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Brief paper

Distributed adaptive coordination for multiple Lagrangian systems under a directed graph without using neighbors' velocity information ☆

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ABSTRACT

In this paper, we study the distributed coordination problem for multiple Lagrangian systems in the presence of parametric uncertainties under a directed graph without using neighbors' velocity information in the absence of communication. We consider two cases, namely, the distributed containment control problem with multiple stationary leaders and the leaderless synchronization problem. In both cases, distributed adaptive control algorithms without using neighbors' velocity information are proposed. The control gains in the algorithms are varying with distributed updating laws. Furthermore, necessary and sufficient conditions on the directed graph are presented, respectively, such that all followers converge to the stationary convex hull spanned by the stationary leaders asymptotically in the containment control problem and the systems synchronize asymptotically in the leaderless synchronization problem. Finally, simulation examples are provided to show the effectiveness of the proposed control algorithms.

1. Introduction

Due to the broad applications in sensor networks, unmanned aerial vehicles, and multiple autonomous robots, cooperative control of multi-agent systems has attracted a considerable amount of attention in recent years. One of the major research foci in the study of multi-agent systems is the leaderless consensus/synchronization problem, in which the agents update their own states by the interactive information from their neighbors, so that they achieve a common value (see Olfati-Saber, Fax, and Murray (2007) and Ren, Beard, and Atkins (2007) and the references therein). In addition, the leader-following problem has also been widely studied (Cao & Ren, 2012; Cao, Stuart, Ren, & Meng, 2011; Hong, Chen, & Bushnell, 2008; Hong, Hu, & Gao, 2006; Ji, Ferrari-Trecate, Egerstedt, & Buffa, 2008; Lou & Hong, 2010; Notarstefano, Egerstedt, & Haque, 2011; Peng & Yang, 2009; Ren, 2010). In this latter problem, two issues are most noteworthy: the coordinated tracking problem with one single leader (Cao & Ren, 2012; Hong et al., 2008, 2006; Peng & Yang, 2009; Ren, 2010), and the containment control problem with multiple leaders (Cao et al., 2011; Ji et al., 2008; Lou & Hong, 2010; Notarstefano et al., 2011). It is notable that the above literature focuses on linear systems with single-integrator dynamics (Cao & Ren, 2012; Hong et al., 2006; Ji et al., 2008; Lou & Hong, 2010; Notarstefano et al., 2011; Peng & Yang, 2009; Ren, 2010) and double-integrator dynamics (Cao & Ren, 2012; Cao et al., 2011; Hong et al., 2008).

Motivated by the fact that Lagrangian systems can be used to represent a large class of mechanical systems, including autonomous vehicles, walking robots and robotic manipulators to name a few, in this paper, we study the distributed coordination problem for multiple Lagrangian systems. Since Lagrangian systems represent nonlinear dynamics, our present work differs from those alluded to above, and the considerable results available therein cease to be applicable. On the other hand, similar to coordination for linear multi-agent systems, recent work on distributed

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coordination for multiple Lagrangian systems also focuses on the leaderless consensus/synchronization problem (Cheng, Hou, & Tan, 2008; Nuno, Ortega, Basanez, & Hill, 2011; Ren, 2009), the coordinated tracking problem with one single leader (Cheah, Hou, & Slotine, 2009; Chung & Slotine, 2009; Hokayem, Stipanovic, & Spong, 2009; Liu & Chopra, 2010; Mei, Ren, & Ma, 2011; Nuno et al., 2011; Spong & Chopra, 2007; Sun, Zhao, & Feng, 2007), and the containment control problem with multiple leaders (Mei, Ren, & Ma, 2012; Meng, Ren, & You, 2010). In Cheng et al. (2008) and Ren (2009), the authors studied the leaderless consensus/synchronization problem for multiple Lagrangian systems under an undirected graph. Control algorithms were proposed in Ren (2009) to cope with actuator saturation and unavailability of velocity measurements. Alternatively, parametric uncertainties were addressed in Cheng et al. (2008). In Nuno et al. (2011), the authors studied the leaderless synchronization problem and the coordinated tracking problem under a directed graph containing a directed spanning tree, wherein both parametric uncertainties and communication delays were considered. The coordinated tracking problem with one single leader is studied in Chung and Slotine (2009) and Sun et al. (2007) under an undirected ring graph, in Cheah et al. (2009) under an general undirected graph, in Spong and Chopra (2007) under a strongly connected and balanced directed graph, in Liu and Chopra (2010) under a strongly connected directed graph, and in Nuno et al. (2011) under a directed graph containing a directed spanning tree. A common assumption in the above works, however, is that all the followers have access to the leader. This assumption is rather restrictive and indeed, unrealistic from a practical standpoint. Under the constraint that the leader is a neighbor of only a subset of the followers, the coordinated tracking problem is studied in Hokayem et al. (2009) with one single leader subject to communication delays and limited data rates. This problem can be viewed as a coordinated tracking problem with a stationary leader in the absence of network effects. In the authors' prior work (Mei et al., 2011), the distributed coordinated tracking problem for multiple Lagrangian systems with a dynamic leader is studied under the constraints that only a subset of followers has access to the leader. Furthermore, while the followers have local interaction, and no acceleration measurements are available. For the containment control problem with multiple leaders, distributed finite-time containment control algorithms were proposed in Meng et al. (2010) for multiple Lagrangian systems with multiple stationary and dynamic leaders. This was done under the assumption that the interaction graph associated with the followers is undirected. The case of a directed interaction graph is studied in Mei et al. (2012).

