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Reliable decentralized control of interconnected discrete delay systems[☆]

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ABSTRACT

In this paper, we study the problem of designing decentralized reliable state-feedback controllers under a class of actuator failures for a class of linear interconnected discrete-time systems having subsystem and coupling time delays. The failures take into consideration possible outages or partial failures in every single actuator. A decentralized stabilizing reliable feedback controller is derived at the subsystem level to give the closed-loop subsystem delay-dependent robust internal stability with a γ-level ℓ_2-gain. The results developed are tested on a representative water quality control example.

1. Introduction

Interconnected systems appear in a variety of engineering applications for which decentralized control schemes present effective means for designing control algorithms based on the individual subsystems (see (Mahmoud, 2010), where several design issues are delineated). The issue reliability is indeed of prime importance in the case of interconnected systems since failures could occur independently in each subsystem or actuator channel in the form of total outages or partial degradations. Approaches for designing reliable controllers for single systems include (Cheng & Zhao, 2004; Geromel, Bernussou, & de Oliveira, 1999; Hsieh, 2003; Veillette, 1995; Yang, Wang, & Soh, 2004; Zhao & Jiang, 1998). The literature on designing reliable controllers for interconnected systems is quite limited (see (Pujol, Rossell, & Pozo, 2007) where an initial effort at decentralized reliable control was developed for a class of interconnected systems). To the best of my knowledge, there are no results available in the literature coping with reliable and robust decentralized control for uncertain interconnected discrete-time delay systems that guarantee asymptotic stability with prescribed performance. The purpose of this paper is to bridge this gap and study the reliable decentralized feedback stabilization problem for a class of linear interconnected discrete-time systems with interval delays under a class of control failures. These failures are described by a model that takes into consideration possible outages or partial failures in every single actuator of each decentralized controller. We construct a Lyapunov–Krasovskii functional in order to exhibit the delay-dependent dynamics and develop a reliable decentralized feedback stabilization method which guarantees overall delay-dependent asymptotic stability with a prescribed ℓ_2 gain. The solution is expressed in terms of new parametrized linear matrix inequalities (LMIs) and the results developed are tested on a representative water quality control example.

Notation. We use W^t and W^{-1} to denote the transpose and the inverse of any square matrix W, respectively. We use (<0) to denote a symmetric negative definite matrix W. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. In symmetric block matrices or complex matrix expressions, we use the symbol $\bullet$ to represent a term that is induced by symmetry. Sometimes, the arguments of a function will be omitted when no confusion can arise.

2. The problem statement

We consider a class of linear systems S structurally composed of n_s coupled subsystems S_j and the model of the jth subsystem is described as

$$x_j(k+1) = A_j x_j(k) + D_j x_j(k - d_j(k)) + B_j u_j(k) + c_j(k) + \Omega_j w_j(k) \quad (1)$$

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$$z_j(k) = G_jx_j(k) + H_jx_j(k - d_j(k)) + L_ju_j(k) + \Phi_j w_j(k). \quad (2)$$

In the sequel, we treat the coupling vector $c_j(k) \in \Re^{n_j}$ as a piecewise-continuous vector function in its arguments that satisfies the quadratic inequality

$$c_j^t(k)\, c_j(k) \le \phi_j\, x_j^t(k)F_j^tF_jx_j(k) + \psi_j\, x_j^t(k - \eta_j(k))E_{dj}^tE_{dj}x_j(k - \eta_j(k)) \quad (3)$$

where for $j \in \{1, \dots, n_s\}$, $x_j(k) \in \Re^{n_j}$ is the state vector, $u_j(k) \in \Re^{m_j}$ is the control input, $w_j(k) \in \Re^{q_j}$ is the disturbance input which belongs to $\ell_2[0, \infty)$ and $z_j(k) \in \Re^{q_j}$ is the performance output. The matrices $A_j, B_j, D_j, \Phi_j, \Omega_j, L_j, G_j, H_j, F_j, E_{dj}$ are real and constant, $\phi_j > 0$, $\psi_j > 0$ are adjustable bounding parameters and $d_j(k), \eta_j(k), j \in \{1, \dots, n_s\}$, are unknown time delay factors satisfying

$$0 < d_j^- \le d_j(k) \le d_j^+, \qquad 0 < \eta_j^- \le \eta_j(k) \le \eta_j^+ \quad (4)$$

where the bounds $d_j^-, d_j^+, \eta_j^-, \eta_j^+$ are known constants present to guarantee smooth state trajectories from the initial condition $\phi_j(\cdot) \in \ell_2[-d_j^+, 0], j \in \{1, \dots, n_s\}$.

