Automatica 48 (2012) 1432–1439

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics[☆]

Hongwei Zhang[a], Frank L. Lewis[b,1]

[a] School of Electrical Engineering, Southwest Jiaotong University, Chengdu, Sichuan, 610031, PR China

[b] Automation and Robotics Research Institute, The University of Texas at Arlington, Fort Worth, TX 76118-7115, USA

ARTICLE INFO

Article history:
Received 30 November 2010
Received in revised form
22 July 2011
Accepted 12 December 2011
Available online 8 June 2012

Keywords:
Consensus
Cooperative control
Multi-agent system
Neural adaptive control
Nonlinear system
Synchronization

ABSTRACT

A practical design method is developed for cooperative tracking control of higher-order nonlinear systems with a dynamic leader. The communication network is a weighted directed graph with a fixed topology. Each follower node is modeled by a higher-order integrator incorporating with unknown nonlinear dynamics and an unknown disturbance. The leader node is modeled as a higher-order nonautonomous nonlinear system. It acts as a command generator giving commands only to a small portion of the networked group. A robust adaptive neural network controller is designed for each follower node such that all follower nodes ultimately synchronize to the leader node with bounded residual errors. Moreover, these controllers are distributed in the sense that the controller design for each follower node only requires relative state information between itself and its neighbors. A simulation example demonstrates the effectiveness of the algorithm.

1. Introduction

Research on networked cooperative systems (or multi-agent systems) has attracted much attention in the past two decades. Its widespread applications include spacecraft, mobile robots, sensor networks, etc. Some seminal works are Fax and Murray (2004), Jadbabaie, Lin, and Morse (2003), Olfati-Saber and Murray (2004), Ren and Beard (2005), and Tsitsiklis, Bertsekas, and Athans (1986), to name a few.

Considerable effort has focused on two control problems of networked systems, i.e., *cooperative regulation problem* and *cooperative tracking problem*. For cooperative regulation problem, controllers are designed to drive all the agents/nodes to a common value, i.e., consensus equilibrium, which is not prescribed and depends on initial conditions (Ren, Beard, & Atkins, 2007). This is also known as (leaderless) consensus in the literature. As for the cooperative tracking problem, there is a leader/control node that acts as a command generator, ignoring information from all the other nodes. The leader node only gives commands to a small portion of the networked group. All the follower nodes are trying to track the trajectory of the leader node. This is called leader–follower consensus, consensus with a virtual leader, or synchronization to a leader in the literature. Numerous results on these two topics have been published in the past few years and readers are referred to survey papers (Olfati-Saber, Fax, & Murray, 2007; Ren et al., 2007; Ren, Beard, & Atkins, 2005) and references therein.

This paper studies the cooperative tracking control of higher-order nonlinear systems. Our research is motivated by the following several observations. First, most existing works on multi-agent systems studied the first- and second-order systems. However, in engineering, many systems are modeled by higher-order dynamics. For example, a single link flexible joint manipulator is well modeled by a fourth-order nonlinear system (Khalil, 2002). The jerk (i.e., derivative of acceleration) systems, described by third-order differential equations, are of particular interest in mechanical engineering. Due to the challenges of designing cooperative controls for systems distributed on communication graphs, it is nontrivial to extend results for first- and second-order systems to systems with higher-order dynamics. Therefore, the literature in cooperative control has many papers dedicated to higher-order cooperative control, most of which deal with higher-order linear systems ((Jiang, Wang, & Jia, 2009; Ren, Moore, & Chen, 2007; Wang & Cheng, 2007; Zhang, Lewis, & Das, 2011) etc.).

☆ This work was supported by NSF grant ECCS-1128050, AFOSR grant FA9550-09-1-0278, and ARO grant W91NF-05-1-0314. The material in this paper was partially presented at the IEEE Conference on Decision and Control (CDC), December 15–17, 2010, Atlanta, Georgia, USA. This paper was recommended for publication in revised form by Associate Editor Shuzhi Sam Ge under the direction of Editor Miroslav Krstic.

E-mail addresses: hwzhang@swjtu.edu.cn (H. Zhang), lewis@uta.edu (F.L. Lewis).

[1] Tel.: +1 817 272 5972; fax: +1 817 272 5989.

