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Brief paper

Distributed sequential algorithms for regional source localization[☆,☆☆]

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ABSTRACT

We study the problem of source localization as a multiple hypothesis testing problem, where each hypothesis corresponds to the event that the source belongs to a particular region. We use sequential hypothesis tests based on posterior computations to solve for the correct hypothesis. Measurements corrupted with noise are used to calculate conditional posteriors. We prove that the regional localization problem has asymptotic properties that allow correct detection almost surely in the limit of a large number of measurements. We present the *Sense, Transmit & Test* distributed algorithm that allows sequential sensing, communication and testing and we analyze the accuracy of this distributed algorithm and show that the test ends in a finite time. We also present numerical results illustrating properties of the suggested algorithm.

1. Introduction

Problem description and motivation. Applications where source localization is of great concern, vary between finding the source of oil spills in the ocean, determining cellular locations, detecting an earthquake's epicenter, locating an acoustic source, or simply finding an intruder in a protected environment. For most of these applications, it is sufficient to find a region that contains the source rather than pinpointing the exact source position, which relies most of the time on approximations.

In this work we consider the following problem: A source at an unknown location in a bounded region Q transmits a power signal. N sensors receive noisy and decayed versions of the signal, they can communicate and exchange measurements. The environment Q is divided into M regions W_α, where $\alpha \in \{1, \dots, M\}$. The objective of the sensors is to find which region contains the source.

We pose the problem as a multiple hypothesis testing problem, where hypothesis H_α is true if the source lies in the region W_α. We assume no prior knowledge about the location of the source and therefore model the source location as a uniformly distributed random variable over the environment Q, any prior information about the source location can be incorporated in the location density function. We adopt the log-normal fading model for the propagation of the received signal power. The noise added to the log of the power is Gaussian with zero mean and a known variance σ^2.

Literature review. In the classical source localization problem, a number of sensors collaborate to locate the exact position of a source. The relation between the position of a source and the received signal strength (RSS) is described in Chen, Yao, and Hudson (2002), Proakis and Salehi (2001), Rappoport (1996) and Sayed, Tarighat, and Khajehnouri (2005). Several authors treat localization as a nonconvex optimization problem (Hero & Blatt, 2005; Rabbat & Nowak, 2004a). Gradient descent algorithms and weighted least squares approximations can be used to solve the maximum likelihood estimation problems but such algorithms tend to get stuck at local optima (Mao, Fidan, & Anderson, 2007; Rabbat & Nowak, 2004b). Authors in Meng, Ding, and Dasgupta (2008) approximate the nonlinear nonconvex optimization problem by a linear and convex problem. Hero and Blatt (2005) use a method of projection onto convex sets. A necessary and sufficient condition for the convergence of this algorithm is that the source lies inside the convex hull of the sensors. Properly placing the sensors assumes knowledge of the position of the source.

Designing distributed algorithms is in general a problem specific task, and many researchers from various communities

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have looked at this problem. We refer the reader to Boyd, Ghosh, Prabhakar, and Shah (2006), Lynch (1997), Nedic and Ozdaglar (2009) and references therein for more details.

The multiple hypothesis problems are considerably more difficult than the binary problem and optimality of the proposed algorithms is usually hard to prove. Some tests that have some asymptotic optimality properties were developed in the literature, but these tests tend to be very complex (Armitage, 1950; Baum & Veeravalli, 1994; Savin, 1984). Alternatively ad hoc tests based on repeated pairwise applications of optimal sequential hypothesis tests (Wald, 1945) were developed but these tests have little optimality results, e.g., see Eisenberg (1991). Some work in the literature look at locating a source inside a region in different contexts, such as triangulation and fingerprinting, than the one studied in this paper (You, Yoo, & Cha, 2007; Zhang, Cao, Chen, & Chen, 2009). For a survey on localization algorithms, see Srinivasan and Wu (2007).

Contributions. The contributions of this paper are three-fold.

First, we formulate the source localization problem in a novel multi-hypothesis testing setting. We analyze properties of the Maximum A Posteriori (MAP) algorithm that requires the computation of a finite number of integrals which is to be compared to the need to solve a nonlinear, nonconvex problem in the classical source localization problem. We provide a proof of almost sure convergence of the MAP solution asymptotically in the limit of a large number of measurements, a step that tends to be missing in all of the work presented earlier in the source localization literature.

Second, inspired by the proof of convergence of the MAP solution, we propose and implement a distributed sequential regional localization algorithm: *Sense, Transmit & Test*. This algorithm allows for sequential sensing, transmission and testing at each processor. We allow each processor to have one or multiple regions of responsibility and relate the probability of error for each processor in the case of multiple regions to the probability of error in the case of a single region. We also show that the test ends in a finite time under mild conditions on the sensor locations.

