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Brief paper

Robust sampled-data control of a class of semilinear parabolic systems[☆]

Emilia Fridman, Anatoly Blighovsky

School of Electrical Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel

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ABSTRACT

We develop sampled-data controllers for parabolic systems governed by uncertain semilinear diffusion equations with distributed control on a finite interval. Such systems are stabilizable by linear infinite-dimensional state-feedback controllers. For a realistic design, finite-dimensional realizations can be applied leading to local stability results. Here we suggest a sampled-data controller design, where the sampled-data (in time) measurements of the state are taken in a finite number of fixed sampling points in the spatial domain. It is assumed that the sampling intervals in time and in space are bounded. Our sampled-data static output feedback enters the equation through a finite number of shape functions (which are localized in the space) multiplied by the corresponding state measurements. It is piecewise-constant in time and it may possess an additional time-delay. The suggested controller can be implemented by a finite number of stationary sensors (providing discrete state measurements) and actuators and by zero-order hold devices. A direct Lyapunov method for the stability analysis of the resulting closed-loop system is developed, which is based on the application of Wirtinger's and Halanay's inequalities. Sufficient conditions for the exponential stabilization are derived in terms of Linear Matrix Inequalities (LMIs). By solving these LMIs, upper bounds on the sampling intervals that preserve the exponential stability and on the resulting decay rate can be found. The dual problem of observer design under sampled-data measurements is formulated, where the same LMIs can be used to verify the exponential stability of the error dynamics.

1. Introduction

We develop sampled-data controllers for parabolic systems governed by semilinear diffusion equations with distributed control. Such systems are stabilizable by linear infinite-dimensional state-feedback controllers. For a realistic design, finite-dimensional realizations (Balas, 1985; Candogan, Ozbay, & Ozaktas, 2008; Smagina & Sheintuch, 2006) can be applied. However, finite-dimensional control, which employs e.g. Galerkin truncation, leads to local stability results (Smagina & Sheintuch, 2006). In Hagen and Mezic (2003) the control input has been designed to enter the semilinear diffusion equation through a finite number of shape functions (e.g. step functions) and their respective amplitude values. Sufficient conditions have been derived for the global stabilization of the infinite-dimensional dynamics. For linear parabolic systems mobile collocated sensors and actuators (see Demetriou (2010) and references therein) or adaptive controllers (Krstic & Smyshlyaev, 2008; Smyshlyaev & Krstic, 2005) can be used. The latter methods are not easy to implement.

Sampled-data control of finite-dimensional systems have been studied extensively over the past decades (see e.g. Chen and Francis (1995), Naghshtabrizi, Hespanha, and Teel (2008), Fujioka (2009), Fridman (2010) and the references therein). Three main approaches have been used to control of sampled-data systems: the discrete-time, the time-delay and the impulsive system approaches. Unlike the other approaches, the discrete-time one does not take into account the inter-sampling behavior and seems not to be applicable to time-varying or nonlinear systems.

There are only a few references on sampled-data control of distributed parameter systems (Cheng, Radisavljevic, Chang, Lin, & Su, 2009; Logemann, Rebarber, & Townley, 2003, 2005). All these works use the discrete-time approach for linear time-invariant systems. Observability of parabolic systems under sampled-data measurements has been studied in Khapalov (1993). Recently a model-reduction-based approach to sampled-data control was introduced in Ghantasala and El-Farra (2010), Sun, Ghantasala, and El-Farra (2009), where a finite-dimensional controller was designed on the basis of a finite-dimensional system that captures the dominant (slow) dynamics of the infinite-dimensional system. The latter approach seems to be not applicable to systems with spatially-dependent diffusion coefficients and with uncertain

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E-mail addresses: emilia@eng.tau.ac.il (E. Fridman), tolik_bl@yahoo.com (A. Blighovsky).

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nonlinear terms. The existing sampled-data results are not applicable to the performance analysis of the closed-loop system, e.g. to the decay rate of the exponential convergence.