0005-1098/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2012.05.008

Second, even for the first- and second-order problems with dynamics described by single or double integrators, cooperative control has not been fully investigated. However, almost all physical systems are inherently nonlinear, and cooperative control of nonlinear systems is more challenging. Pinning control was introduced for controlling synchronization of interconnected dynamic systems with identical nonlinear dynamics (Li, Wang, & Chen, 2004; Wang & Chen, 2002). But, in practice, the node dynamics may be non-identical or even unknown. Consensus of nonlinear systems was also studied recently in Lee and Ahn (2010) and Qu, Chunyu, and Wang (2007), where the nonlinear dynamics are assumed to be known precisely. Moreover, external disturbances such as white noise, often neglected by the current research, exist in almost every practical application. Most relevant to this paper are the works (Das & Lewis, 2010, 2011; Hou, Cheng, & Tan, 2009). Ref. (Hou et al., 2009) applied neural adaptive control to leaderless consensus problem of first-order nonlinear systems on undirected graphs. They also showed that the method can be extended to higher-order systems using the backstepping technique. As is well known, backstepping is a recursive design procedure whose complexity increases drastically with the order of the systems. In Das and Lewis (2010), cooperative tracking problems was solved for first-order nonlinear systems with unknown dynamics, and this result was generalized to second-order nonlinear systems in Das and Lewis (2011).

Motivated by the above observations, this paper deals with the cooperative tracking control problem of general higher-order nonlinear systems on directed graphs with a time-varying active leader, and thus further generalizes the results in Das and Lewis (2010, 2011). Each follower node is a higher-order integrator incorporating with unknown nonlinear dynamics and an unknown external disturbance. The node dynamics can all be different. The leader node is a higher-order nonautonomous nonlinear system whose dynamics is unknown to all the follower nodes. This paper proposes distributed neural adaptive controllers for networked higher-order systems, which guarantee the ultimate boundedness of the tracking errors.

Compared with Das and Lewis (2010, 2011), the main contributions of this paper are threefold. First, the node dynamics are extended to general higher-order nonlinear systems in the Brunovsky form, which include first- and second-order systems as special cases. Second, the requirement of graph topology is relaxed such that the augmented graph has a spanning tree. This means the original graph may be disconnected, as long as the leader node pins into the proper nodes in each disconnected component. This is a necessary condition and less stringent than strong connectedness. Finally, fewer assumptions are made. Thus the controller design is more flexible. It is worth mentioning that the above extensions are nontrivial, since the node dynamics get more involved with the graph topology. Also note that Lemma 2 in Das and Lewis (2011), which plays a fundamental role in the stability analysis, does not work for the higher-order case. This results in a Lyapunov stability analysis that is more complicated. In particular, two Lyapunov equations are used in this paper, i.e., one for graph topology and one for control design. This paper is also an improved work of our preliminary results (Zhang & Lewis, 2010; Zhang, Lewis, & Qu, 2012).

2. Basic graph theory and notations

A graph is expressed by $\mathcal{G} = (\mathcal{V}, E)$. $\mathcal{V} = \{v_1, \ldots, v_N\}$ is a nonempty set of nodes/agents and $E \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges/arcs. $(v_i, v_j) \in E$ means there is an edge from node i to node j. The topology of a weighted graph is often represented by the adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$, and $a_{ij} > 0$ if $(v_j, v_i) \in E$; otherwise $a_{ij} = 0$. Throughout this paper, it is assumed that $a_{ii} = 0$ and the topology is fixed, i.e., A is time-invariant. A digraph is a directed graph. Define $d_i = \sum_{j=1}^{N} a_{ij}$ as the weighted in-degree of node i and $D = \mathrm{diag}(d_1, \ldots, d_N) \in \mathbb{R}^{N\times N}$ as the in-degree matrix. The graph Laplacian matrix is $L = [l_{ij}] = D - A$. Let $\underline{1} = [1, \ldots, 1]^T$ with appropriate dimension; then $L\underline{1} = 0$. The set of neighbors of node i is denoted as $N_i = \{j|(v_j, v_i) \in E\}$. If node j is a neighbor of node i, then node i can get information from node j, not necessarily vice versa for directed graph. For undirected graph, neighbor is a mutual relation. A direct path from node i to node j is a sequence of successive edges in the form $\{(v_i, v_k), (v_k, v_l), \ldots, (v_m, v_j)\}$. A digraph has a spanning tree, if there is a node (called the root), such that there is a directed path from the root to every other node in the graph. A digraph is strongly connected, if for any ordered pair of nodes $[v_i, v_j]$ with $i \neq j$, there is a direct path from node i to node j.

