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Brief paper

A generalized framework for robust nonlinear H_∞ filtering of Lipschitz descriptor systems with parametric and nonlinear uncertainties[☆]

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ABSTRACT

In this paper, a generalized robust nonlinear H_∞ filtering method is proposed for a class of Lipschitz descriptor systems, in which the nonlinearities appear both in the state and measured output equations. The system is assumed to have norm-bounded uncertainties in the realization matrices as well as nonlinear uncertainties. We synthesize the H_∞ filter through semidefinite programming and strict LMIs. The admissible Lipschitz constants of the nonlinear functions are maximized through LMI optimization. The resulting H_∞ filter guarantees asymptotic stability of the estimation error dynamics with prespecified disturbance attenuation level and is robust against time-varying parametric uncertainties as well as Lipschitz nonlinear additive uncertainty. Explicit bound on the tolerable nonlinear uncertainty is derived based on a norm-wise robustness analysis.

1. Introduction

State estimation and filtering of nonlinear dynamical systems has been a subject of extensive research in recent years due to its theoretical and practical importance. Generalizing the state space modeling (i.e. pure ODE systems), descriptor systems can characterize a larger class of systems than conventional state space models and can *describe* the physics of the system more precisely. Descriptor systems, also referred to as singular systems or differential–algebraic equation (DAE) systems, arise from an inherent and natural modeling approach, and have vast applications in engineering disciplines such as power systems, network and circuit analysis, and multibody mechanical systems, as well as in social and economic sciences. Many approaches have been developed to design state observers for descriptor systems. In Boutayeb and Darouach (1995), Dai (1989), Darouach and Boutayeb (1995), Darouach, Zasadzinski, and Hayar (1996), Hou and Muller (1999), Lu and Ho (2006), Masubuchi, Kamitane, Ohara, and Suda (1997), Shields (1997), Uezato and Ikeda (1999), Wang, Yung, and Chang (2006), and Zimmer and Meier (1997) various methods of observer design for linear and nonlinear descriptor systems have been proposed. In Boutayeb and Darouach (1995) an observer design procedure is proposed for a class nonlinear descriptor systems using an appropriate coordinate transformation. In Shields (1997), the authors address the unknown input observer design problem dividing the system into two dynamic and static subsystems. Ref. Lu and Ho (2006) studies the full order and reduced order observer design for Lipschitz descriptor nonlinear systems.

In this paper, we study the *robust* nonlinear H_∞ filtering for continuous-time Lipschitz descriptor systems in the presence of disturbance and model uncertainties, in the LMI framework. The linear matrix inequalities proposed here are developed in such a way that the admissible Lipschitz constant of the system is maximized through LMI optimization. This adds the important extra feature to the filter, making it robust against a class of *nonlinear uncertainty*. Securing the same filter features, the LMI optimization approach to nonlinear H_∞ filtering for the conventional state space models can be found in Abbaszadeh and Marquez (2007, 2008, 2009) in continuous and discrete

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time domains, respectively. The results given here generalize our previous results in that:

(i) Extend the model from conventional state space to descriptor models. The problem is first formulated as a Semidefinite Programming (SDP) and then is converted into strict LMIs.
(ii) Consider nonlinearities in both the state and output equations.
(iii) Generalize the filter structure by proposing a novel general dynamical filtering framework that can easily capture both dynamic and static-gain filter structures.

Stability of nonlinear ODE systems is established through Lyapunov theory, while the stability of DAE systems is established through LaSalle's invariant set theory. The results on ODEs, such as in Abbaszadeh and Marquez (2007, 2008, 2009), are directly cast into strict LMIs while the results here are a set of linear matrix equations and inequalities leading into a semidefinite programming (SDP). The developed SDP problem is then smartly converted into a strict LMI formulation without any approximations, which is efficiently solvable by readily available LMI solvers. The proposed dynamical structure has additional degrees of freedom compared to conventional static-gain filters and consequently is capable of robustly stabilizing the filter error dynamics for systems for which an static-gain filter cannot be found. Besides, for the cases that both static-gain and dynamic filters exist, the maximum admissible Lipschitz constant obtained using the proposed dynamical filter structure can be much larger than that of the static-gain filter. The result is an H_∞ filter with a prespecified disturbance attenuation level which guarantees asymptotic stability of the estimation error dynamics and is robust against Lipschitz nonlinear uncertainties as well as time-varying parametric uncertainties, simultaneously.

The rest of the paper is organized as follows. In Section 2, the problem statement and some preliminaries are mentioned. In Section 3, we propose a new method for robust H_∞ filter design for nonlinear descriptor uncertain systems based on semidefinite programming. In Section 4, the SDP problem of Section 3 is converted into strict LMIs. Section 5, is devoted to robustness analysis in which an explicit bound on the tolerable nonlinear uncertainty is derived. In Section 6, we show the effectiveness of the proposed filter design method through an illustrative example.

