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Stabilization of networked control systems with multirate sampling☆

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ABSTRACT

In this paper, we study the stabilization of networked control systems with multirate sampling. The input channels are modeled in two different ways. First, each of them is modeled as the cascade of a downsampling circuit, an ideal transmission system together with an additive norm bounded uncertainty, and a discrete zero-order hold. Then each input channel is modeled as the cascade of a downsampling circuit, an ideal transmission system together with a feedback norm bounded uncertainty, and a discrete zero-order hold. For each channel model, different downsampling rates are allowed in different input channels. The minimum total channel capacity required for stabilization is investigated. The key idea of our approach is the channel resource allocation, i.e., given the total capacity of the communication network, we do have the freedom to allocate the capacities among different input channels. With this new idea, we successfully show that the multirate networked control system with each channel model can be stabilized by state feedback under an appropriate resource allocation, if and only if the total network capacity is greater than the topological entropy of the plant. We also apply the result to multirate quantized control systems. Both the commonly used logarithmic quantizer and the alternative logarithmic quantizer are considered. For each case, a sufficient condition for stabilization is obtained which involves a trade-off between the densities of time quantization and spatial quantization.

1. Introduction

Arising from the cross-pollination of control, network and information theories, the networked control systems (NCSs) have attracted great attention nowadays. They are control systems wherein the feedback loop is closed over a communication network. Applications of NCSs have been found in more and more areas. Examples include mobile sensor networks (Ogren, Fiorelli, & Leonard, 2004), multi-agent systems (Moreau, 2005) and automated highway systems (Seiler & Sengupta, 2001). In special issues, Antsaklis and Baillieul (2004, 2007), and many survey papers, e.g., Goodwin, Quevedo, and Silva (2006); Goodwin, Silva, and Quevedo (2010) and Nair, Fagnani, Zampieri, and Evans (2007), much information of the current status of NCSs research has been presented.

In the NCSs, different kinds of information constraints and uncertainties appear due to the imperfect communication networks, such as quantization (Elia & Mitter, 2001; Fu & Xie, 2005), packet drop (Elia, 2005; Sinopoli et al., 2004; Xiao, Xie, & Qiu, 2012), limited data rate (Matveev & Savkin, 2005; Nair & Evans, 2003), delay (Nilsson, Bernhardsson, & Wittenmark, 1998; Zhang, Branicky, & Phillips, 2001), *etc.* Numerous results have been reported in the literature addressing the stabilization of NCSs under these constraints and uncertainties. For discrete-time single-input NCSs, Fu and Xie (2005) considers logarithmic quantization of the control inputs as a sector uncertainty. It is shown that the largest uncertainty bound which renders stabilization possible is given in terms of the Mahler measure of the system, i.e., the absolute product of the unstable poles. Elia (2005) studies the NCS with a multiplicative stochastic input channel. It is shown that the NCS can be mean-square stabilized by state feedback, if and only if the mean-square channel capacity exceeds the topological entropy of the plant which is the logarithm of the Mahler measure. The networked stabilization over additive white Gaussian noise (AWGN) channel is studied in Braslavsky, Middleton, and Freudenberg (2007), where the minimum channel capacity rendering stabilization possible for the single-input case is given again in terms of the topological entropy of the plant.

For discrete-time multi-input NCSs, Qiu, Gu, and Chen (2013) models each input channel in three different ways, i.e., the signal-to-error ratio (SER) model, the received signal-to-error ratio

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(R-SER) model and the AWGN channel model. The main contribution there is in the introduction of the channel resource allocation to solve the networked stabilization problem. It is assumed that the information constraints in the input channels are determined by the total network recourse available to the channels which can be allocated by the controller designer. This additional design freedom gives rise to channel/controller co-design, under which a uniform analytic solution is obtained for the minimum total channel capacity required for stabilization with each channel model given again in terms of the topological entropy of the plant. Xiao et al. (2012) studies the multi-input NCSs with multiplicative stochastic input channels. The authors generalize the stabilization condition for the single-input case (Elia, 2005) to the multi-input case by applying the channel resource allocation.

Researchers have also devoted much effort to the continuous-time networked stabilization. Xiao and Xie (2010) studies stabilization of a continuous-time LTI system over multiplicative stochastic input channels with channel resource allocation leading to the minimum total capacity required for stabilization also given by the topological entropy of the plant. A distributed control system is investigated in Brockett (1995) where a central controller communicates sequentially with the subsystems through one shared communication network under some periodic communication pattern. Both the communication pattern and the control law are to be designed, leading to channel/controller co-design for periodic multirate linear systems. Another work involving multirate operations in NCSs can be seen in Ishii and Hara (2008), where a subband coding scheme is proposed to efficiently use the available bit rates and to account for message losses. The trade-off between the required densities of time quantization and spatial quantization for stabilization of NCSs has also been studied in the literature, which is closely related to our work in this paper. For the single-input case, Elia and Mitter (2001) considers the situation of uniform sampling and infinite-level logarithmic spatial quantization. There a trade-off between the densities is obtained in terms of the Mahler measure. In the case when a finite-level spatial quantizer is used, the trade-off is studied in Li and Baillieul (2004, 2007). There it is concluded that the minimum data rate for stabilization could only be achieved by binary control. Unfortunately, so far, no efficient result has been reported on the trade-off for the multi-input case.

