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Brief paper

Controller design for disturbance decoupling of Boolean control networks[☆]

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ABSTRACT

We investigate a type of disturbance decoupling problem (DDP) of Boolean control networks. Using the semi-tensor product of matrices, the dynamics of a Boolean control network is expressed in its algebraic form. Under the framework of output-friendly subspace, we give a necessary and sufficient condition for the solvability of DDP by analyzing the redundant variables, and we present a computationally feasible method to construct all the valid feedback control matrices. The logical functions of each controller can be recovered from the obtained feedback control matrix. Finally, an example is provided to show the effectiveness of the proposed method.

1. Introduction

The Boolean network (BN) is often used as a model for gene regulation which treats genes as binary nodes that are either expressed or unexpressed (Dee & Ghil, 1984; Kauffman, 1969, 1993). It is also a candidate for representing a class of behaviors observed in large regulatory networks (see Kauffman, Peterson, Samuelsson, & Troein, 2003). In order to manipulate networks, the control of BNs is a very important topic (e.g., Akutsu, Hayashida, Ching, & Ng, 2007; Ideker, Galitski, & Hood, 2001). A Boolean control network (BCN) can be considered as a BN with additional binary inputs and outputs. One can use various existing approaches to investigate BCNs.

Recently, Cheng and colleagues have proposed a semi-tensor product (STP) technique, which can express the dynamics of BNs in an algebraic state-space form and is hence useful for studying the dynamics and control of BNs (Cheng & Qi, 2010; Cheng, Qi, & Li, 2011). With this technique, the structures of attractors and basins of BNs were analyzed in Cheng (2009) and Yang and Chu (2012), and the complete synchronization of BNs was examined in Li and Chu (2012). Some control problems concerning BNs, such as controllability and observability (Cheng & Qi, 2009; Li & Sun, 2011a), realization (Cheng, Li, & Qi, 2010), stability and stabilization (Li & Sun, 2011b) were addressed in the same manner. Besides, the STP technique has also extended to other problems in a wider range of applications (Li & Wang, 2012; Xu & Hong, 2012).

Disturbance decoupling problem (DDP) is a fundamental problem in control theory. Most existing studies of DDP are concerned with linear systems (Wonham, 1979), smooth nonlinear systems (Isidori, 1995), and switched systems (Yurtseven, Heemels, & Camlibel, 2012; Zhang, Cheng, & Li, 2005). The geometric control theory has been widely used to solve the DDPs for these systems. However, for the case of BCNs, there have been only a few results on DDPs available to date. In Cheng (2011), a type of DDP for BCNs is investigated based on the STP technique. The basic idea for solving DDP proposed in Cheng (2011) consists of two key issues. Firstly, finding a coordinate transformation such that in the new coordinate frame, the outputs are only involved in a set of coordinates, and these coordinates form a output-friendly subspace. Secondly, finding controllers such that the dynamic equations of the output-friendly coordinate variables are disturbance independent. Cheng (2011) proposed an algorithm to solve the first issue. As for the second issue, the condition for the solvability of DDP was derived and how to solve DDP by constant controls based on canalizing Boolean mapping was discussed. However, to the best of our knowledge, no direct method of finding all the existent controllers has been presented in Cheng (2011) and the references therein.

The aim of this paper is to find all the existent controllers to solve DDP, which is also referred to as controller design. We first derive the algebraic form of the BCN by using the STP technique. Then, for a given output-friendly subspace, we design the

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controllers such that the output-friendly subspace is disturbance invariant.

The rest of the paper is organized as follows. Section 2 gives preliminaries concerning the algebraic expression of BCNs and the coordinate transformation. Section 3 presents the DDP formulation. A method for designing DDP controllers is proposed in Section 4. An example is given in Section 5 to illustrate the proposed method. Section 6 offers concluding remarks.

2. Preliminaries

This section gives a brief introduction to the STP of matrices, the algebraic form and the coordinate transformation of BCNs.

The STP of matrices was first proposed by Cheng et al. (2011).

Definition 1 (*Cheng & Qi, 2010*)**.** Given an $m \times n$ matrix A and a $p \times q$ matrix B, the STP of A and B is defined as

$$A \ltimes B = (A \otimes I_{l/n})(B \otimes I_{l/p}),$$

where "$\otimes$" is the Kronecker product of matrices and l is the least common multiple of n and p.

