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Brief paper

Optimal location of mouse sensors on mobile robots for position sensing☆

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ABSTRACT

Optical mouse sensors have been utilized recently to measure the position and orientation of a mobile robot. This work provides a systematic solution to the problem of locating N optical mouse sensors on a mobile robot with the aim of increasing the quality of the position measurements. The developed analysis gives insights on how the selection of a particular configuration influences the estimation of the robot position, and it allows to compare the effectiveness of different configurations. The results are derived from the analysis of the singular values of a particular matrix obtained by solving the sensor kinematics problem. Moreover, given any mobile robot platform, an end-user procedure is provided to select the best location for N optical mouse sensors on such a platform. The procedure consists of solving a feasible constrained optimization problem.

1. Introduction

The problem of using optical sensors to detect the position of a mobile robot has been recently considered in the literature. These sensors, which are utilized in every computer mouse, can be installed on the bottom of the mobile robot platform to detect its motion relative to the ground. The use of optical sensors as dead-reckoning sensors has the advantage that the position measurements are not dependent on the kinematics of the robot or on the rotation of the robot wheels (Bonarini, Matteucci, & Restelli, 2005). Therefore, the position measurements are not affected by two of the most common sources of measurement errors: (1) slipping, which occurs when a rotation of the traction wheels does not generate a corresponding motion of the robot and (2) crawling, which corresponds to a motion of the robot not measurable by incremental encoders. Two of the main advantages of optical sensors over GPS and camera-based sensors are their low cost and high resolution sensing capability. However, like every dead-reckoning method, the position measurement based on optical mice can be affected by systematic errors (due to the miscalculation of the exact location and orientation of the mice on the robot, and imprecise knowledge of their resolution) and non-systematic errors (due to imperfections of the surface on which the robot is moving) (Bonarini et al., 2005).

Even though the use of two optical mice to detect the position of the robot results in a significant reduction of measurement errors with respect to the classic dead-reckoning method based on incremental encoders (Sekimori & Miyazaki, 2005), several strategies to automatically detect and reduce systematic and non-systematic measurement errors have been developed in Bonarini et al. (2005), Hyun, Yang, Hye Ri Park, and Hyuk-Sung Park (2009), Kim and Lee (2008) and Sekimori and Miyazaki (2005). The solution proposed in Hyun et al. (2009) provides consistent position measurements by optical sensors when changes in the distance between the surface and the sensors occur. The solutions provided in Kim and Lee (2008) and Sekimori and Miyazaki (2005) are obtained with the use of a redundant number of optical mice, the measurements of which are utilized to minimize certain cost functions formed from the holonomic constraints of the robot. As a result, it has been shown that the use of additional optical mice is beneficial to efficiently minimize measurement errors.

In this work, we consider the problem of optimizing the location of mouse sensors on a robot platform in order to increase the sensibility of measurements from the sensors to a robot displacement. The questions that this work addresses are: (1) does the orientation of the mouse optical sensors affect the measurement quality? (2) what is the best location of the optical mice on the robot? and (3) is such a location unique? An attempt to answer these questions was considered in Lee and Song (2004) for the case of only two optical mice. In particular, the notion of absolute deviation of a function was utilized to determine the best location of the optical mice on the robot. However, even in the case of just two optical mice, the high number of unknown parameters did not allow to express the solution in a compact form, and to compare different configurations. Moreover, the analysis in Lee and Song (2004)

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does not allow to determine whether the orientation of the mice is relevant to the considered problem. One of the contributions of this work consists of providing a systematic analysis to answer the above questions, not just in the case of two mice, but also in the more general case of N optical mice (where N can be any positive integer). In this work, a systematic procedure is also provided to locate N mice on any robot platform. The solution of the developed procedure can be obtained by solving a constrained optimization problem. The procedure is based on the analysis of the singular values of a matrix based on the sensor kinematics. This matrix by construction is similar to the Fisher information matrix, which is often utilized in the sensor-target localization problems (Bishop, Fidan, Anderson, Dogancay, & Pathirana, 2010; Martínez & Bullo, 2006). Note that we do not consider any disturbances such as stochastic noise in the relation between the position of the robot and the output of the sensors. The results presented in this paper serve to revise our initial work in Cimino and Pagilla (2010) and provide additional insights and simulation results.

This work is organized as follows. The sensor kinematics is discussed in Section 2. Section 3 shows the singular value analysis and the procedure to obtain the best location of N sensors on a mobile robot platform. Such procedure is applied to some practical examples in Section 4. Conclusion and future work are given in Section 5.

2. Sensor kinematics

Solving the sensor kinematics problem is important to determine the linear and angular absolute position of the robot with respect to the measurements from the mice. The kinematic equations obtained in this section will be utilized in Section 3 to determine the best location for N optical mouse sensors. To facilitate understanding, the sensor kinematics problem for only one mouse sensor is first discussed, followed by generalization to the N mice case.

Consider the mobile robot shown in Fig. 1, where three coordinate frames are considered to describe the robot motion. In particular, the subscripts A, R and S are utilized to refer, respectively, to the absolute frame placed at the fixed point O_A, the robot frame placed at the geometric center O_R of the robot, and the sensor frame placed at a generic point O_S on the robot platform. In the following, we will assume that the X_R axis is always aligned with the axis of the wheels, so that the Y_R axis always points toward the forward direction of robot motion. The orientation of the robot corresponds to the angle θ formed by the axes X_A and X_R, and positive rotations are given by the right-hand rule. We will denote with (r, ψ) the polar coordinates of the sensor position O_S with respect to the robot frame, and with the angle ϕ the fixed orientation of the sensor with respect to the robot frame (i.e., the angle formed by the axis X_R and X_S). In the following, the superscripts A, R and S will be utilized for each vector to denote the frame with respect to which the vector itself is expressed.

