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Brief paper

A constrained control strategy for the shape control in thermonuclear fusion tokamaks[☆]

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ABSTRACT

The paper deals with the application of the so-called Reference (or Command) Governor constrained control strategy to the shape control of plasmas in thermonuclear fusion reactors with the main scope of optimizing tokamak operations also in conditions very close to the operating envelope limits. A primal inner loop controlling the plasma–wall distance is first designed; the Reference Governor device is then tuned to modify, whenever necessary, the reference signals to the inner loop, on the basis of constraints due to voltage saturations on the power supply converters, limitations of currents in the active control coils, minimum clearance between the plasma surface and the vacuum chamber wall, maximum induced magnetic fields and forces on coils. As usual in model predictive paradigms, the reference signal modification is accomplished through an on-line optimization procedure which embodies plasma model forecasts computed along a finite time virtual receding horizon. The ITER (International Thermonuclear Experimental Reactor) tokamak is assumed as the case study. Numerical simulations are carried out on a finite elements nonlinear model taking into account induced currents in the passive structures. The proposed application shows how almost a hundred constraints can be managed on-line by the Reference Governor.

1. Introduction

In tokamak devices, Wesson (1997), plasma control has gained relevance due to increasing performance demand. To this extent modern tokamaks like ITER are designed to obtain plasmas which are significantly elongated and vertically unstable, placed as close as possible to the metallic facing components. This ensures a good passive stabilization, due to the eddy currents induced in these metallic structures, and an efficient use of the available volume.

Contact between plasma and chamber walls is always a major concern during operations. The desired plasma–wall clearance is obtained by regulating currents in a number of PF (Poloidal Field) coils surrounding the plasma ring.

Fig. 1 shows a poloidal section of ITER: this is a 500 MW fusion power reactor under construction in Cadarache (France). Voltages applied to the 12 PF active coils are generated by power supplies driven by a feedback control system. The feedback action can be divided in two parts: position control guarantees vertical stability of plasma that would be naturally unstable if elongated, whereas shape control guarantees the control of plasma geometrical parameters such as elongation, triangularity, and plasma–wall distance.

Usually both plasma position and shape controllers have quite a simple structure and are mainly based on multi-loop Proportional Integral Derivative (PID) actions. In order to maximize the tokamak performance, the distance between the plasma boundary and the vessel at some specific points (gaps) is usually controlled together with the vertical speed to ensure stability.

Multiple-input–multiple-output control approaches have been deeply investigated in the last decade to improve shape control performances, Ariola et al. (1999), Crisanti et al. (2003), and Humphreys et al. (2005). The most successful are linear model based approaches which however do not take into account the possibility that some physical variables of interest can exceed their operating limits.

In Ambrosino, Ariola, Pironti, and Walker (2001), Varano et al. (2010), and Vitelli et al. (2010), some attempts to deal with control inputs saturation are described. However, during tokamak operations, not only voltages and currents, but also induced

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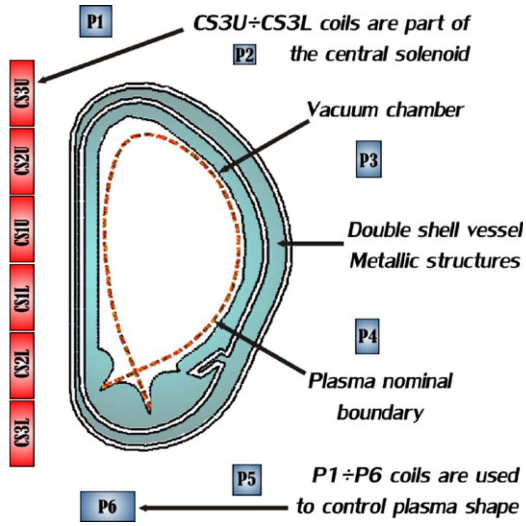

Fig. 1. ITER poloidal section.

electromagnetic fields and forces, and shape parameters must stay within prescribed ranges. An important motivation for the present work is that active tokamaks have been designed with large operational margins. This does not happen in ITER, and will not happen in future large machines where the impact of margins on costs grows very rapidly.

The need to satisfy input and/or state dependent constraints is in general a relevant problem in control theory and practice. Anti-Windup (AW), Bumpless methods, AW/LQR, and AW/H2, are feedback control methodologies dealing with the presence of input constraints in an indirect manner. Recently, techniques based on invariant sets arguments and predictive control ideas, Kothare, Balakrishnan, and Morari (1996) and Mayne, Rawlings, Rao, and Scokaert (2000), have gained in popularity due to their inherent capability to take the presence of constraints directly into account during the design phase. The control action is computed through the solution of a sequence of optimization problems based on the prediction of the plant state evolution. The objective is to jointly maximize the control performance and enforce the satisfaction of prescribed constraints. The interest towards a practical application of such methodologies has been growing in the last decade due to the availability of fast computing units, Diel, Bock, and Schloder (2005).