In practice, for Lagrangian systems, relative velocity measurements between neighbors are generally more difficult to obtain than relative position measurements. Even if each system can measure its absolute velocity, to communicate the velocity measurements between neighbors will require the systems to be equipped with the communication capability and raise the communication burden. But in some applications, it might not be realistic to have communication among the systems. Unfortunately, to the best of our knowledge, all the work in the distributed coordination for multiple Lagrangian systems requires neighbors' velocity information except (Ren, 2009), where a passivity-based estimator is proposed to solve the leaderless consensus problem. However, the approach in Ren (2009) relies on the assumptions that the graph is undirected and there do not exist parametric uncertainties. In this paper, we study the distributed coordination for multiple Lagrangian systems under three challenges: (i) directed graphs; (ii) parametric uncertainties; and (iii) absence of neighbors' velocity information and absence of communication. These challenges make the problem more difficult to tackle. Specifically, we extend our prior work (Mei et al., 2012) to address both the distributed containment control problem with multiple stationary leaders and the leaderless synchronization problem in the presence of parametric uncertainties under a directed graph without using neighbors' velocity information in the absence of communication. In our proposed algorithms, only the relative position measurements between the neighbors and the absolute velocity measurements are required. These measurements can be obtained by the sensing devices carried by the agents and hence the need for communication is removed.

Notations. Let $\mathbf{1}_m$ and $\mathbf{0}_m$ denote, respectively, the $m \times 1$ column vector of all ones and all zeros. Let $\mathbf{0}_{m\times n}$ denote the $m \times n$ matrix with all zeros and I_m denote the $m \times m$ identity matrix. Let $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote, respectively, the maximal and minimum eigenvalue of a square real matrix with real eigenvalues. Let $\sigma_{\max}(\cdot)$ denote the maximal singular value of a matrix. Let $\mathrm{diag}(z_1, \ldots, z_p)$ be the diagonal matrix with diagonal entries z_1 to z_p. For symmetric square real matrices A and B with the same order, $A > B$ ($A \geq B$) means that $A - B$ is symmetric positive definite (semidefinite). For a point x and a set M, let $d(x, M) \triangleq \inf_{y\in M} \|x-y\|$ denote the distance between x and M. For a vector function $f(t) : \mathbb{R} \mapsto \mathbb{R}^n$, we say that $f(t) \in \mathbb{L}_2$ if $\int_0^\infty f(\tau)^T f(\tau) d\tau < \infty$ and $f(t) \in \mathbb{L}_\infty$ if for each element of $f(t)$, denoted as $f_i(t)$, $\sup_t |f_i(t)| < \infty$, $i = 1, \ldots, n$. Throughout the paper, we use $\|\cdot\|$ to denote the Euclidean norm.

2. Background

Suppose that there exist m followers, labeled as agents 1 to m, and $n - m$ ($n \geq m, n \geq 2$) leaders, labeled as agents $m + 1$ to n, in a team. The m followers are represented by Euler–Lagrange equations of the form

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \quad i = 1, \ldots, m, \tag{1}$$

where $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{p\times p}$ is the symmetric positive-definite inertia matrix, $C_i(q_i, \dot{q}_i)\dot{q}_i \in \mathbb{R}^p$ is the vector of Coriolis and centrifugal torques, $g_i(q_i)$ is the vector of gravitational torque, and $\tau_i \in \mathbb{R}^p$ is the vector of control torque on the ith agent. We assume that the leaders' motions are independent of those of the followers.

Throughout the subsequent analysis we assume that the following assumptions hold (Kelly, Santibanez, & Loria, 2005; Spong, Hutchinson, & Vidyasagar, 2006):

- (A1) For any i, there exist positive constants $k_{\underline{m}}$ and $k_{\overline{m}}$ such that $0 < k_{\underline{m}} I_p \leq M_i(q_i) \leq k_{\overline{m}} I_p$.
- (A2) $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric.
- (A3) $M_i(q_i)x + C_i(q_i, \dot{q}_i)y + g_i(q_i) = Y_i(q_i, \dot{q}_i, x, y)\Theta_i$ for all vectors $x, y \in \mathbb{R}^p$, where $Y_i(q_i, \dot{q}_i, x, y)$ is the regressor and Θ_i is the constant parameter vector associated with the ith agent.

We use a directed graph to describe the network topology between the n agents. Let $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$ be a directed graph with the node set $\mathcal{V} \triangleq \{1, \ldots, n\}$ and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An edge $(i, j) \in \mathcal{E}$ denotes that agent j can obtain information from agent i, but not vice versa. Here, node i is the parent node while node j is the child node. Equivalently, node i is a neighbor of node j. A directed path from node i to node j is a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \ldots,$ in a directed graph. A directed graph is strongly connected if there exists a directed path between any two distinct nodes. A directed tree is a directed graph, where every node has exactly one parent except for one node, called the root, and the root has directed paths to every other node. A directed spanning tree of a directed graph is a direct tree that contains all nodes of the directed graph. A directed graph contains a spanning tree if there exists a directed spanning tree as a subset of the directed graph.

The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\mathcal{G}$ is defined as $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. In this paper, self edges are not allowed, i.e., $a_{ii} = 0$. The (nonsymmetric) Laplacian matrix $\mathcal{L}_A = [l_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\mathcal{A}$ and hence $\mathcal{G}$ is defined as $l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$.