We suggest a sampled-data controller design for a one-dimensional semilinear diffusion equation, where the sampled-data in time measurements of the state are taken in a finite number of fixed sampling spatial points. It is assumed that the sampling intervals in time and in space may be variable, but bounded. The sampling instants (in time) may be uncertain. The diffusion coefficient and the nonlinearity may be unknown, but they satisfy some bounds. The sampled-data static output feedback controller is piecewise-constant in time. It can be implemented by a finite number of stationary sensors and actuators and by zero-order hold devices. Sufficient conditions for exponential stabilization are derived in terms of LMIs in the framework of time-delay approach to sampled-data systems. By solving these LMIs, *upper bounds on the sampling intervals that preserve the stability and on the resulting decay rate* can be found. Finally, the dual problem of observer design under sampled-data measurements is discussed.

We note that the LMI approach has been introduced in Fridman and Orlov (2009a), Fridman and Orlov (2009b) for some classes of distributed parameter systems, leading to simple finite-dimensional sufficient conditions for stability. The method in the present paper is based on the novel combination of Lyapunov–Krasovskii functionals with Wirtinger's and Halanay's inequalities. A numerical example illustrates the efficiency of the method. Some preliminary results will be presented in Fridman and Blighovsky (2011).

Notation. Throughout the paper $\mathbf{R}^n$ denotes the n dimensional Euclidean space with the norm $|\cdot|$, $\mathbf{R}^{n\times m}$ is the set of all $n\times m$ real matrices, and the notation $P>0$ with $P\in\mathbf{R}^{n\times n}$ means that P is symmetric and positive definite. The symmetric elements of the symmetric matrix will be denoted by $*$. Functions, continuous (continuously differentiable) in all arguments, are referred to as of class C (of class C^1). $L_2(0,l)$ is the Hilbert space of square integrable functions $z(\xi)$, $\xi\in[0,l]$ with the corresponding norm $\|z\|_{L_2}=\sqrt{\int_0^l z^2(\xi)d\xi}$. $\mathcal{H}^1(0,l)$ is the Sobolev space of absolutely continuous scalar functions $z:[0,l]\to R$ with $\frac{dz}{d\xi}\in L_2(0,l)$. $\mathcal{H}^2(0,l)$ is the Sobolev space of scalar functions $z:[0,l]\to R$ with absolutely continuous $\frac{dz}{d\xi}$ and with $\frac{d^2z}{d\xi^2}\in L_2(0,l)$.

2. Problem formulation and useful inequalities

Consider the following semilinear scalar diffusion equation

$$z_t(x,t)=\frac{\partial}{\partial x}[a(x)z_x(x,t)]+\phi(z(x,t),x,t)z(x,t)+u(x,t),\quad t\ge t_0,\ x\in[0,l],\ l>0,\tag{1}$$

with Dirichlet boundary conditions

$$z(0,t)=z(l,t)=0,\tag{2}$$

or with mixed boundary conditions

$$z_x(0,t)=\gamma z(0,t),\qquad z(l,t)=0,\quad \gamma\ge 0,\tag{3}$$

where subindexes denote the corresponding partial derivatives and γ may be unknown. In (1) $u(x,t)$ is the control input. The functions a and ϕ are of class C^1 and may be unknown. These functions satisfy the inequalities $a\ge a_0>0$, $\phi_m\le\phi\le\phi_M$, where a_0, ϕ_m and ϕ_M are known bounds.

It is well-known that the open-loop system (1) under the above boundary conditions may become unstable if ϕ_M is big enough (see Curtain and Zwart (1995) for $\phi\equiv\phi_M$). Moreover, a linear infinite-dimensional state feedback $u(x,t)=-Kz(x,t)$ with big enough $K>0$ exponentially stabilizes the system (see Proposition 1). In the present paper we develop a sampled-data controller design.

