Automatica 48 (2012) 879–885

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Target containment control of multi-agent systems with random switching interconnection topologies[☆]

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ARTICLE INFO

Article history:
Received 19 December 2010
Received in revised form
8 June 2011
Accepted 30 September 2011
Available online 20 March 2012

Keywords:
Multi-agent systems
Containment control
Multiple leaders
Random switching topologies
Markov chain

ABSTRACT

In this paper, the distributed containment control is considered for a second-order multi-agent system guided by multiple leaders with random switching topologies. The multi-leader control problem is investigated via a combination of convex analysis and stochastic process. The interaction topology between agents is described by a continuous-time irreducible Markov chain. A necessary and sufficient condition is obtained to make all the mobile agents almost surely asymptotically converge to the static convex leader set. Moreover, conditions on the tracking estimation are provided for the convex target set determined by moving multiple leaders.

1. Introduction

Recent years have witnessed a huge and rapidly growing literature concerned with multi-agent problems due to the broad applications in various disciplines. The leader–follower coordination, as one of the important problems of multi-agent networks, has been studied in the last decade, with significant results obtained for first-order or second-order multi-agent systems. Static-leader cases were studied with jointly-connected interaction topologies in Jadbabaie, Lin, and Morse (2003). Moreover, potential function approaches were used to drive the agents to follow a desired trajectory in Olfati-Saber (2006) and similar results under relaxed assumptions were obtained in Ren and Beard (2008) and Su, Wang, and Lin (2009). To follow a moving leader with unmeasurable velocity, distributed observers were designed for second-order multi-agent systems in Hong, Chen, and Bushnell (2008). Also, an estimator-based tracking problem was investigated for a leader–follower system with measurement noises in Hu and Feng (2010).

Due to the practical demand, multi-agent coordination with multiple leaders becomes more and more important since multiple leaders may be useful to achieve effectively the containment or guidance of an agent group in a target region (see Couzin, Krause, Franks, and Levin (2005)). Target aggregation or containment with multiple leaders was developed, aiming at containing a group of agents in a specific target region. Containment control schemes were proposed to make the agents stay in the convex set spanned by the multiple leaders in Ji, Ferrari-Trecate, Egerstedt, and Buffa (2008). The target containment of nonlinear multi-agent systems with different switching topologies was considered to contain a group of agents guided by leaders in a given target set in Shi and Hong (2009). Also, a distributed control method was reported for multi-agent containment in Cao and Ren (2010). Additionally, the attitude containment control was studied in Dimarogonas, Tsiotras, and Kyriakopoulos (2009), while finite-time control law was designed for containment in Meng, Ren, and You (2010).

Random switching topologies were also investigated for multi-agent coordination algorithms due to many practical backgrounds including gossip algorithms and communication patterns (for example, Boyd, Ghosh, Prabhakar, and Shah (2006) and Matei, Martins, and Baras (2009)). In fact, during the information transmission, packet drop and node failure phenomena can be described as random switching graph processes, and multi-agent consensus with various random graph processes was also important. To solve the related coordination problems, different approaches were proposed. For example, the asymptotic almost sure consensus is achieved over random information networks in Porfiri and Stilwell (2007), where the existence of any edge in

☆ This work has been supported in part by the NNSF of China under Grants No. 60874018 and 61174071. The material in this paper was partially presented at the 49th IEEE Conference on Decision and Control, December 15–17, 2010, Atlanta, Georgia, USA. This paper was recommended for publication in revised form by Associate Editor Giancarlo Ferrari-Trecate under the direction of Editor Ian R. Petersen.

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0005-1098/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2012.02.032

a topology is probabilistic and independent from the existence of any other edge. Moreover, similar results were obtained for stationary and ergodic graph processes in Tahbaz-Salehi and Jadbabaie (2010), while the mean square consensus problem was discussed for a second-order discrete-time system with Markovian graphs in Zhang and Tian (2009). Additionally, Liu, Lu, and Chen (2011) also investigated consensus problem based on adapted stochastic processes.

To our knowledge, there is no theoretical result on containment of second-order multi-agent systems with random switching interconnections. The objective of the paper is to study the containment control for a second-order multi-agent system with a target set specified by multiple leaders. Here, we develop a new method to solve the problem with the help of both convex analysis and stochastic process analysis, because the existing methods on random consensus used in Porfiri and Stilwell (2007), Tahbaz-Salehi and Jadbabaie (2010) and Zhang and Tian (2009), or the containment methods for deterministic systems proposed in Cao and Ren (2010) and Ji et al. (2008), cannot be applied to solve our problem; we solve the containment of the second-order agent systems with switching topologies, which is more complicated than the first-order agent model with deterministic switching studied in Shi and Hong (2009). Additionally, we investigate set containment for continuous-time systems, different from many existing random consensus results for discrete-time systems.

