Automatica 48 (2012) 1447–1454

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Infinite horizon H_2/H_∞ control for discrete-time time-varying Markov jump systems with multiplicative noise[☆]

Hongji Ma[a,b], Weihai Zhang[b,1], Ting Hou[b]

[a] The Seventh Research Division, Beijing University of Aeronautics and Astronautics, Beijing, CO 100191, China

[b] College of Information and Electrical Engineering, Shandong University of Science and Technology, Qingdao, CO 266510, China

ARTICLE INFO

Article history:
Received 18 November 2010
Received in revised form
21 September 2011
Accepted 18 December 2011
Available online 8 June 2012

Keywords:
H_2/H_∞ control
Discrete-time systems
Time-varying systems
Markov parameters
Multiplicative noise
Riccati equations

ABSTRACT

In this paper we consider the infinite horizon H_2/H_∞ control problem for discrete-time time-varying linear systems subject to Markov jump parameters and state-multiplicative noise. A stochastic version of a bounded real lemma is firstly developed for a general class of discrete-time time-varying Markov jump systems with state- and disturbance-multiplicative noise. By which we present a necessary and sufficient condition for the solvability of the H_2/H_∞ control problem in terms of four coupled discrete-time Riccati equations. Moreover, the obtained design is applied to a macroeconomic problem to verify its effectiveness.

1. Introduction

Based on Zames' foundational work (Zames, 1981), Doyle, Glover, Khargonekar, and Francis (1989) showed that the H_∞ norm of a transfer function is equivalent to the $\mathcal{L}_2$-induced norm of the input–output operator with initial state zero. So far, H_∞ control has been one of the central issues in robust control literature; see Petersen, Ugrinovskii, and Savkin (2000), Xu and Lam (2006) and Zhou, Doyle, and Glover (1996). On the other hand, mixed H_2/H_∞ control for deterministic systems has also attracted considerable attention and is now widely applied to various practical fields; see Basar and Bernhar (1995) and Limebeer, Anderson, and Hendel (1994). As for the stochastic case, the research of stochastic H_∞ control can be traced back at least to Hinrichsen and Pritchard (1998) and Ugrinovskii (1998). In Hinrichsen and Pritchard (1998), an infinite horizon H_∞ control problem was tackled for a class of stochastic Itô systems with state- and control-multiplicative noise and a stochastic bounded real lemma (SBRL) was presented for the first time in the form of linear matrix inequalities (LMIs). Inspired by this work, Chen and Zhang (2004) addressed the stochastic H_2/H_∞ control problem about stochastic Itô systems with state-multiplicative noise and settled it for both the finite and infinite horizon case, which, to some extent, may be viewed as a stochastic counterpart of Limebeer et al. (1994). In the past two decades, the stochastic H_∞ and H_2/H_∞ control theories have been well developed for continuous- and discrete-time linear control systems with multiplicative noise. For the latest progress, a good introduction can be found in Gershon, Shaked, and Yaesh (2005), and interested readers are also referred to Dragan, Morozan, and Shi (2002), Gershon and Shaked (2008), Gershon, Shaked, and Berman (2007), Muradore and Picci (2005), and Zhang, Huang, and Xie (2008), among others. Via carefully examining the existing work, it can be found that although fruitful results have been contributed to infinite horizon H_2/H_∞ control design, most of them are focused on time-invariant systems.

Due to a great many applications of Markov jump systems, the relevant research has become a very active issue in the control community (Costa, Fragoso, & Marques, 2005; Fragoso & Rocha, 2006; Wang, Lam, & Liu, 2004). Currently, there exists a growing interest in studying stochastic linear systems subject to both independent random perturbations and Markov jumps; see the

☆ This work was supported by the National Natural Science Foundation of China (Nos. 60874032, 61174078), the Mathematical Tianyuan Youth Foundation of China (No. 11126094), the Key Project of Natural Science Foundation of Shandong Province (No. ZR2009GZ001), the Specialized Research Fund for the Doctoral Program of Higher Education (No. 20103718110006), and the Chunlei Project of SDUST (No. 2009AZZ074). The material in this paper was partially presented at the 18th IFAC World Congress, August 28–September 2, 2011, Milano, Italy. This paper was recommended for publication in revised form by Associate Editor Tongwen Chen under the direction of Editor Ian R. Petersen.

E-mail addresses: ma_math@163.com (H. Ma), w_hzhang@163.com (W. Zhang), ht_math@sina.com (T. Hou).

