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Brief paper

Adaptive sliding mode control for stochastic Markovian jumping systems with actuator degradation☆

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ABSTRACT

This paper investigates the problem of sliding mode control for stochastic Markovian jumping systems, in which there may happen actuator degradation. By on-line estimating the loss of effectiveness of actuators, an adaptive sliding mode controller is designed such that the effect of the actuator degradation can be effectively attenuated. Besides, both the reachability of the specified sliding surfaces and the stability of sliding mode dynamics are ensured despite the actuator degradation and Markovian jumping. Finally, theoretical results are supported by numerical simulations.

1. Introduction

Markovian jumping systems (MJSs) have recently received considerable attention, since they may effectively represent a class of plants with abrupt variations in their structures, due to random failures of components, sudden environmental disturbances, changing subsystem interconnections and abrupt variations in the operating point of a nonlinear plant. Both the analysis and synthesis for MJSs have been extensively studied, see Liu, Ho, and Niu (2009), Ma and Boukas (2011), Shu, Lam, and Xiong (2010), Wu, Xie, Shi, and Xia (2009), Xu and Chen (2005), Zhang and Boukas (2009) and Zhang, Chen, and Tseng (2005) and the references therein.

More recently, the application of sliding mode control (SMC) is also extended to MJSs in Ma and Boukas (2009), Niu, Ho, and Wang (2007), Shi, Xia, Liu, and Rees (2006) and Wu and Ho (2010). SMC is an effective robust control approach for uncertain systems, whose main feature is its insensitiveness to variations of system parameters and external disturbances. In Shi et al. (2006), the design of SMC was investigated for a class of linear systems with Markovian jumping parameters. By the state transformation, a set of linear sliding surfaces corresponding to every mode was constructed, and both the reachability and the stability for every sliding mode dynamics were analyzed. Niu et al. (2007) further considered the problem of SMC for Itô stochastic systems with Markovian switching, and these sliding surfaces involving in the connections among various modes were constructed such that the reachability of sliding surfaces can be successionally ensured when the system mode changes from one to another. The aforementioned works have shown the effectiveness of SMC method for MJSs. Nevertheless, it is worthy of noting that, the aforementioned works were made under the assumption that the actuator or sensor worked normally, i.e., there did not exist the degradation of actuator or sensor, which may apparently result in limited application fields.

As is well-known, the actuator degradation in actual physical systems is usually inevitable, and often yields performance degradation or even instability. Therefore, how to maintain an acceptable stability/performance for the closed-loop systems against actuator or sensor failures has been a long-standing and active research topic (Veillette, Medanic, & Perkins, 1992). In the past decades, various reliable control methodologies have been proposed, e.g., linear–quadratic state-feedback control (Veillette, 1995), pre-compensator (Zhao & Jiang, 1998), H_∞ disturbance attenuation (Yang, Wang, & Soh, 2001), fuzzy logic method (Wu & Zhang, 2007), variable structure scheme (Niu & Wang, 2009), and so on. Recently, a reliable H_∞ controller with *adaptive* mechanism was proposed by Yang and Ye (2010). Based on the on-line estimation of eventual faults, the parameters of the reliable controller in

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Yang and Ye (2010) were updated automatically to compensate the fault effects on the system performance. Zuo, Ho, and Wang (2010) further extended the above idea to the singular systems. The above works have shown that the on-line estimating method is effective for the systems subject to actuator degradation.

However, to the author's best knowledge, the problem of SMC for stochastic MJSs with *actuator degradation* has not been well addressed and remains open. Moreover, these existing works cannot be simply extended to the present case with Markovian jumping and actuator degradation. These motivate the present study.

In this work, the problem of SMC has been investigated for a class of Itô stochastic MJSs subject to *actuator degradation*. Firstly, the model of *actuator degradation* is presented, in which only the bounds of *actuator degradation* are known. And then, the on-line estimation will be made for the loss of effectiveness of actuators. Due to the characteristic of MJSs, one has to consider how to establish the connections among sliding surfaces corresponding to each mode. To this end, some specified matrices are employed in the design of sliding surfaces such that the necessary connections among various sliding surfaces are ensured. And then, an *adaptive* sliding mode controller is synthesized, which cannot only ensure the reachability of the specified sliding surfaces, but also effectively compensate the effect of actuator faults on the system performance by adaptively updating the controller's parameters. Finally, the sufficient conditions for the stability of the closed-loop systems are derived. It is shown from simulation results that, the effect of both *Markovian switching* and *actuator degradation* can be effectively attenuated by the present SMC method.

Notations: $\|\cdot\|$ and $\|\cdot\|_1$ denote, respectively, the Euclidean norm and 1-norm of a vector (sum of absolute values) or its induced matrix norm. For a real matrix, $M > 0$ means that M is symmetric and positive definite, and I is used to represent an identity matrix of appropriate dimensions. $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space with Ω the sample space, and $\mathcal{F}$ the σ-algebra of subsets of the sample space, and $\mathcal{P}$ is the probability measure. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation and preliminaries

Let $\{r_t, t \geq 0\}$ be a right-continuous Markov process on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ which takes values in a finite state space $\mathcal{S} = \{1, 2, \ldots, N\}$ with generator $\Pi = (\lambda_{ij})$ $(i, j \in \mathcal{S})$ given by:

$$\mathcal{P}\{r_{t+\Delta} = j | r_t = i\} = \begin{cases} \lambda_{ij}\Delta + o(\Delta), & i \neq j, \\ 1 + \lambda_{ii}\Delta + o(\Delta), & i = j, \end{cases} \quad (1)$$

where $\Delta > 0$ and $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$, $\lambda_{ij} > 0$ (for $i \neq j$) is the transition rate from mode i to j with $\lambda_{ii} = -\sum_{j\neq i}\lambda_{ij}$ $(i, j \in \mathcal{S})$.