Notation. I_n is the $n \times n$ identity matrix; For a given vector x, x^T stands for its transpose, $\|x\|_2$ for its Euclidean norm; For a given matrix F, $\|F\|_\infty$ stands for its infinite norm, $\exp(F)$ for its matrix exponential, $(F)_{ij}$ for its i-th row and j-th column entry; $(W)_{**}$ denotes the $2n \times 2n$ left upper block of matrix $W \in R^{(2n+l)\times(2n+l)}$; $\otimes$ denotes Kronecker product.

2. Preliminaries and formulation

In this section, we introduce preliminary knowledge about graph theory and stochastic process, and then our problem formulation.

It is known that the interaction topology of a multi-agent system consisting of n agents (followers) and l leaders can be described by a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with the set of nodes $\mathcal{V} = \mathcal{I} \bigcup \mathcal{L}$ and the set of arcs $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$. Without loss of generality, we assume the first n agents as the followers and the last l agents as the leaders. Let $\mathcal{I} = \{1, \ldots, n\}$ and $\mathcal{L} = \{n+1, \ldots, n+l\}$ denote the index sets of followers and leaders, respectively. $(i, j) \in \mathcal{E}$ means that there is an arc from node i to node j (or equivalently, node j is a neighbor of node i). The adjacency matrix associated with the graph is denoted as $A = [a_{ij}]_{(n+l)\times(n+l)}$ with nonnegative adjacency elements a_{ij}. The element a_{ij} of matrix A associated with arc (i, j) is positive, i.e., $a_{ij} > 0$ if and only if $(i, j) \in \mathcal{E}$. There is no self-loop in $\mathcal{G}$, i.e., $a_{ii} = 0$ for all $i \in \mathcal{V}$. In our problem, $a_{ij} = 0$ for all $i \in \mathcal{L}$ and $j \in \mathcal{V}$. A path from i to j in $\mathcal{G}$ is a sequence $i_0, i_1, \ldots, i_\iota$ of distinct nodes such that $(i_{k-1}, i_k) \in \mathcal{E}$ for $k = 1, \ldots, \iota$, where $i_0 = i, i_\iota = j$. Node j is reachable from node i if there is at least one path from i to j. Leader set $\mathcal{L}$ is reachable from node i if there exists at least one leader $j \in \mathcal{L}$ such that j is reachable from i. Moreover, $\mathcal{L}$ is globally reachable in $\mathcal{G}$ if it is reachable from every node of $\mathcal{I}$.

Given digraph $\mathcal{G}$, $\mathcal{E}(\mathcal{G})$ and $A(\mathcal{G})$ denote the set of arcs and the adjacency matrix of $\mathcal{G}$, respectively. The set of neighbors of node i in $\mathcal{I}$ and $\mathcal{L}$ are denoted by $\mathcal{N}_{if}(\mathcal{G}) = \{j|(i, j) \in \mathcal{E}(\mathcal{G}), j \in \mathcal{I}\}$, $\mathcal{N}_{il}(\mathcal{G}) = \{j|(i, j) \in \mathcal{E}(\mathcal{G}), j \in \mathcal{L}\}$, respectively. $\bigcup_{1\le r\le p} \mathcal{G}_r$ denotes the union graph with nodes set $\mathcal{V}$ and arcs set $\bigcup_{1\le r\le p} \mathcal{E}(\mathcal{G}_r)$. Let $\mathcal{G}^f$ be the induced subgraph of $\mathcal{G}$ with all followers as nodes. The degree matrix of $\mathcal{G}^f$ is a diagonal matrix $D^f = \mathrm{diag}\{d^f_1, \ldots, d^f_n\}$ with $d^f_i = \sum_{1\le j\le n} a_{ij} (1 \le i \le n)$ and the Laplacian matrix of $\mathcal{G}_f$ is defined as $L^f = D^f - A^f$, where A^f is the adjacency matrix of $\mathcal{G}^f$ (referring to Godsil and Royle (2001) for details). Moreover, A^{fl} and D^{fl} denote the adjacency and degree matrix between followers and leaders, respectively, i.e., $(A^{fl})_{ir} = a_{i(n+r)}$, $D^{fl} = \mathrm{diag}\{d^{fl}_1, \ldots, d^{fl}_n\}$, where $d^{fl}_i = \sum_{1\le r\le l} a_{i(n+r)} (1 \le i \le n)$.