1 Tel.: +86 532 80691786; fax: +86 532 86057187.

0005-1098/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2012.05.006

discussions of the linear quadratic optimization problem (Costa & Wanderlei, 2007; Dragan & Morozan, 2004), eigenvalue sets and convergence rate (Li, Zhou, Wang, & Duan, 2011), robust stabilization and control problem (Dragan & Morozan, 2002; Xu & Chen, 2005). In a recent monograph (Dragan, Morozan, & Stoica, 2010), the infinite horizon robust H_∞ control problem has been elaborately discussed for a broad class of discrete-time Markov jump systems with multiplicative noise. However, to the best of our knowledge, the topic on infinite horizon H_2/H_∞ control remains unexplored for such dynamics. Compared with the sole H_∞ control, mixed H_2/H_∞ control may give consideration to both the average performance and disturbance attenuation index of the closed-loop system, and therefore appears more attractive in practice (Du, Xie, Teoh, & Guo, 2005).

In this paper, we will handle the infinite horizon stochastic H_2/H_∞ control problem, paralleling the work of Chen and Zhang (2004), for discrete-time time-varying linear systems with Markov jump parameters and state-multiplicative noise. Roughly speaking, time-varying systems can be utilized to model more realistic dynamics and are also more challenging in mathematics than time-invariant ones. Our first main result (Theorem 1) consists of a SBRL for a class of discrete-time time-varying Markov jump systems with state- and disturbance-multiplicative noise. As is well known (e.g., see Dragan et al., 2010; Gershon et al., 2005), the SBRL is a fundamental tool to investigate the H_∞ control and estimation problems for stochastic systems. The second main result (Theorem 2) is dedicated to providing a necessary and sufficient condition for the existence of an infinite horizon H_2/H_∞ controller by means of four coupled discrete-time Riccati equations (CDTREs). Since the system coefficients are allowed to be time-varying, the obtained design takes that of the time-invariant case as its special case. It is worth mentioning that a finite horizon H_2/H_∞ control problem has been treated for analogous systems with (x, u, v)-multiplicative noise (Hou, Zhang, & Ma, 2010). While in contrast to the finite horizon case, the infinite horizon H_2/H_∞ controller is more difficult to achieve due to the requirement of stabilizing the closed-loop system internally. Hence, this research is by no means a trivial extension of Hou et al. (2010) to the infinite horizon case. In addition, the efficiency of our proposed technique is illustrated through an example (Example 1) of a macroeconomic problem, where a comprehensive comparison is made among the optimal H_2, H_∞, and mixed H_2/H_∞ controllers.

The rest of this paper is organized as follows. Section 2 presents some adequate preliminaries and useful definitions. Section 3 is devoted to developing a SBRL for the discrete-time time-varying systems with state- and disturbance-multiplicative noise. Based on the obtained SBRL, Section 4 proceeds with the discussion of the infinite horizon H_2/H_∞ control problem under the condition of stochastic detectability. Finally, we end this paper in Section 5 with a brief concluding remark and some interesting topics that remain open.

Notations. R^n: n-dimensional space with the usual Euclidean norm $\|\cdot\|$; $R^{n\times m}$: the space of all $n \times m$ real matrices with the operator norm $\|\cdot\|_2$; S^n: the set of all $n \times n$ symmetric matrices; $A > 0 (\geq 0)$: A is a positive (semi-positive) definite matrix; $R^N_{n\times m}(S^N_n)$: the set of all N sequences $V = (V_1, \ldots, V_N)$ with $V_i \in R^{n\times m}(S^n)$; S^{N+}_n: the set of all N sequences $V = (V_1, \ldots, V_N)$ where $V_i \in S^n$ and $V_i \geq 0$; A': the transpose of a matrix (vector) A; I_n: the $n \times n$ identity matrix; $\mathbb{Z}_+ := \{0, 1, \ldots\}$ and $\mathbb{Z}_{1+} := \{1, 2, \ldots\}$; $D = \{1, 2, \ldots, N\}$; $\delta(\cdot)$: the Kronecker function.

