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Brief paper

Minimal time bioremediation of natural water resources[☆]

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ABSTRACT

We study minimal time strategies for the treatment of pollution in large water volumes, such as lakes or natural reservoirs, with the help of an autonomous bioreactor. The control consists of feeding the bioreactor from the resource, with clean output returning to the resource with the same flow rate. We first characterize the optimal policies among constant and feedback controls under the assumption of a uniform concentration in the resource. In the second part, we study the influence of inhomogeneity in the resource, considering two measurement points. With the help of the Maximum Principle, we show that the optimal control law is non-monotonic and terminates with a constant phase, in contrast to the homogeneous case in which the optimal flow rate decreases with time. This study allows decision makers to identify situations in which the benefit of using non-constant flow rates is significant.

1. Introduction

The fight against eutrophication of lakes and natural reservoirs (that is, the excessive development of living organisms associated with an excess of nutrients) constitutes a major challenge. This ecological issue has given rise to many studies over the past 30 years (see, for instance, the surveys Gulati and van Donk (2002) or Sondergaard et al. (2007) and the references therein). To remediate eutrophication, various techniques such as bio-manipulation or ecological control have been proposed for mitigation. A common point across the proposed remediation approaches is that they are usually based on "biotic" actions on the lake trophic chain dedicated to the restoration of equilibrium in local ecosystems. To do so, most studies are based on empirical knowledge. However, since the 1970s, the use of eutrophication models (ranging from heuristic data-based models at a steady state to more recent dynamic mass-balance-based models) together with optimal control techniques have been proposed (see Estrada, Parodi, and Diaz (2009) and the references herein).

In the present paper, an alternative to these techniques is studied using a very simple model of the lake. It is assumed that a bioreactor is available to treat the polluted water by removing a substrate considered to be in excess. Particularly, we consider a natural water resource of volume V polluted with a substrate of concentration S_l. As underlined above, typical examples of natural water resources in need of treatment are lakes or water tables that have been contaminated with diffused pollutants, such as organic matter or nutrients. The objective of the treatment is to make the concentration of the pollutant S_l decrease as quickly as possible to a prescribed value $\underline{S}_l$, with the help of a continuous stirred bioreactor of volume V_r. The reactor is fed from the resource with a flow rate Q, and its output returns to the resource with the same flow rate Q after separation of biomass and substrate in a settler (see Fig. 1). The settler avoids the presence of excessive biomass used for treatment in the natural resource, which could result in undesirable sludge and possibly lead to an increase of eutrophication. We assume that during the entire treatment, the volume V of the resource does not change.

Since the pioneering work by D'Ans, Gottlieb, and Kokotovic (1972), the optimization of bioreactor operation has received great attention in the literature; see Alford (2006), Banga, Balsa-Canto, Moles, and Alonso (2003) and Rani and Rao (1999) for reviews of the different optimization techniques that have been used in bioprocesses. Among them, the theory of optimal control

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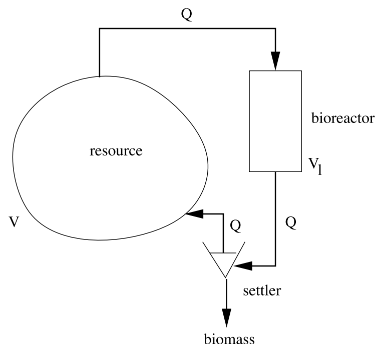

Fig. 1. Interconnection of the bioreactor with the resource.

has proven to be a generic tool for deriving practical optimal rules (Smets, Claes, November, Bastin, & Van Impe, 2004; Smets & Van Impe, 2002; Van Impe & Bastin, 1998). Clearly, one can distinguish two different kinds of problems depending on the continuous or discontinuous operation mode of the process. On the one hand, if the process is operated in feed-batch, the control objective is usually to optimize trajectories to attain a prescribed target in finite time or to maximize production at a given time (Gajardo, Ramírez, & Rapaport, 2008; Hong, 1986; Irvine & Ketchum, 1989; Johnson, 1987; Kurtanjek, 1991; Lim, Tayeb, Modak, & Bonte, 1986; Mazouni, Harmand, Rapaport, & Hammouri, 2010; Moreno, 1999; Shioya, 1992; Srinivasan, Palanki, & Bonvin, 2003; Tsoneva, Patarinska, & Popchev, 1998). On the other hand, the optimal control of continuous processes usually involves a two-step procedure. First, the optimal steady state is determined as a nominal set point that maximizes a criterion (Soukkou, Khellaf, Leulmi, & Boudeghdegh, 2008; Spitzer, 2004). The benefit of operating a periodic control about the nominal point can be analyzed (Abulesz & Lyberatos, 1987; Ruan & Chen, 1996). Then, a control strategy that drives the state about the nominal set point from any initial condition is searched for Kittisupakorn and Hussain (2000), possibly in the presence of model uncertainty using extremum seeking techniques (Bastin, Nesic, Tan, & Mareels, 2009; Marcos, Guay, Dochain, & Zhang, 2004; Wang, Krstic, & Bastin, 1999; Zhang, Guay, & Dochain, 2003). Concerning these strategies, the problem studied in the present paper leads to several original contributions with respect to the existing literature as follows.