Consider the following stochastic MJSs of the Itô form:

$$\begin{aligned} dx(t) &= [(A(r_t) + \Delta A(r_t))\, x(t) + B(r_t)u(t)]\, dt \\ &\quad + D(r_t)g(t, x(t), r_t)dw(t), \end{aligned} \quad (2)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $w(t)$ is a 1-dimensional Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$.

For each $r_t = i \in \mathcal{S}$, let $A(r_t) = A_i$, $\Delta A(r_t) = \Delta A_i(t)$, $B(r_t) = B_i$, $D(r_t) = D_i$, and $g(t, x(t), r_t) = g(t, x(t), i)$. Then, the system (2) can be rewritten as:

$$\begin{aligned} dx(t) &= [(A_i + \Delta A_i(t))\, x(t) + B_i u(t)]\, dt \\ &\quad + D_i g(t, x(t), i)dw(t). \end{aligned} \quad (3)$$

Here, A_i, B_i and D_i are known real constant matrices, the parameter uncertainty $\Delta A_i(t)$ and the unknown function $g(t, x(t), i) \in \mathbb{R}^l$ (with $l < n$), respectively, satisfying:

$$\Delta A_i(t) = M_i F_i(t) N_i, \quad (4)$$

$$\text{trace}\left[g(t, x(t), i)^T g(t, x(t), i)\right] \leq \|H_i x(t)\|^2, \quad \text{with } g(t, x(t_0), i) = 0, \quad (5)$$

where M_i, N_i, and H_i are known real constant matrices, and $F_i(t)$ is an unknown matrix function satisfying $F_i(t)^T F_i(t) \leq I$, for any $i \in \mathcal{S}$. Without loss of generality, it is assumed that the pair (A_i, B_i) is controllable and the input matrix B_i has full column rank, i.e., rank $(B_i) = m$.

As discussed in the Introduction, the actuator degradation is usually inevitable in actual application. Hence, in this work, it is assumed that, the actuator degradation may happen according to the following model:

$$u^F(t) = (I - \rho)\, u(t), \quad (6)$$

with $\rho = \text{diag}(\rho_1, \ldots, \rho_m)$ satisfying:

$$0 \leq \underline{\rho}_k \leq \rho_k \leq \bar{\rho}_k < 1, \quad k = 1, 2, \ldots, m, \quad (7)$$

where the unknown parameter ρ_k $(k = 1, \ldots, m)$ denotes the loss of effectiveness of the kth actuator. Moreover, it is assumed that, both the lower and upper bounds of ρ_k are known. Define $\underline{\rho} = \text{diag}(\underline{\rho}_1, \ldots, \underline{\rho}_m)$, $\bar{\rho} = \text{diag}(\bar{\rho}_1, \ldots, \bar{\rho}_m)$.

Then, the system (3) subject to actuator degradation (6) is described by:

$$\begin{aligned} dx(t) &= [(A_i + \Delta A_i(t))x(t) + B_i(I - \rho)u(t)]dt \\ &\quad + D_i g(t, x(t), i)dw(t). \end{aligned} \quad (8)$$

Remark 1. It can be seen that the actuator model in (6) covers the normal operation case (as $\underline{\rho}_k = \bar{\rho}_k = 0$) and partial degradation case (as $0 < \underline{\rho}_k \leq \bar{\rho}_k < 1$), while in the aforementioned works, the actuator degradation was usually assumed to occur among a pre-specified subset of actuators. Hence, the case considered in this work is more general.

The objective of this work is to design a sliding mode controller such that the stability of the resultant closed-loop system can be ensured despite actuator degradation and external disturbance.

In the sequel, some concepts and lemmas are introduced, which are useful for the development of the main results.

Definition 1. The equilibrium solution, $x_t = 0$, of the stochastic differential equation (8) with $u(t) = 0$ is said to be globally asymptotically stable (with probability one) if for any $s \geq 0$ and $\varepsilon > 0$,

$$\lim_{x\to 0} P\left\{\sup_{s<t} \left|x_t^{s,x}\right| > \varepsilon\right\} = 0, \qquad P\left\{\lim_{t\to+\infty} \left|x_t^{s,x}\right| = 0\right\} = 1,$$

where $x_t^{s,x}$ denotes the solution at time t of a stochastic differential equation starting from the state x at time s for $s \leq t$.

Lemma 1 (Lin & Cai, 1995). *The trivial solution of the stochastic differential equation*

$$dx(t) = a(t, x)dt + b(t, x)dw(t)$$

with $a(t, x)$ and $b(t, x)$ sufficiently differentiable maps, is globally asymptotically stable (with probability one) if there exists a positive-definite radially unbounded function $V(t, x)$, and satisfies

$$\begin{aligned} \mathcal{L}V &= \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} \cdot a(t, x) + \frac{1}{2}\text{trace}\left\{b^T(t, x)\frac{\partial^2 V}{\partial x^2}b(t, x)\right\} \\ &< 0, \quad \text{for } x \neq 0. \end{aligned} \quad (9)$$

3. Sliding surface

In this section, the sliding surface will be firstly constructed. It is worthy of noting that, under Markov jumping, all modes of system