2. Preliminaries and problem statement

Consider the following class of continuous-time uncertain nonlinear descriptor systems:

$$(\Sigma_s):\mathbf{E}\dot{x}(t) = (A+\Delta A(t))x(t) + \Phi(x,u) + Bw(t)$$
$$y(t) = (C+\Delta C(t))x(t) + \Psi(x,u) + Dw(t),$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^p$ and $\Phi(x,u)$ and $\Psi(x,u)$ contain nonlinearities of second order or higher. $\mathbf{E}, A, B, C$ and D are constant matrices with compatible dimensions; $\mathbf{E}$ may be singular. When the matrix $\mathbf{E}$ is singular, the above form is equivalent to a set of differential–algebraic equations (DAEs), Dai (1989). In other words, the dynamics of descriptor systems comprise a set of differential equations together with a set of algebraic constraints. Unlike conventional state space systems in which the initial conditions can be freely chosen in the operating region, in the descriptor systems, initial conditions must be *consistent*, i.e. they should satisfy the algebraic constraints. Consistent initialization of descriptor systems naturally happens in physical systems but should be taken into account when simulating such systems (Pantelides, 1988). Without loss of generality, we assume that $0 < \text{rank}(\mathbf{E}) = s < n$; $x(0) = x_0$ is a consistent (unknown) set of initial conditions. If the matrix $\mathbf{E}$ is non-singular (i.e. full rank), then the descriptor form reduces to the conventional state space. The number of algebraic constraints that must be satisfied by x_0 equals $n-s$. We assume the pair $(\mathbf{E}, A)$ to be *regular*, i.e. $\det(s\mathbf{E}-A) \neq 0$ for some $s \in \mathbb{C}$ and the triplet $(\mathbf{E}, A, C)$ to be observable, i.e., Ishihara and Terra (2002)

$$\text{rank}\begin{bmatrix} s\mathbf{E}-A \\ C \end{bmatrix} = n, \quad \forall s \in \mathbb{C}.$$

We also assume that the system is locally Lipschitz with respect to x in a region $\mathcal{D}$ containing the origin, uniformly in u, i.e.:

$$\Phi(0,u^*) = \Psi(0,u^*) = 0,$$
$$\|\Phi(x_1,u^*) - \Phi(x_2,u^*)\| \leqslant \gamma_1\|x_1 - x_2\|, \quad \forall x_1, x_2 \in \mathcal{D}$$
$$\|\Psi(x_1,u^*) - \Psi(x_2,u^*)\| \leqslant \gamma_2\|x_1 - x_2\|, \quad \forall x_1, x_2 \in \mathcal{D}$$

where $\|\cdot\|$ is the induced 2-norm, u^* is any admissible control signal and $\gamma_1, \gamma_2 > 0$ are the Lipschitz constants of $\Phi(x,u)$ and $\Psi(x,u)$, respectively. If the nonlinear functions $\Phi(x,u)$ and $\Psi(x,u)$ satisfy the Lipschitz continuity condition globally in $\mathbb{R}^n$, then the results will be valid globally. $w(t) \in \mathcal{L}_2[0,\infty)$ is an unknown exogenous disturbance, and $\Delta A(t)$ and $\Delta C(t)$ are unknown matrices representing time-varying parameter uncertainties, and are assumed to be of the form

$$\begin{bmatrix} \Delta A(t) \\ \Delta C(t) \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} F(t)N, \tag{1}$$

where M_1, M_2 and N are known real constant matrices and $F(t)$ is an unknown real-valued time-varying matrix satisfying $F^T(t)F(t) \leq I, \ \forall t \in [0,\infty)$.

2.1. Filter structure

We propose the general filtering framework of the following form

$$\begin{aligned}(\Sigma_o):\mathbf{E}\dot{x}_F(t) &= A_F x_F(t) + B_F y(t) + \mathcal{E}_1\Phi(x_F,u) \\ &\quad + \mathcal{E}_2\Psi(x_F,u) \\ z_F(t) &= C_F x_F(t) + D_F y(t) + \mathcal{E}_3\Psi(x_F,u).\end{aligned} \tag{2}$$

The proposed framework can capture both dynamic and static-gain filter structures by proper selection of $\mathcal{E}_1$, $\mathcal{E}_2$ and $\mathcal{E}_3$. Choosing $\mathcal{E}_1 = I$, $\mathcal{E}_2 = 0$ and $\mathcal{E}_3 = 0$ leads to the following dynamic filter structure:

$$\begin{aligned}\mathbf{E}\dot{x}_F(t) &= A_F x_F(t) + B_F y(t) + \Phi(x_F,u) \\ z_F(t) &= C_F x_F(t) + D_F y(t).\end{aligned} \tag{3}$$

Furthermore, for the static-gain filter structure we have:

$$\begin{aligned}\mathbf{E}\dot{x}_F(t) &= Ax_F(t) + \Phi(x_F,u) \\ &\quad + L[y(t) - Cx_F(t) - \Psi(x_F,u)] \\ z_F(t) &= x_F(t).\end{aligned} \tag{4}$$

Hence, with $A_F = A - LC,\ B_F = L,\ C_F = I,\ D_F = 0,\ \mathcal{E}_1 = I,\ \mathcal{E}_2 = -L,\ \mathcal{E}_3 = 0$, the general filter captures the well-known static-gain observer filter structure as a special case. We prove our result for the general filter of class (Σ_o). Now, suppose that $z(t) = Hx(t)$ stands for the controlled output for states to be estimated where H is a known matrix. The estimation error is defined as

$$\begin{aligned}e(t) \triangleq z(t) - z_F(t) &= -C_F x_F + (H - D_F C - D_F\Delta C)x \\ &\quad - D_F\Psi(x,u) - \mathcal{E}_3\Psi(x_F,u) - D_F Dw.\end{aligned} \tag{5}$$