Third, we illustrate the results of the *Sense, Transmit & Test* and show how the expected decision time for a network increases with the required accuracy and noise. We also provide numerical results illustrating how it is possible to increase the level of localization accuracy at the expense of the expected decision time for the network for a fixed decision accuracy.

Paper organization. The paper proceeds as follows: we formulate the problem as a multi-hypotheses testing problem in Section 2. We present a distributed algorithm for the problem in Section 3. We present in Section 4 numerical results showing the performance of the algorithm as various parameters are changed. We conclude in Section 5.

2. Source localization as multi-hypothesis testing

We start this section by introducing the model and the problem definition.

2.1. Model and problem definition

Consider a compact connected environment $Q \subset \mathbb{R}^2$. Suppose that there are N sensors placed at positions $q_i \in Q$ with $i \in \{1, \dots, N\}$, and that the source located at an unknown location $s \in Q$ transmits a signal whose power undergoes log-normal shadowing summarized as follows. The average power loss for an arbitrary Transmitter–Receiver separation is expressed as a function of distance by using a path loss exponent $\rho > 2$. Recall that the standard ideal model of log-normal fading states that the received power at a sensor i is $P_i = \frac{P}{\|q_i - s\|^\rho}$, where ρ is the rate at which the power loss increases with distance and where $\|q_i - s\|$ is measured in appropriate units. In this paper, we adopt a more realistic version of this ideal model. Specifically, we assume the received power is

$$\ln P_i = \ln(P) - \ln(1 + \|q_i - s\|^\rho) + n_i, \tag{1}$$

where (1) the unit additive term in the fading term is introduced so that the received power is well defined near the source and equal to the transmitted power P at the source, and (2) the variable n_i is the noise associated with sensor i. We assume all n_i are independent and identically distributed (i.i.d) Gaussian random variables with zero mean and known variance σ^2. The joint probability density function of the received power $P_r = [P_1, \dots, P_N]^T$, conditioned on the source location $y \in Q$ is

$$\mathbb{p}(P_1, \dots, P_N|y) = \frac{1}{(2\pi\sigma^2)^{N/2}} \times \exp\left(- \frac{\sum_{i=1}^{N} \left(\ln P_i - \ln\left(\frac{P}{1+\|q_i - y\|^\rho} \right) \right)^2}{2\sigma^2} \right). \tag{2}$$

Problem 2.1 (*MAP Point Localization Problem*)**.** Compute the position that maximizes the posterior of the joint observations, that is compute

$$y^* = \arg\max_{y \in Q} \mathbb{p}(P_1, \dots, P_N|y)\mathbb{P}(y).$$

Here $\mathbb{p}(P_1, \dots, P_N|y)$ is the joint conditional probability and $\mathbb{P}(y)$ is the prior probability.

Problem 2.1 is a nonlinear nonconvex optimization problem. Attempts to solve this problem, usually revert to relaxing the problem or approximating its solution without providing a convergence analysis. In this paper we look for a regional localization, so the conditioning on the exact position y in (2) is replaced by a conditioning on the source being in a region W_i. The environment Q with area A is divided into M regions $\{W_1, \dots, W_M\}$ with positive areas $\{A_1, \dots, A_M\}$. Each W_α has a positive measure, each intersection $W_\alpha \cap W_\beta$ has zero measure, and $\cup_{\alpha=1}^{M} W_\alpha = Q$. The hypothesis H_α is true if and only if $s \in W_\alpha$.

Problem 2.2 (*MAP Regional Localization Problem*)**.** Compute the hypothesis H_α that maximizes the posterior of the joint observations, that is, compute

$$\alpha^* = \arg\max_{\alpha \in \{1,\dots,M\}} \mathbb{p}(P_1, \dots, P_N|H_\alpha)\mathbb{P}(H_\alpha). \tag{3}$$

2.2. Regional posterior density

Assuming no prior knowledge about the location of the source, the density describing $s \in Q$ is

$$\mathbb{p}(s) = \begin{cases} 1/A, & \text{if } s \in Q, \\ 0, & \text{otherwise.} \end{cases}$$

Definition 2.3 (*Repeated Measurements*)**.** The ith sensor takes k repeated i.i.d. noisy measurements and computes the average of the logarithms of the measurements

$$\ln \mathbf{P}_i(k) = \sum_{t=1}^{k} \frac{\ln P_i(l)}{k}. \tag{4}$$