- The actual control problem is dedicated to the optimization of transient trajectories (as in the case of feed-batch processes), while it is actually a continuous process. In a standard optimal minimal time problem for a bioprocess, the volume of water to be processed is completely decoupled from the bioreactor. In other words, the problem is to use a biological reactor to process a given volume of "substrate", which is finally released in the environment after processing, whether it is operated continuously or discontinuously. In the present problem, treated water is immediately recycled into the lake. From a modeling point of view, this introduces an original coupling via the dilution of the treated water with the polluted water.
- The lake and the reactor are isolated in the sense that no biomass is supposed to be present in the water resource. The biomass used as a catalyst in the bioreactor is separated from the treated water and withdrawn from the overall process. Thus, the quantity of available biomass is not a limiting parameter. We consider the usual chemostat model to describe the dynamics of the bioreactor:

$$\begin{cases} \dot{S}_r = -\mu(S_r)X_r + \dfrac{Q}{V_r}(S_l - S_r) \\ \dot{X}_r = \mu(S_r)X_r - \dfrac{Q}{V_r}X_r \end{cases} \tag{1}$$

where S_r and X_r indicate the concentrations of substrate and biomass, respectively. For the sake of simplicity, we assume that the yield coefficient of this reaction is equal to one (at the price of changing the unitary value of the biomass concentration, which is always possible).

The growth rate function $\mu(\cdot)$ fulfills the following properties.

Assumption A1. a. Function $\mu(\cdot)$ is increasing such that $\mu(0) = 0$.
b. Function $\mu(\cdot)$ is concave.

A reasonable hypothesis is to assume that the volume of the resource is much larger than the bioreactor one, $V \gg V_r$, and that the possible variations of the manipulated variable Q are slow as compared to the time scale of bioreactor dynamics. Consequently, one may consider that these dynamics (1) are fast, with trajectories at the quasi-steady state $(S_r^\star, X_r^\star) = (S_r(Q), S_l - S_r(Q))$, where $S_r(Q)$ fulfills $\mu(S_r(Q)) = Q/V_r$ (see the usual equilibria analysis of the chemostat in Smith and Waltman (1995)). We shall also neglect any external input flow in the resource.

Problem. The optimization problem consists of driving down the concentration of the resource to a prescribed value $\underline{S}_l > 0$ in a minimal amount of time by modifying the control variable $Q > 0$. In Section 2, we assume that this concentration is uniform in the resource, while in Section 3, we study the effect of spatial inhomogeneity. For each case, we characterize the optimal policy $Q^\star$ (resp. $Q^{opt}(\cdot)$) among constants (resp. feedback controls). Section 4 is devoted to simulations and discussion.

2. The homogeneous case

The dynamics of the resource concentration is simply

$$\dot{S}_l = \frac{Q}{V}(S_r(Q) - S_l). \tag{2}$$

Notice that under Assumption A1.a, choosing Q is equivalent to choosing S_r as a control variable. Then

$$\dot{S}_l = \alpha\mu(S_r)(S_r - S_l), \quad S_r \in (0, S_l) \tag{3}$$

where we denote $\alpha = V_r/V$.

Proposition 1. *Under Assumption A1, the best constant control $Q^\star$ is defined as $Q^\star = V_r\mu(S_r^\star)$, where $S_r^\star$ is the unique minimum on the interval $(0, \underline{S}_l)$ of the following function*

$$T_f(S_r) = \frac{1}{\alpha\mu(S_r)} \ln\left(\frac{S_l(0) - S_r}{\underline{S}_l - S_r}\right). \tag{4}$$

Proof. For a constant control S_r, the explicit solution of (2) is $S_l(t) = S_r(Q) + (S_l(0) - S_r(Q))e^{-\frac{Q}{V}t}$. Therefore, the time $T_f(S_r)$ for reaching the target with $Q = V_r\mu(S_r)$ is given by (4). The function $T_f(\cdot)$ tends toward $+\infty$ when S_r tends toward 0 or $\underline{S}_l$. Consequently, its infimum is reached on the interval $(0, \underline{S}_l)$. Let $T_f^\star$ denote its minimum, which we fix as follows. For each constant control S_r, one can deduce, computing $d^2S_l(T_f^\star)/dS_r^2$ and taking into account the Assumption A1, that the map $S_r \mapsto S_l(T_f^\star)$ is strictly convex (for the sake of space, details of the proof are omitted but are available in Gajardo, Ramirez, Rapaport, and Harmand (2010)). Note that necessarily $S_l(T_f^\star) \geq \underline{S}_l$ and $S_l(T_f^\star) = \underline{S}_l$ when $S_r = S_r^\star$ realizes the minimum of the function $T_f(\cdot)$. Consequently, the optimal control $S_r^\star$ is unique. □

Proposition 2. *Under Assumption A1, the optimal feedback fulfills $Q^{opt}(S_l) = V_r\mu(S_r^{opt}(S_l))$ with*

$$S_r^{opt}(S_l) \in \underset{S_r \in (0, S_l)}{\operatorname{argmax}}\, \mu(S_r)(S_l - S_r). \tag{5}$$

Moreover, $t \mapsto Q^{opt}(S_l(t))$ is decreasing along any optimal trajectory.