Throughout this paper, the following notations are used. $|\cdot|$ is the absolute value of a real number; $\|\cdot\|$ is the Euclidean norm of a vector; $\|\cdot\|_F$ is the Frobenius norm of a matrix; $tr\{\cdot\}$ is the trace of a matrix; $\sigma(\cdot)$ is the set of singular values of a matrix, with the maximum singular value $\bar{\sigma}(\cdot)$ and the minimum singular value $\underline{\sigma}(\cdot)$; matrix $P > 0$ $(P \geq 0)$ means P is positive definite (positive semidefinite); I denotes the identity matrix with appropriate dimensions; and $\mathcal{N} = \{1, \ldots, N\}$.

3. Problem formulation

Consider $N (N \geq 2)$ agents with distinct dynamics. Dynamics of the ith $(i = 1, \ldots, N)$ agent is described in the Brunovsky form

$$\begin{aligned} \dot{x}_{i,m} &= x_{i,m+1}, \quad m = 1, \ldots, M-1 \\ \dot{x}_{i,m} &= f_i(x_i) + u_i + \zeta_i, \quad m = M \end{aligned} \tag{1}$$

where $x_{i,m} \in \mathbb{R}$ is the mth state of node i; $x_i = [x_{i,1}, \ldots, x_{i,M}]^T$ is the state vector of node i; $f_i(\cdot) : \mathbb{R}^M \to \mathbb{R}$ is locally Lipschitz in $\mathbb{R}^M$ with $f_i(0) = 0$, and it is assumed to be unknown; $u_i \in \mathbb{R}$ is the control input/protocol; and $\zeta_i \in \mathbb{R}$ is an external disturbance, which is unknown but bounded. Define $x^m = [x_{1,m}, \ldots, x_{N,m}]^T$. Then one has

$$\begin{aligned} \dot{x}^m &= x^{m+1}, \quad m = 1, \ldots, M-1 \\ \dot{x}^m &= f(x) + u + \zeta, \quad m = M \end{aligned}$$

where $f(x) = [f_1(x_1), \ldots, f_N(x_N)]^T$, $u = [u_1, \ldots, u_N]^T$ and $\zeta = [\zeta_1, \ldots, \zeta_N]^T$. Specifically, when $M = 3$, x^1, x^2 and x^3 can be the global position vector, global velocity vector and global acceleration vector, respectively.

The time-varying dynamics of the leader node, labeled 0, is described by

$$\begin{aligned} \dot{x}_{0,m} &= x_{0,m+1}, \quad m = 1, \ldots, M-1 \\ \dot{x}_{0,m} &= f_0(t, x_0), \quad m = M \end{aligned} \tag{2}$$

where $x_{0,m} \in \mathbb{R}$ is the mth state; $x_0 = [x_{0,1}, \ldots, x_{0,M}]^T$ is the state vector; and $f_0(t, x_0) : [0, \infty) \times \mathbb{R}^M \to \mathbb{R}$ is piecewise continuous in t and locally Lipschitz in x_0 with $f_0(t, 0) = 0$ for all $t \geq 0$ and $x_0 \in \mathbb{R}^M$, and it is unknown to all other nodes. System (2) is assumed to be forward complete, i.e., for every initial condition, the solution $x_0(t)$ exists for all $t \geq 0$. In other words, there is no finite escape time. The leader node dynamics (2) can be considered as an exosystem that generates a desired command trajectory.

Define the mth order tracking error (or disagreement variable) for node i $(i \in \mathcal{N})$ as $\delta_{i,m} = x_{i,m} - x_{0,m}$. Let $\delta^m = [\delta_{1,m}, \ldots, \delta_{N,m}]^T$; then $\delta^m = x^m - \underline{x}_{0,m}$, where $\underline{x}_{0,m} = [x_{0,m}, \ldots, x_{0,m}]^T \in \mathbb{R}^N$. The objective of this paper is to design distributed controllers for all follower nodes, such that the tracking error δ^m converges to a small neighborhoods of zero, for all $m = 1, \ldots, M$. This is illustrated by the following definition, which extends the standard concept of uniform ultimate boundedness (Khalil, 2002; Lewis, Yeşildirek, & Liu, 1996) to cooperative control systems.

Definition 1 (*Cooperative Uniform Ultimate Boundedness*). For any $m (m = 1, \ldots, M)$, the tracking error δ^m is said to be cooperatively