The absolute position of the robot, O_R^A, can be expressed as a function of the absolute position of the sensor, O_S^A, by the expression $O_S^A = O_R^A + \mathcal{R}(\theta) O_S^R$, where $\mathcal{R}(\theta)$ is an isomorphic transformation that rotates vectors in the $X_A Y_A$ plane counterclockwise by an angle of θ. By taking the time-derivative of O_S^A, and considering that the position of the sensor with respect to the robot does not change (i.e., $O_S^R = [r\cos(\psi), r\sin(\psi)]^T$ is constant, where T denotes the transpose operator), the absolute velocity vector of the robot, $v_{O_R}^A$, and of the sensor, $v_{O_S}^A$, can be related by the expression

$$\begin{aligned} v_{O_S}^A &= v_{O_R}^A + \mathcal{S}(\omega)\mathcal{R}(\theta)O_S^R \\ &= v_{O_R}^A + \|\omega\|_2 \begin{bmatrix} -r\cos(\theta+\psi) \\ r\sin(\theta+\psi) \end{bmatrix} \end{aligned} \tag{1}$$

where $\|\omega\|_2 = \mathrm{d}\theta/\mathrm{d}t$, and $\mathcal{S}(\omega)$ is a 2×2 skew-symmetric matrix such that $\mathrm{d}\mathcal{R}(\theta)/\mathrm{d}t = \mathcal{S}(\omega)\mathcal{R}(\theta)$. Considering that the output of the optical sensor updates at a certain rate depending on the sensor characteristics, we can rewrite (1) in the form

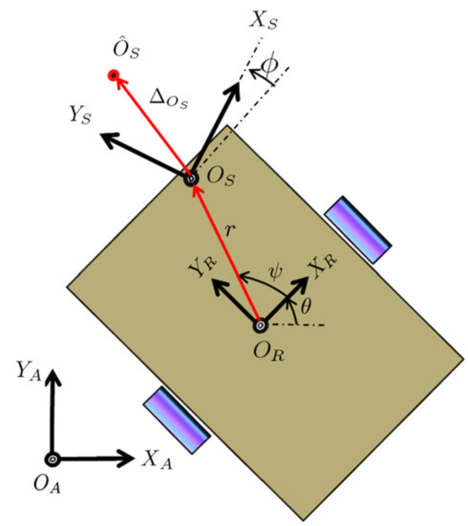

Fig. 1. Sensor kinematics: coordinate axis.

$$\Delta_{O_S}^A = \Delta_{O_R}^A + \Delta\theta \begin{bmatrix} -r\sin(\theta+\psi) \\ r\cos(\theta+\psi) \end{bmatrix} \tag{2}$$

where $\Delta\theta$ denotes the incremental angular position of the robot, and the symbol $\Delta_{\mathcal{P}}^{\mathcal{F}}$ denotes the increment of motion of the point $\mathcal{P}$ with respect to the coordinate frame $\mathcal{F}$. Considering that $\Delta_{O_S}^A = \mathcal{R}(\theta+\phi)\Delta_{O_S}^S$, where $\Delta_{O_S}^S$ is the measurement from the sensor, (2) can be rewritten as

$$\mathcal{R}(\theta+\phi)\Delta_{O_S}^S = \Delta_{O_R}^A + \Delta\theta \begin{bmatrix} -r\sin(\theta+\psi) \\ r\cos(\theta+\psi) \end{bmatrix}. \tag{3}$$

Eq. (3) relates the robot linear and angular absolute motion (i.e., $\Delta_{O_R}^A$ and $\Delta\theta$) to the sensor measurements $\Delta_{O_S}^S$. Since it is desired to express the absolute motion of the robot as function of the sensor measurements, Eq. (3) can be rewritten in the more convenient form

$$F(r,\theta,\psi)u = b(\theta,\phi,\Delta_{O_S}^S), \tag{4}$$

where $u \triangleq [\Delta_{O_R,x}^A, \Delta_{O_R,y}^A, \Delta\theta]^T$ and

$$F \triangleq \begin{bmatrix} 1 & 0 & -r\sin(\theta+\psi) \\ 0 & 1 & r\cos(\theta+\psi) \end{bmatrix}, \qquad b \triangleq \mathcal{R}(\theta+\phi)\Delta_{O_S}^S. \tag{5}$$

The matrices in (5) correspond to a system with two equations in the three unknowns $\Delta_{O_R,x}^A$, $\Delta_{O_R,y}^A$ and $\Delta\theta$. This implies that the robot displacement cannot be determined with the use of only one mouse sensor. If $N \geq 2$ sensors are utilized, the matrices F and b in (4) are given by

$$F \triangleq \begin{bmatrix} 1 & 0 & -r_1\sin(\theta+\psi_1) \\ 0 & 1 & r_1\cos(\theta+\psi_1) \\ \vdots & \vdots & \vdots \\ 1 & 0 & -r_N\sin(\theta+\psi_N) \\ 0 & 1 & r_N\cos(\theta+\psi_N) \end{bmatrix}$$

$$b \triangleq \mathrm{diag}\{\mathcal{R}(\theta+\phi_1), \ldots, \mathcal{R}(\theta+\phi_N)\} \begin{bmatrix} \Delta_{O_{S_1}}^{S_1} \\ \vdots \\ \Delta_{O_{S_N}}^{S_N} \end{bmatrix}$$

where the subscript $i = 1, \ldots, N$ refers to the i-th sensor. This system is now overdetermined because the number of equations