To deal with random switching of multi-agent systems, we have to consider stochastic processes (referring to Chow and Teicher (1997), Norris (1997) and Ross (1983)). Given a probability space $(\Xi, \mathcal{F}, \mathbf{P})$. The elements of Ξ are called sample events. For $Q \in \mathcal{F}$, the indicator function $\chi_Q : \Xi \to R$ is defined by $\chi_Q(w) = 1$ if $w \in Q$, and $\chi_Q(w) = 0$ otherwise. $\mathbf{E}x$ denotes the expectation of random variable x. Let $\{\varphi_k, k = 0, 1, \ldots\}$ be an ergodic stationary sequence, and g an infinite dimensional Borel measurable function. Then $\{\xi_k, k = 0, 1, \ldots\}$ is also an ergodic stationary sequence if $\mathbf{E}|\xi_0| < +\infty$, where $\xi_k = g(\varphi_k, \varphi_{k+1}, \ldots)$. According to the strong law of large numbers of ergodic stationary sequence,

$$\lim_{k\to\infty} (\xi_0 + \xi_1 + \cdots + \xi_k)/(k + 1) = \mathbf{E}\xi_0 \quad a.s.$$

Let $\{\sigma(t), t \ge 0\}$ be a homogeneous irreducible continuous-time Markov chain taking values in a finite set $\mathcal{S} = \{1, \ldots, s^*\}$ of positive recurrent states. Define random variable sequence $t_0 = 0$,

$$t_{k+1} = \min\{t|t > t_k, \sigma(t) \ne \sigma(t_k)\}, \quad k = 0, 1, \ldots. \tag{1}$$

Then $\{t_{k+1} - t_k, k = 0, 1, \ldots\}$ are independent, conditional on $\{\sigma(t_k), k = 0, 1, \ldots\}$ and, for each $r \in \mathcal{S}$, there is a scalar $0 < \rho_r < \infty$[2] such that $t_{k+1} - t_k$ has the exponential distribution with parameter ρ_r, conditional on $\sigma(t_k) = r$. In addition, $t_k \to \infty$ as $k \to \infty$ with probability one. The embedded Markov chain is defined as $\{\sigma(t_k), k = 0, 1, \ldots\}$, which is homogeneous, irreducible and takes values in $\mathcal{S}$. Let $\varpi = (\varpi_{ij}) \in R^{s^*\times s^*}$ be its transition probability matrix. According to (1), $\varpi_{ii} = 0$ for all $i \in \mathcal{S}$. Moreover, let $\pi = (\pi_1, \ldots, \pi_{s^*})$ be its unique stationary distribution, i.e., $\mathbf{P}(\sigma(t_k) = r) = \mathbf{P}(\sigma(0) = r) = \pi_r$ for any k, where $\pi_r > 0$ for all $1 \le r \le s^*$. Here $\mathbf{P} = \mathbf{P}_\pi$ is the probability measure generated by the unique stationary distribution and transition probability matrix ϖ, and then under $\mathbf{P}$ the embedded Markov chain $\{\sigma(t_k), k = 0, 1, \ldots\}$ is an ergodic stationary sequence. $\mathbf{E} = \mathbf{E}_\pi$ is the expectation corresponding to $\mathbf{P}$. In fact, the obtained conclusions also hold for $\mathbf{P} = \mathbf{P}_{\bar\pi}$, where $\bar\pi$ is any given initial distribution.

Discrete-time Markovian random graphs were discussed in Matei et al. (2009), and here we give a corresponding concept for continuous-time cases.

Definition 1. Let $\mathcal{P} = \{G_r, r = 1, \ldots, s^*\}$ be a set of digraphs with n followers and l leaders. By a continuous-time Markovian random graph process we understand a map $\mathbf{G} : \mathcal{S} \to \mathcal{P}$ such that $\mathbf{G}(\sigma(t)) = G_{\sigma(t)}$ for any $t \ge 0$, where $\{\sigma(t), t \ge 0\}$ is a continuous-time homogeneous irreducible Markov chain taking values in a finite set $\mathcal{S} = \{1, \ldots, s^*\}$ of positive recurrent states.

In our multi-agent problem, the dynamic of leader i $(i = n + 1, \ldots, n + l)$ is expressed as:

$$\dot h_i = f_i(h, t), \quad h = (h^T_{n+1}, \ldots, h^T_{n+l})^T \tag{2}$$

where $h_i \in R^m$ is the position of leader i, and $f_i(h, t) : R^{lm} \times R \to R^m$ is its velocity, piecewise continuous in (h, t). The dynamic of agent i $(i = 1, \ldots, n)$ is described by:

$$\dot x_i = v_i, \qquad \dot v_i = u_i \tag{3}$$

[2] The irreducibility of Markov chain implies that $\rho_r > 0$ for all $1 \le r \le s^*$. A state r for which $\rho_r = \infty$ means that it is instantaneously left once entered. Without loss of generality, in this paper we assume $\rho_r < \infty$ for all $1 \le r \le s^*$.