Consider (1) under the boundary conditions (2) or (3). Let the points $0=x_0<x_1<\cdots<x_N=l$ divide $[0,l]$ into N sampling intervals. We assume that N sensors are placed in the middle $\bar{x}_j=\frac{x_{j+1}+x_j}{2}$ $(j=0,\ldots,N-1)$ of these intervals. Let $t_0<t_1<\cdots<t_k\ldots$ with $\lim_{k\to\infty}t_k=\infty$ be sampling time instants. The sampling intervals in time and in space may be variable but bounded

$$0\le t_{k+1}-t_k\le h,\qquad x_{j+1}-x_j\le\Delta.\tag{4}$$

Sensors provide discrete measurements of the state:

$$y_{jk}=z(\bar{x}_j,t_k),\quad \bar{x}_j=\frac{x_{j+1}+x_j}{2},\quad j=0,\ldots,N-1,\ t\in[t_k,t_{k+1}),\ k=0,1,2\ldots\tag{5}$$

Our objective is to design for (1) an exponentially stabilizing (sampled-data in space and in time) controller

$$u(x,t)=-Kz(\bar{x}_j,t_k),\quad \bar{x}_j=\frac{x_{j+1}+x_j}{2},\quad x\in[x_j,x_{j+1}),\ j=0,\ldots,N-1,\quad t\in[t_k,t_{k+1}),\ k=0,1,2\ldots\tag{6}$$

with the gain $K>0$. The closed-loop system (1), (6) has the form:

$$\begin{aligned}z_t(x,t)=&\frac{\partial}{\partial x}[a(x)z_x(x,t)]+\phi(z(x,t),x,t)z(x,t)\\&-Kz(\bar{x}_j,t_k),\quad t\in[t_k,t_{k+1}),\ k=0,1,2\ldots\\&x_j\le x<x_{j+1},\ j=0,\ldots,N-1.\end{aligned}\tag{7}$$

By using the relation $z(\bar{x}_j,t_k)=z(x,t_k)-\int_{\bar{x}_j}^x z_\zeta(\zeta,t_k)d\zeta$, (7) can be represented as

$$\begin{aligned}z_t(x,t)=&\frac{\partial}{\partial x}[a(x)z_x(x,t)]+\phi(z(x,t),x,t)z(x,t)\\&-K[z(x,t_k)-\int_{\bar{x}_j}^x z_\zeta(\zeta,t_k)d\zeta],\\&x_j\le x<x_{j+1},\ j=0,\ldots,N-1,\\&t\in[t_k,t_{k+1}),\ k=0,1,2\ldots\end{aligned}\tag{8}$$

We will start with the sampled-data in space and continuous in time controller

$$u(x,t)=-Kz(\bar{x}_j,t),\quad x_j\le x<x_{j+1},\ j=0,\ldots,N-1.\tag{9}$$

Also a more general controller of the form

$$u(x,t)=-Kz(\bar{x}_j,t_k-\eta_k),\quad t\in[t_k,t_{k+1}),\ k=0,1,2\ldots,\quad x_j\le x<x_{j+1},\ j=0,\ldots,N-1,\ u(x,t)=0,\ t<t_0,\tag{10}$$

where $\eta_k\in[0,\eta_M]$ is an additional (control or measurement) delay, will be studied. Such a controller models e.g. network-based stabilization, where variable and uncertain sampling instants t_k may appear due to data packet dropouts, whereas η_k is network-induced delay (Gao, Chen, & Lam, 2008; Zhang, Branicky, & Phillips, 2001). Representing $t_k-\eta_k=t-\tau(t)$, where $\tau(t)=t-t_k+\eta_k$, we have $\tau(t)\in[0,\tau_M]$ with $\tau_M=h+\eta_M$. Finally, the dual problem of the observer design for semilinear diffusion equations under the sampled-data measurements is considered.

Remark 1. Our results will be applicable to convection–diffusion equation

$$z_t(x,t)=a_0z_{xx}(x,t)-\beta z_x(x,t)+\phi(z(x,t),x,t)z(x,t)+u(x,t),\quad t\ge t_0,\ x\in[0,l],\ l>0,\tag{11}$$