structure in Mirkin and Raskin (2003) and the H_∞ approach in Tadmor (2000).

The reduction approach applies to cases where the delays are too large for being neglected, as done for instance in Mazenc, Malisoff, and Lin (2008): the one-dimensional system

$$\dot{X}(t) = X(t) + U(t-\tau), \tag{1}$$

where U is the input, can be exponentially stabilized through the reduction approach for any constant delay $\tau \geq 0$ although, when τ is larger than a certain value, there is no continuous function φ such that the feedback $U(t-\tau) = \varphi(X(t-\tau))$ asymptotically stabilizes (1). Moreover, this technique applies to cases where the delays are either pointwise or distributed, and significantly simplifies stabilization problems for systems with delay by reducing them to similar problems for ordinary differential equations (see, for instance Fiagbedzi and Pearson (1986) and Wang, Lee, and Tan (1998) for further discussions).

Although Lyapunov functionals are tools whose importance is more and more recognized by the researchers who work in delay area (see for instance (Bekiaris-Liberis & Krstic, 2011), Karafyllis and Jiang (2011), Pepe and Verriest (2003) and Zhou, Lin, and Duan (2010)), strict Lyapunov–Krasovskii functionals for linear systems in closed-loop with feedbacks resulting from

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the reduction approach have been constructed only recently in Bekiaris-Liberis and Krstic (2011) using a novel approach which relies on the introduction of a hyperbolic PDE. This result is motivated by the important benefits which can be derived from the knowledge of a strict Lyapunov–Krasovskii functional. In particular, strict Lyapunov–Krasovskii functionals are frequently ISS or iISS Lyapunov–Krasovskii functionals as defined and discussed in Pepe and Jiang (2006) for systems with disturbances, which straightforwardly implies that the systems possess the desirable ISS or iISS property with respect to these disturbances (see Sontag (2007) for information on the celebrated ISS notion).

In the present work, we revisit the problem of constructing Lyapunov–Krasovskii functionals for two main families of closed-loop systems with additive disturbances: the first class is associated with the classical reduction approach and the second is employing dynamic feedback to overcome the instability that arises in some implementations of control laws specific to the classical reduction approach. The new construction we propose shares some features with the one of Mazenc and Niculescu (2011) which relies on the representation of a system with delay as an ordinary differential equation interconnected with an integral equation. However, the Lyapunov functionals we propose here are by no means straightforwardly deduced from Mazenc and Niculescu (2011). Indeed, by contrast with the feedbacks resulting from the reduction approach, the control laws considered in Mazenc and Niculescu (2011) do not have distributed terms. Furthermore, our ISS Lyapunov–Krasovskii functionals do not rely on the introduction of hyperbolic PDEs and therefore are significantly different from those proposed in Bekiaris-Liberis and Krstic (2011), Krstic (2008) and Krstic and Smyshlaev (2008).

The paper is organized as follows. In Section 2, a construction of functionals for a general family of systems is presented. From the latter result, Lyapunov–Krasovskii functionals for three families of systems stabilized via control laws provided by the reduction model approach are deduced in Section 3. Finally, some conclusions are drawn in Section 4.

Notation and definitions. • The notation will be simplified whenever no confusion can arise from the context. • For any integer p, we denote by Id_p or simply Id the identity matrix in $\mathbb{R}^{p\times p}$. • We let $|\cdot|$ denote the Euclidean norm of matrices and vectors of any dimension. • Given $\phi : \mathcal{I} \to \mathbb{R}^p$ defined on an interval $\mathcal{I}$, let $|\phi|_{\mathcal{I}}$ denote its (essential) supremum over $\mathcal{I}$. • For any integer p, we let $C_{in} = C([-\tau, 0], \mathbb{R}^p)$ denote the set of all continuous $\mathbb{R}^p$-valued functions defined on a given interval $[-\tau, 0]$. • For a function $x : [-\tau, +\infty) \to \mathbb{R}^k$, for all $t \geq 0$, the function x_t is defined by $x_t(\ell) = x(t+\ell)$ for all $\ell \in [-\tau, 0]$. • Let $\mathcal{K}_\infty$ denote the set of all continuous functions $\rho : [0,\infty) \to [0,\infty)$ for which (i) $\rho(0) = 0$ and (ii) ρ is strictly increasing and unbounded. • We adopt a definition of ISS Lyapunov–Krasovskii functional for coupled retarded functional differential equations and functional equations, which is an adaptation to this family of systems of the definitions given in Dashkovskiy and Naujok (2010) and Pepe, Karafyllis, and Jiang (2008).