2.1. The failure model

An approach to actuator failure modeling is presented in Cheng and Zhao (2004), where a matrix of ones and zeros is introduced to reflect the actual working conditions of the actuators along with a fault-induced offset vector and a weighting matrix. As indicated in Cheng and Zhao (2004), this model is generally of the stuck type, which includes the so called lock-in-place, float and hard-over failure types, which are very common in practical systems such as aircraft and robotic manipulators. This approach is not directly amenable to the decentralized setting treated hereafter. In the sequel, we follow a different approach, by extending the representation of Veillette (1995). In such a representation, an independent outage or partial degradation in any single actuator of every decentralized controller is allowed. In this way, a rather general scenario is defined for the design of a reliable decentralized control structure for a class of interconnected systems facing simultaneously uncertainties and failures. Let $u_j^f \in \Re^{f_j}$ denote the vector of signals from the f_j actuators that control the jth subsystem. We consider the following failure model:

$$u_j^f(k) = \Sigma_ju_j(k) + \beta_j(u_j) \quad (5)$$

where $0 < \Sigma_j \in \Re^{f_j \times f_j} = \mathrm{diag}(\sigma_{j1}, \dots, \sigma_{jf_j})$ and the function $\beta_j(u_j) = [\beta_{j1}(u_{j1}), \dots, \beta_{jf_j}(u_{jf_j})]^t$ is uncertain and satisfies, for each j,

$$\beta_{jk}^2(u_j) \le \gamma_{jk}^2\, u_{jk}^2, \quad k = 1, \dots, f_j, \gamma_{jk}^2 \ge 0. \quad (6)$$

When (6) holds, then

$$\|\beta_j(u_j)\|^2 \le \|\Gamma_ju_j\|^2, \quad j = 1, \dots, n_s \quad (7)$$

where $0 \le \Gamma_j = \mathrm{diag}(\gamma_{j1}, \dots, \gamma_{jf_j}) \in \Re^{f_j \times f_j}$. The value of σ_{jk}, $k = 1, \dots, f_j$, represents the percentage of failure in the actuator j controlling subsystem S_j, and thus each subsystem actuator can fail independently.

Remark 2.1. Note that in this model the case $\sigma_{jm} = 1, \gamma_{jm} = 0$ corresponds to the normal case for the mth actuator of the jth subsystem $u_{jm}^f = u_{jm}$. When $\sigma_{jm} = \gamma_{jm}$, it is seen that (5)–(6) cover the outage case $u_j^f = u_j$ since $\beta_{jk} = -\gamma_{jk}\, u_{jk}$ satisfies (6). Obviously, the case $\beta_j(u_j) = -\Sigma_ju_j$ corresponds to the outage of the whole controller of the jth system. Partial failures or partial degradations of the actuators can be treated in a similar way.

Our objective in this paper is to develop a robust decentralized state-feedback control scheme in the presence of failures so as to guarantee the overall system's asymptotic stability with a prescribed performance measure.