Note that if $n = p$, then $A \ltimes B = AB$. Hence, the STP of matrices is a generalization of the conventional matrix product. Based on this, the symbol "$\ltimes$" is omitted in most cases hereafter.

Let δ_n^i be the i-th column of the identity matrix I_n. Set $\Delta_n = \{\delta_n^i \mid 1 \le i \le n\}$. For notational ease, denote $\Delta = \Delta_2$. Denote by Col(A) the set of columns of a matrix A. An $n \times s$ matrix L is called a logical matrix if $\mathrm{Col}(L) \subset \Delta_n$. The set of $n \times s$ logical matrices is denoted by $\mathcal{L}_{n\times s}$. We simply write an $n \times s$ logical matrix $L = [\delta_n^{i_1}, \delta_n^{i_2}, \ldots, \delta_n^{i_s}]$ as $L = \delta_n[i_1, i_2, \ldots, i_s]$.

Let $\mathcal{D} = \{1, 0\}$. A logical function is a mapping from $\mathcal{D}^n$ to $\mathcal{D}$. A logical mapping $F : \mathcal{D}^n \to \mathcal{D}^k$ is defined by k logical functions. Let $F_l\{x_1, \ldots, x_n\}$ denote the set of logical functions with logical arguments $x_1, \ldots, x_n$.

To use the algebraic expression of logical mappings, we identify the elements in $\mathcal{D}$ with 2-dimensional vectors as: $1 \sim \delta_2^1$, $0 \sim \delta_2^2$. Then a logical mapping $F = (f_1, \ldots, f_k) : \mathcal{D}^n \to \mathcal{D}^k$ can be regarded as a mapping from Δ^n to Δ^k. Referring to Cheng and Qi (2010), there exists a unique matrix $M_F \in \mathcal{L}_{2^k\times 2^n}$ such that

$$\ltimes_{i=1}^{k} f_i(x_1, \ldots, x_n) = M_F \ltimes_{i=1}^{n} x_i$$

for every $(x_1, \ldots, x_n) \in \Delta^n$. We call M_F the structure matrix of the logical mapping F.

Now we introduce some special logical matrices that will be used in the later discussion. Let M_c, M_d, M_i, M_e, and M_n denote, respectively, the structure matrices of the logical operators conjunction $\wedge$, disjunction $\vee$, conditional $\to$, biconditional $\leftrightarrow$, and negation $\neg$. Then

$$M_c = \delta_2[1, 2, 2, 2], \qquad M_d = \delta_2[1, 1, 1, 2],$$
$$M_i = \delta_2[1, 2, 1, 1], \qquad M_e = \delta_2[1, 2, 2, 1], \qquad M_n = \delta_2[2, 1].$$

For each $n \in \mathbb{N}$, let

$$\Psi(n) = \delta_{2^{2n}}[1, 2^n + 2, 2 \cdot 2^n + 3, \ldots, (2^n - 1)2^n + 2^n, 2^{2n}].$$

A straightforward computation shows that

$$x^2 = \Psi(n)x, \quad x \in \Delta_{2^n}.$$

Let $m, n \in \mathbb{N}$. The swap matrix $W_{[m,n]}$ is defined as

$$W_{[m,n]} = \delta_{mn}[1, m+1, \ldots, (n-1)m+1, 2, m+2, \ldots, (n-1)m+2, \ldots, m, 2m, \ldots, nm].$$

It is easy to verify that

$$x_2x_1 = W_{[m,n]}x_1x_2, \quad x_1 \in \Delta_m,\ x_2 \in \Delta_n.$$

The dummy matrix E_d is defined as

$$E_d = \delta_2[1, 2, 1, 2].$$

Since $E_d\delta_2^1 = E_d\delta_2^2 = I_2$, it follows that $E_dx = I_2$ for $x \in \Delta$, so that

$$E_dx_1x_2 = x_2, \qquad E_dW_{[2,2]}x_1x_2 = x_1, \quad x_1, x_2 \in \Delta.$$

We are now ready to introduce the algebraic form of BCNs. In general, the dynamics of a BCN with n nodes, m inputs, and p outputs can be expressed as