Inspired by the existing results discussed above, we in this paper study stabilization of continuous-time NCSs with multirate sampling. Partial results of this study have been reported in Chen and Qiu (2011). In this work, two different channel models are adopted. The first one is the cascade of a downsampling circuit, an ideal transmission system together with an additive norm bounded uncertainty, and a discrete zero-order hold. Although this model is motivated from the logarithmic quantizer studied in Elia and Mitter (2001) and Fu and Xie (2005), it also has the capability to address other network features. The second model is the cascade of a downsampling circuit, an ideal transmission system together with a feedback norm bounded uncertainty, and a discrete zero-order hold. This model is motivated from an alternative logarithmic quantizer. Each channel model consists of three components with the second component inherited from the SER model or the R-SER model proposed in Qiu et al. (2013). Different from the work of Qiu et al. (2013) which focuses on discrete-time NCSs, in this paper, we start with a continuous-time multi-input system. The additional downsampling and hold components in the input channels enable multirate sampling leading to a multirate NCS. The main novelty of this work is to investigate the minimum total channel capacity required for stabilization with channel resource allocation, i.e., the capacities can be allocated among different input channels. A lifting technique is employed to transform the multirate system to an equivalent LTI system. We show that for each channel

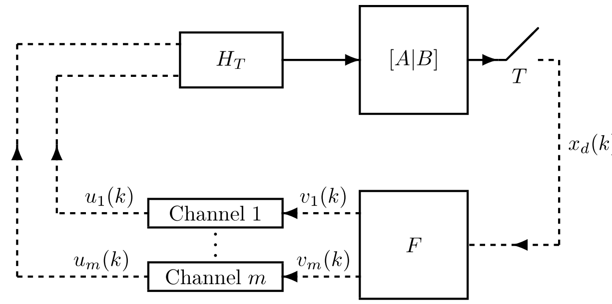

Fig. 1. A multirate NCS.

model, the multirate NCS could be stabilized by state feedback under an appropriate resource allocation, if and only if the total network capacity is greater than the topological entropy of the plant. We further apply this result to multirate quantized control systems. Both the commonly used logarithmic quantizer and the alternative logarithmic quantizer are considered. For each case, a sufficient condition for stabilization is obtained which shows a trade-off between the densities of time quantization and spatial quantization.

Note that the idea of channel resource allocation was first proposed in the conference paper, Gu and Qiu (2008), to study the stabilization of multi-input NCSs and then extended in Qiu et al. (2013). Following this idea, several other works have been carried out, e.g., Chen and Qiu (2011), Chen, Zheng, and Qiu (2012) Xiao and Xie (2010), Xiao et al. (2012) and Zheng, Chen, Shi, and Qiu (2012).

The remainder of this paper is organized as follows. The multirate NCS is formulated in Section 2. Some preliminary knowledge on multirate systems and lifting technique are presented in Section 3. The main result on minimum capacity required for stabilization is stated and proved in Section 4. Section 5 applies the result to the trade-off between the densities of time quantization and spatial quantization. Section 6 gives an illustrative example. Finally, some conclusion remarks follow in Section 7. The notations in this paper are more or less standard and will be made clear as we proceed.

2. Problem formulation

The setup of a multirate NCS studied in this paper is shown in Fig. 1. We use solid lines for continuous-time signals and dotted lines for discrete-time signals. The plant is a continuous-time LTI system with state space realization $[A|B]$:

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad x(0) = x_0,$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$. The sampled states $x_d(k) = x(kT)$ are available for feedback with sampling interval T. Assume that all hold and sampling circuits are synchronized at time 0. The control signal $v(k)$ generated by a static state feedback gain F is transmitted through a multirate communication network before reaching the plant. In many practical applications, the actuators are located separately from each other and from the controller. To fit this case, a parallel transmission strategy is adopted, i.e., each element $v_i(k)$ of the control signal is separately sent through an independent communication channel. The received control signal is finally converted to a continuous-time signal by a zero-order hold with period T.

In this paper, the communication channels are modeled in two different ways. The first model, depicted in Fig. 2, is the cascade of a downsampling circuit, an ideal transmission system with a unity transfer function together with an additive norm bounded uncertainty, and a discrete zero-order hold. The uncertainty $\mathbf{\Delta}_i$ can be a nonlinear, time-varying and dynamic system. The only