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Discrete-time l_∞ and l_2 norm vanishment and low gain feedback with their applications in constrained control[☆]

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ABSTRACT

The existing low gain feedback, which is a parameterized family of stabilizing state feedback gains whose magnitudes approach zero as the parameter decreases to zero, has been designed in very specific ways. In this paper, by recognizing the l_∞ and l_2 slow peaking phenomenon that exists in discrete-time systems under low gain feedback, more general notions of l_∞ and l_2 norm vanishment are considered so as to provide a full characterization of the nonexistence of slow peaking phenomenon in some measured signals. Low gain feedback that does not lead to l_∞ and l_2 slow peaking in the control input are respectively referred to as l_∞ and l_2 low gain feedback. Based on the notions of l_∞ and l_2 vanishment, not only can the existing low gain feedback be recognized as an l_∞ low gain feedback, but also a new design approach referred to as the l_2 low gain feedback approach is developed for discrete-time linear systems. Parallel to the effectiveness of l_∞ low gain feedback in magnitude constrained control, the l_2 low gain feedback is instrumental in the control of discrete-time systems with control energy constraints. The notions of l_∞ and l_2-vanishment also result in a systematic approach to the design of l_∞ and l_2 low gain feedback by providing a family of solutions including those resulting from the existing design methods.

1. Introduction

Linear systems subject to actuator saturation have been very well studied during the past several decades. The interest in this problem is mainly motivated by the facts that every practical control system is subject to actuator saturation and such saturation is a source of limit cycles, parasitic equilibrium points and even instability of the closed-loop system (He, Chen, & Wu, 2007; Lin, 1998; Sussmann, Sontag, & Yang, 1994). Among the problems studied for control systems subject to actuator saturation, stabilization is the most important and fundamental one. The first question asked is under what conditions a linear system can be globally stabilized in the presence of actuator saturation. It is now well-known that global asymptotic stabilization is possible if and only if the linear system is asymptotically null controllable with bounded controls (ANCBC), that is, the linear system is stabilizable in the usual linear systems sense and all its open-loop poles are located in the closed left-half plane (inside or on the unit circle for discrete-time systems). It is also well known that, even under the ANCBC assumption, nonlinear feedback is usually needed for global asymptotic stabilization (Sussmann et al., 1994).

On the other hand, it has been shown that semi-global stabilization can be achieved by linear feedback (Lin & Saberi, 1993). The linear feedback laws that achieve semi-global stabilization for general ANCBC linear systems subject to actuator saturation were initially proposed in Lin and Saberi (1993) in the continuous-time setting and then extended to the discrete-time setting in Lin and Saberi (1995). The resulting feedback laws are known as low gain feedback (Lin, 1998), which has found applications in solving several other problems such as H_2 and H_∞ control (Chu, Liu, & Tan, 2002; Lin, 1998), global stabilization with input saturation (Grognard, Sepulchre, & Bastin, 2002; Sepulchre, 2000), stability region analysis in the presence of actuator saturation (Turner & Postlethwaite, 2001), nonlinear stabilization (Battilotti, 2001; Sepulchre, 2000), nonlinear H_∞ control (Lin, 1998) and stabilization of time-delayed systems (Lin, 2007; Zhou & Lin, 2011). Since the initial de-

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sign of low gain feedback in Lin and Saberi (1993, 1995), several other low gain design methods have been developed (Lin & Saberi, 1995; Lin, Saberi, & Stoorvogel, 1996; Teel, 1995; Zhou, Lin, & Duan, 2009a).

Even though low gain feedback has been widely used in the literature, as we have noted in our recent papers (Zhou, Lin, & Duan, 2009b,c, 2011), its characteristics have not yet been fully understood. Slow peaking is one of them: as the state of a continuous-time ANCBC linear system is made to converge to zero arbitrarily slowly by placing the poles of the closed-loop system close to the imaginary axis, its magnitude will climb slowly to an arbitrarily high value during the convergence process. Roughly speaking, the state cannot converge to zero very slowly without experiencing expansion in its magnitude. In a continuous-time setting, the slow peaking phenomenon has been recognized in Lin (1998) and utilized in semi-global stabilization of cascade nonlinear systems (Sepulchre, 2000).

As has been observed in the very beginning of the development of low gain feedback (Lin & Saberi, 1993), although the slow peaking phenomenon cannot be avoided in the state evolution of the closed-loop system under low gain feedback, the control signal that results from the multiplication of the state and the feedback gain can however be kept under an arbitrarily low level by decreasing the value of the low gain parameter. Therefore, the success of low gain design can be intuitively understood as the peaking in the different components of the state canceling each other when they are summed with feedback gains as weighting factors. Prompted by such an observation, the more general problems called norm vanishment were initially studied in Zhou et al. (2009b,c, 2011) in the continuous-time setting. The notion of norm vanishment not only results in a rigorous definition of the existing (L_∞) low gain feedback, but also reveals some differences and connections between the traditional L_∞ low gain feedback and the newly proposed L_2 low gain feedback, which has been shown to play a very important role in the stabilization of linear systems with the control energy constraints. By characterizing the necessary and sufficient conditions for norm vanishment, necessary and sufficient conditions for testing whether a feedback gain is an L_∞ or an L_2 low gain feedback were proposed. As a by-product, a systematic approach to the design of low gain feedback was established, which results in a family of solutions to the L_∞ and L_2 low gain feedback design.