$$\begin{cases} x_i(t+1) = f_i(u_1(t), \ldots, u_m(t), x_1(t), \ldots, x_n(t)), \\ \qquad i = 1, \ldots, n, \\ y_j(t) = h_j(x_1(t), \ldots, x_n(t)), \quad j = 1, \ldots, p, \end{cases} \tag{1}$$

where $x_i(t)$, $u_k(t)$, and $y_j(t)$ are the state, control input, and output at the time t, respectively. Suppose $M_{f_i} \in \mathcal{L}_{2\times 2^{m+n}}$ and $M_{h_j} \in \mathcal{L}_{2\times 2^n}$ are the structure matrices of the logical functions f_i and h_j, respectively. Let $x(t) = \ltimes_{i=1}^n x_i(t)$ and $u(t) = \ltimes_{k=1}^m u_k(t)$. It then follows from (1) that

$$\begin{cases} x_i(t+1) = M_{f_i}u(t)x(t), \quad i = 1, \ldots, n, \\ y_j(t) = M_{h_j}x(t), \quad j = 1, \ldots, p. \end{cases} \tag{2}$$

Moreover, by multiplying all the state equations and output equations in (2) respectively, the dynamics of the BCN (1) can further be expressed as

$$\begin{cases} x(t+1) = Lu(t)x(t), \\ y(t) = Hx(t), \end{cases}$$

where $L = M_{f_1} \ltimes_{i=2}^n [(I_{2^{m+n}} \otimes M_{f_i})\Psi(n+m)] \in \mathcal{L}_{2^n\times 2^{n+m}}$ and $H = M_{h_1} \ltimes_{j=2}^p [(I_{2^n} \otimes M_{h_j})\Psi(n)] \in \mathcal{L}_{2^p\times 2^n}$.

The coordinate transformation (or coordinate change) of a logical dynamics was first introduced in Cheng et al. (2010).

Definition 2. Let $(x_1, \ldots, x_n)$ be the state variables of nodes of a Boolean (control) network. A mapping $F : \mathcal{D}^n \to \mathcal{D}^n$ is said to be a logical coordinate transformation, if F is a bijective mapping.

Definition 3. Let $y_1, \ldots, y_k \in F_l\{x_1, \ldots, x_n\}$. $\{y_1, \ldots, y_k\}$ is said to be a k-dimensional regular subspace with regular sub-basis $\{y_1, \ldots, y_k\}$ if there exist $y_{k+1}, \ldots, y_n \in F_l\{x_1, \ldots, x_n\}$ such that $\{x_i \mid i = 1, \ldots, n\} \mapsto \{y_i \mid i = 1, \ldots, n\}$ is a coordinate transformation.

3. Problem formulation

Assume the BCN (1) is disturbed by some disturbance inputs. Then its dynamics becomes

$$\begin{cases} x_i(t+1) = f_i(u_1(t), \ldots, u_m(t), x_1(t), \ldots, x_n(t), \\ \qquad \xi_1(t), \ldots, \xi_q(t)), \quad i = 1, \ldots, n, \\ y_j(t) = h_j(x_1(t), \ldots, x_n(t)), \quad j = 1, \ldots, p, \end{cases} \tag{3}$$

where $\xi_i(t)$, $i = 1, \ldots, q$ are the disturbances. Roughly speaking, the DDP is to find suitable controllers such that the outputs of the closed-loop system are not affected by the disturbances.

Definition 4. Consider system (3). The DDP is solvable, if we can find a coordinate transformation $\{x_i \mid i = 1, \ldots, n\} \mapsto \{z_i \mid i = 1, \ldots, n\}$ and feedback controls

$$u_i(t) = \Phi_i(z_1(t), \ldots, z_n(t)), \quad i = 1, \ldots, m, \tag{4}$$

such that the closed-loop system becomes

$$\begin{cases} z_i(t+1) = F_i^1(z_1(t), \ldots, z_r(t)), \quad i = 1, \ldots, r, \\ z_k(t+1) = F_k^2(z_1(t), \ldots, z_n(t), \xi_1(t), \ldots, \xi_q(t)), \\ \qquad k = r+1, \ldots, n, \\ y_j(t) = H_j(z_1(t), \ldots, z_r(t)), \quad j = 1, \ldots, p. \end{cases} \tag{5}$$