2. Definitions and preliminaries

On a given probabilistic space $(\Omega, \mathscr{F}, P)$, we consider the following discrete-time time-varying linear systems subject to Markov jump parameters and multiplicative noises:

$$\begin{cases} x(t+1) = A_0(t, \eta_t)x(t) + G_0(t, \eta_t)u(t) \\ \qquad + \sum_{k=1}^{r} [A_k(t, \eta_t)x(t) + G_k(t, \eta_t)u(t)]w_k(t), \\ z(t) = C(t, \eta_t)x(t), \qquad x(0) = x_0 \in R^n, \quad t \in \mathbb{Z}_+, \end{cases} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^{n_u}$ and $z(t) \in R^{n_z}$ are the system state, control input and system output, respectively. $\{w(t)|w(t) = (w_1(t), \ldots, w_r(t))', t \in \mathbb{Z}_+\}$ is a sequence of independent random vectors, and the triple $(\{\eta_t\}_{t\in\mathbb{Z}_+}, P_t, D)$ is a time-varying Markov chain with the transition probability matrix denoted by $P_t = (p_t(i,j))_{N\times N}$, in which $p_t(i,j) = P(\eta_{t+1} = j|\eta_t = i)$. It is unknown *a priori* when jumps will occur but the current mode of η_t is measurable. Let $\tilde{H}_k$ be a σ-algebra defined by $\sigma\{\eta_t, w(s)|0 \leq t \leq k, 0 \leq s \leq k-1\}$. In the case $k = 0$, we set $\tilde{H}_0 = \sigma\{\eta_0\}$. $l^2(0, \infty; R^m)$ represents the space of R^m-valued, nonanticipative square summable stochastic processes $\{y(t, \omega) : \mathbb{Z}_+ \times \Omega \to R^m\}$, which are $\tilde{H}_k$-measurable for all $k \in \mathbb{Z}_+$ and $\sum_{t=0}^{\infty} E\|y(t)\|^2 < +\infty$. It is clear that $l^2(0, \infty; R^m)$ is a real Hilbert space with the norm induced by the usual inner product: $\|y(\cdot)\|_{l^2(0,\infty;R^m)} = \left(\sum_{t=0}^{\infty} E\|y(t)\|^2\right)^{\frac{1}{2}} < +\infty$. For any $T \in \mathbb{Z}_{1+}$, $l^2(0, T; R^m)$ may be defined in a similar way.

Throughout this paper, we make four underlying assumptions.

Hypothesis 1. (A1) For each $t \in \mathbb{Z}_+$, the two σ-algebras $\sigma\{w(0), \ldots, w(t)\}$ and $\sigma\{\eta_0, \ldots, \eta_t\}$ are mutually independent;

(A2) $E[w(t)] = 0$, $E[w(t)w(s)'] = I_r\delta(t-s)$, $t, s \in \mathbb{Z}_+$;

(A3) The transition probability matrix P_t is nondegenerate and $\inf_{t\in\mathbb{Z}_+} \pi_t(i) > 0$ where $\pi_t(i) := P(\eta_t = i)$, $i \in D$;

(A4) All the coefficients of the considered systems are bounded matrix-valued sequences with appropriate dimensions.

Firstly, we state the definitions of stabilization and detectability (Dragan et al., 2010) that are essential in the subsequent discussions.

Definition 1. Under Hypothesis 1, the zero state equilibrium of discrete-time linear stochastic systems:

$$x(t+1) = A_0(t, \eta_t)x(t) + \sum_{k=1}^{r} A_k(t, \eta_t)x(t)w_k(t) \tag{2}$$

or $(A_0, \mathbb{A}, \mathbb{P})(\mathbb{A} := (A_1, \ldots, A_r),\ \mathbb{P} := \{P_t\}_{t\in\mathbb{Z}_+})$ is called strongly exponentially stable in the mean square (SESMS) if there exist $\beta \geq 1, q \in (0, 1)$ such that for any sequence of independent random vectors $\{w(t)\}_{t\in\mathbb{Z}_+}$ and any Markov chain satisfying (A1) and (A2), there holds that for all $t \geq s \geq 0$, $i \in D$, $x_0 \in R^n$,

$$E[\|\Phi(t,s)x_0\|^2|\eta_s = i] \leq \beta q^{t-s}\|x_0\|^2,$$

where $\Phi(t, s)$ is the fundamental matrix solution of (2). Further, system (1) is stochastically stabilizable if there exists a bounded sequence $\{K(t, i)\}_{t\in\mathbb{Z}_+} \in R^{n\times n_u} (i \in D)$ such that the zero state equilibrium of the closed-loop system:

$$x(t+1) = [A_0(t, \eta_t)x(t) + G_0(t, \eta_t)K(t, \eta_t)]x(t) + \sum_{k=1}^{r} [A_k(t, \eta_t)x(t) + G_k(t, \eta_t)K(t, \eta_t)]x(t)w_k(t) \tag{3}$$

is SESMS for any $(x_0, \eta_0) \in R^n \times D$, where $u(t) = K(t, \eta_t)x(t)$ is called a stabilizing feedback.