The aim of the present paper is to extend the results obtained in Zhou et al. (2009b,c, 2011) to discrete-time setting. After clarifying that the slow peaking phenomenon that exists in a continuous-time setting also exists in the discrete-time linear systems under low gain feedback design, we will introduce the notions of l_∞ and l_2 norm vanishment. A couple of equivalent characterizations of these notions in terms of the coefficient matrices will be established. As applications of these new notions, the l_∞ and l_2 low gain feedback are introduced for discrete-time linear systems subject to input magnitude constraints and input energy constraints, respectively. It is shown that the l_2 low gain feedback can be utilized to achieve semi-global stabilization of discrete-time ANCBC linear systems with control energy constraints. Moreover, by utilizing the proposed characterizations of l_∞ and l_2 norm vanishment, a systematic approach is also established to the design of l_∞ and l_2 low gain feedback for discrete-time ANCBC linear systems. This new approach yields a family of solutions rather than a particular one such as the eigenstructure assignment based approach (Lin & Saberi, 1995), by introducing more design freedom that can be utilized to achieve other requirements, for example, improved transient performances of the closed-loop systems.

Although the work of this paper has been motivated by its continuous-time counterparts (Zhou et al., 2009b,c, 2011), the development of the results involve techniques that are very different from those used in Zhou et al. (2009b,c, 2011). For example, in comparison with the continuous-time setting considered in Zhou et al. (2009b,c, 2011), the necessary condition for characterizing the l_∞ and l_2 norm vanishment in a discrete-time setting is harder to prove and requires the establishment of quite a different technique. In the continuous-time setting, the necessary condition is proven via contradiction by showing that the peaking phenomenon will appear in the measured signals at the time that is specified as a continuous function of the low gain parameter. However, in the discrete-time setting, as the time variable is varying discretely but the low gain parameter is varying continuously, we must find a new approach to detect the peaking phenomenon in the measured signals when the low gain parameter approaches zero (see the proof of Lemma 2 for details).

As another example, in the development of the systematic approach to the L_ϕ low gain design in the continuous-time setting (Zhou et al., 2009b,c, 2011), the poles of the closed-loop system are obtained by shifting the open loop poles to their left by ε, the low gain parameter, and correspondingly, the Jordan form of the closed-loop system is obtained by shifting the open loop Jordan by $-\varepsilon I$. In developing the l_ϕ low gain design in the present discrete-time setting, while the poles of the closed-loop system are obtained by shrinking the poles of the open-loop system by a factor of $1-\varepsilon$, the Jordan form of the closed-loop system is *not* obtained from the Jordan form of the open-loop system by a factor of $(1-\varepsilon)I$, as one may expect. As a result, some nonsingular parametric matrices should be constructed explicitly to transform the closed-loop system matrices into their Jordan canonical forms (see the proof of Theorem 5 for details).

The remainder of this paper is organized as follows. The l_∞ and l_2 slow peaking phenomena that result from the low gain feedback control for discrete-time linear systems will be examined in Section 2, where the problems of low gain feedback analysis and design are also given formally. In Section 3, we give solutions to the problem of low gain analysis by formally defining of the notions of l_∞-vanishment and l_2-vanishment and developing a series of necessary and sufficient conditions to test these two properties. Consequently, solutions to the problem of low gain design are proposed in Section 4. A numerical example is presented in Section 5 to illustrate the design procedure of l_∞ and l_2 low gain feedback. The paper is concluded in Section 6.

Notation. In this paper, we will use fairly standard notation. We use $A^T, \lambda(A)$, $\det(A)$ and $\text{rank}(A)$ to denote the transpose, the eigenvalue set, the determinant, and the rank of matrix A, respectively. The symbol $\mathbf{I}[p,q]$, where p and q are two integers with $p \leq q$, denotes the set $\{p, p+1, \ldots, q\}$, $\mathbf{C}^{\odot} \triangleq \{z : |z| < 1\}$, $\mathbf{Z} \triangleq \{0, 1, 2, \ldots\}$, $\text{sat}(u_i) \triangleq \text{sign}(u_i)\min\{1, |u_i|\}$, $\mathscr{F} \triangleq \{f(\varepsilon) : [0,1] \to [0,1]\}$, and $\binom{k}{n} \triangleq \frac{n!}{k!(n-r)!}$ denotes the number of combinations. For a nonnegative number a, the symbol $\text{ceil}(a)$ rounds the elements of a to the nearest integers greater than or equal to a. For two matrices $A \in \mathbf{R}^{m\times n}$ and $B \in \mathbf{R}^{p\times q}$, $A \otimes B$ denotes their Kronecker product. The notation $\|A\|$ refers to any norm of matrix A if the subscript is omitted and $\|A\|_F$ refers to the Frobenius norm of A. We use $\text{Jord}_n\{a\}$ to denote the (block) Jordan matrix whose n diagonal elements are a. Furthermore, we use $\mathscr{N}_n \triangleq \text{Jord}_n\{0\} \in \mathbf{R}^{n\times n}$ to denote a nilpotent matrix. We denote by $\text{diag}\{A_i\}_{i=p}^{q}$ a (block) diagonal matrix whose diagonal elements are A_i, $i \in \mathbf{I}[p,q]$. For a function $F(k) : \mathbf{Z} \to \mathbf{R}^{m\times n}$, the l_∞ norm and l_2 norm of $F(k)$, if they exist, are respectively defined as

$$\|F(k)\|_{l_\infty} \triangleq \sup_{k\in\mathbf{Z}} \{\|F(k)\|\},$$

and

$$\|F(k)\|_{l_2} \triangleq \left(\sum_{k=0}^{\infty} \|F(k)\|^2\right)^{\frac{1}{2}}.$$