Some of the predictive control methodologies are mainly devoted to constraints fulfillment leaving control performance satisfaction to traditional regulation frameworks. Such a family of control strategies is known in the literature as the *Reference Governor* (RG) approach. The RG is a nonlinear device which is added to a pre-compensated plant, designed so as to exhibit stability and good tracking performance in the RG's absence. At each time instant t_k, it computes a modified reference command which, if applied from t_k onward, does not produce constraint violations. Such a modified reference command is computed to minimize its distance from the actual desired reference signal, according to an on-line constrained procedure on a receding horizon finite time interval. Many mature assessments of the RG *state of the art* for linear systems can be found in Bemporad (1998), Bemporad and Mosca (1995), Borrelli, Falcone, Pekar, and Stewart (2009), Casavola, Mosca, and Angeli (2000), Gilbert and Kolmanovsky (2002), Gilbert, Kolmanovsky, and Tin Tan (1995), Kolmanovsky and Sun (2006), and Vahidi, Kolmanovsky, and Stefanopolou (2007).

2. Mathematical modeling of the plasma response

Three main subsystems need to be considered to obtain a control oriented model of plasma shape and position evolution in a tokamak: the plasma, the control circuits, and the passive conductors.

The physical phenomenon to be controlled is governed by Maxwell's equations in their quasi-stationary form where the electric field does not vary too rapidly and the current density is *divergence free* with the constitutive relationships. In axial-symmetric geometry, with cylindrical coordinates (r, φ, z), the magnetic field and current density vectors B and J can be expressed in terms of two scalar functions, namely, the poloidal magnetic flux per radians ψ and the poloidal current function $f = rB_\varphi$, B_φ being the toroidal magnetic field.

At the time scale of interest for current, position, and shape control, because of the low plasma mass density, inertial effects can be neglected. Hence the plasma momentum balance becomes $J \times B = \nabla p$ at equilibrium, that can be rewritten as the well known *Grad–Shafranov equation*, Wesson (1997):

$$\begin{aligned}\Delta^*\psi &= r\frac{\partial}{\partial r}\left(\frac{1}{\mu_r r}\frac{\partial\psi}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\mu_r}\frac{\partial\psi}{\partial z}\right)\\ &= -f\frac{df}{d\psi} - \mu_0 r^2\frac{dp}{d\psi}.\end{aligned} \tag{1}$$

$\mu_r = \mu/\mu_0$ being the relative magnetic permeability, and μ_0 the permeability of free space.

To complete modeling with the interaction between plasma and surrounding passive structures and active coils the following partial differential equation problem can be written taking into account the axial-symmetry

$$\begin{cases}\Delta^*\psi = -f\dfrac{df}{d\psi} - \mu_0 r^2\dfrac{dp}{d\psi} & \text{in plasma region}\\ \Delta^*\psi = -\mu_0 r j_{ext} & \text{in conductors}\\ \Delta^*\psi = 0 & \text{elsewhere}\end{cases} \tag{2}$$

with the initial and boundary conditions

$$\begin{cases}\psi(r, z, 0) = \psi_0(r, z),\\ \psi(0, z, t) = 0,\\ \lim_{r^2+z^2\to\infty}\psi(r, z, t) = 0.\end{cases} \tag{3}$$

j_{ext} being the toroidal current density in the external conductors and coils. Both j_{ext} and ψ depend on space (r and z in axial-symmetric conditions), and time.

The above equations are used to calculate the poloidal flux function at time t provided that the plasma boundary can be determined, the toroidal current density in the PF is known and the functions $p(\psi)$ and $f(\psi)$ are defined. The plasma boundary is determined by means of an iterative numerical procedure. Functions $p(\psi)$ and $f(\psi)$ can be expressed in terms of global plasma parameters, for example I_{pl}, β_{pol} and l_i, namely plasma current, poloidal beta, and internal inductance. As for the toroidal current density j_{ext}, it can be expressed as a linear combination of the PF circuit currents. The dependence of $p(\psi)$ and $f(\psi)$ functions on I_{pl}, β_{pol} and l_i is assigned implicitly with a suitable parameterization of such functions. In practice this is obtained by adding constraints to the numerical solution of problem (2) and (3).

Therefore, the magnetic flux and the plasma configuration can be determined when prescribing the vector of currents, including poloidal field currents and plasma current along with β_p and l_i. The time evolution of currents is governed by circuit equations driven by voltages in the active coils. Induced eddy currents in the metallic