3. State-feedback reliable design

In this section, we develop new criteria for LMI-based characterization of decentralized reliable stabilization by local state feedback. The criteria include some parameter matrices, where the aim is expanding the range of applicability of the conditions developed. Using the local state feedback $u_j(k) = K_jx_j(k), j = 1, \dots, n_s$, the local closed-loop subsystem under failure becomes

$$x_j(k + 1) = \widehat{A}_jx_j(k) + D_jx_j(k - d_j(k)) + B_j\beta_j(u_j) + c_j(k) + \Omega_jw_j(k), \quad \widehat{A}_j = A_j + B_j\Sigma_jK_j \quad (8)$$

$$z_j(k) = \widehat{G}_jx_j(k) + H_jx_j(k - d_j(k)) + L_j\beta_j(u_j) + \Phi_jw_j(k), \quad \widehat{G}_j = G_j + L_j\Sigma_jK_j. \quad (9)$$

Observe that by using the failure model inequality (7) with $u_j(t) = K_jx_j(t)$, we have

$$\beta_j^t(x_j)\beta_j(x_j) \le x_j^tK_j^t\Gamma_j^t\Gamma_jK_jx_j. \quad (10)$$

For the time being, we consider that the gains K_j are specified. Let $d_j^* = (d_j^+ - d_j^- + 1)$, $\eta_j^* = (\eta_j^+ - \eta_j^- + 1)$ represent the respective numbers of samples. The following theorem establishes the asymptotic stability conditions of the feedback design for subsystem S_j.

Theorem 3.1. *Given the bounds $d_j^+ > 0, d_j^- > 0, \eta_j^+ > 0, \eta_j^- > 0, j = 1, \dots, n_s$, the global system S with subsystem S_j given by (1)–(2) has delay-dependent internal stability with an ℓ_2-gain $< \gamma_j$ if there exist weighting matrices $0 < \mathbf{P}_j, 0 < \mathbf{Q}_j, 0 < \mathbf{Z}_{jm}, \forall j = 1, \dots, n_s, m = 1, \dots, n_s$, and scalars $\gamma_j > 0$ satisfying the following LMIs:*

$$\widehat{\Xi}_j = \begin{bmatrix} \widehat{\Xi}_{aj} & \Xi_{cj} & \widehat{\Xi}_{zj} \\ \bullet & -\widehat{\Xi}_{oj} & \widehat{\Xi}_{wj} \\ \bullet & \bullet & -I_j \end{bmatrix} < 0,$$

$$\widehat{\Xi}_{aj} = \begin{bmatrix} \Xi_{1j} & 0 & 0 & 0 \\ \bullet & -\mathbf{Q}_j & 0 & 0 \\ \bullet & \bullet & -I_j & 0 \\ \bullet & \bullet & \bullet & -I_j \end{bmatrix},$$

$$\Xi_{cj} = \begin{bmatrix} 0 & 0 & \widehat{A}_j^t\mathbf{P}_j & K_j^t\Gamma_j^t & F_j^t & 0 \\ 0 & 0 & D_j^t\mathbf{P}_j & 0 & 0 & E_{dj}^t \\ 0 & 0 & B_j^t\mathbf{P}_j & 0 & 0 & 0 \\ 0 & 0 & \mathbf{P}_j & 0 & 0 & 0 \end{bmatrix},$$

$$\widehat{\Xi}_{oj} = \begin{bmatrix} \widehat{\Xi}_{o1j} & 0 \\ \bullet & \Xi_{o2j} \end{bmatrix}$$

$$\widehat{\Xi}_{o1j} = \begin{bmatrix} -\gamma^2 I_j & 0 & \Omega_j^t\mathbf{P}_j \\ \bullet & -\sum_{k=1,k\neq j}^{n_s} \mathbf{Z}_{jk} & 0 \\ \bullet & \bullet & -\mathbf{P}_j \end{bmatrix}, \qquad \widehat{\Xi}_{zj} = \begin{bmatrix} \widehat{G}_j^t \\ H_j^t \\ L_j^t \\ 0 \end{bmatrix}$$

$$\Xi_{o2j} = \begin{bmatrix} -I_j & 0 & 0 \\ \bullet & -\phi_jI & 0 \\ \bullet & \bullet & -\psi_jI \end{bmatrix},$$

$$\widehat{\Xi}_{wj} = \begin{bmatrix} \Phi_j & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^t$$

$$\Xi_{1j} = -\mathbf{P}_j + d_j^*\mathbf{Q}_j + \sum_{k=1,k\neq j}^{n_s} \mathbf{Z}_{jk}. \quad (11)$$