Definition 1. We consider a system composed by a retarded functional differential equation coupled with a functional equation:

$$\begin{cases} \dot{x}_1(t) = f_1(x_{1t}, x_{2t}, u(t)), \\ x_2(t) = f_2(x_{1t}, x_{2t}), \\ (x_1(r), x_2(r)) = (x_{10}(r), x_{20}(r)), \quad \forall r \in [-\tau, 0], \end{cases} \tag{2}$$

where $t \in [0, +\infty)$, $x_1(t) \in \mathbb{R}^{N_1}$, $x_2(t) \in \mathbb{R}^{N_2}$, $u(t) \in \mathbb{R}^{N_3}$ is an essentially bounded measurable input and τ is the maximum involved delay and the functionals f_1 and f_2 are locally Lipschitz continuous on any bounded set such that all the solutions of (2) with initial function in C_{in} are defined and of class C^1 over $[0, +\infty)$.

A locally Lipschitz continuous functional $V : C_{in} \to [0, +\infty)$ is called an ISS Lyapunov–Krasovskii functional for (2) if (i) there are functions of class $\mathcal{K}_\infty$, α_1 and α_2 such that, for all functions $(\phi_1, \phi_2) \in C_{in}$ the inequalities

$$\alpha_1(|(\phi_1(0), \phi_2(0))|) \leq V(\phi_1, \phi_2) \leq \alpha_2(|(\phi_1, \phi_2)|_{[-\tau,0]}) \tag{3}$$

are satisfied,

(ii) it is continuously differentiable along the trajectories of (2) and satisfies:

$$\dot{V}(t) \leq -\alpha_3(V(x_{1t}, x_{2t})) + \alpha_4(|u(t)|), \quad \forall t \in [0, +\infty), \tag{4}$$

where α_3 and α_4 are functions of class $\mathcal{K}_\infty$.

2. Technical result

The result of this section is instrumental in establishing our main results. However, it is of interest for its own sake.

2.1. System and assumptions

We consider the system

$$\Sigma_{z,v} : \begin{cases} \dot{z}(t) = f(z(t)) + \delta(t), & \forall t \geq 0, \\ v(t) = Nz(t), & \forall t \geq 0, \end{cases} \tag{5}$$

with $z \in \mathbb{R}^n$, $v \in \mathbb{R}^m$, where the initial conditions $\phi_z \in C_{in}$ and $\phi_v \in C_{in}$ are such that $\phi_v(0) = N\phi_z(0)$, where $N \in \mathbb{R}^{m\times n}$ is a constant matrix, $N \neq 0$, where f is a function of class C^1 and where δ is a continuous function. Consider also the system

$$\Sigma_x : x(t) = g(x_t, z_t, v_t), \tag{6}$$

where g is a locally Lipschitz continuous functional.

We introduce two assumptions:

Assumption H1. There exists a function S of class C^1, positive definite and radially unbounded, a function κ_1 of class $\mathcal{K}_\infty$ and a positive real number a_1 such that

$$\frac{\partial S}{\partial z}(z)[f(z) + \delta] \leq -a_1 S(z) + \kappa_1(|\delta|), \tag{7}$$

for all $z \in \mathbb{R}^n$ and $\delta \in \mathbb{R}^n$.

Assumption H2. There exists a nonnegative function θ such that

$$\theta(Nz) \leq \frac{a_1}{2} S(z), \tag{8}$$

for all $z \in \mathbb{R}^n$, all the solutions $(z(t), x(t))$ of the system $\Sigma_{z,v} - \Sigma_x$ are defined and of class C^1 over $[0, +\infty)$ and are such that, for all $t \geq 0$, the inequality

$$\max\{|x(t)|, |\dot{x}(t)|\} \leq \sqrt{a_2\left(S(z(t)) + \overline{\theta}(v_t) + \kappa_2(|\delta(t)|)\right)}, \tag{9}$$

where a_2 is a positive real number, κ_2 is a function of class $\mathcal{K}_\infty$ and

$$\overline{\theta}(v_t) = \theta(v(t-\tau)) + \int_{t-\tau}^{t} \theta(v(m))dm, \tag{10}$$

is satisfied.

2.2. Discussion of the assumptions

1. The inequality (7) implies that the z-subsystem in (5) is ISS with respect to δ.

2. The z-subsystem is written as an ordinary differential equation, however, we regard it as a subsystem of the system with delay