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Recursive predictor design for state and output feedback controllers for linear time delay systems ☆

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ABSTRACT

This paper presents a recursive method to design state and output feedback controllers for MIMO, block-feedforward linear systems with delays in the inputs, outputs, and interconnections between the blocks. The resulting controller is of predictor-type, which means that it contains finite integrals over past state and input values. The method is a generalization of the well-known model reduction approach for systems with input delay. A recursive procedure replaces delay terms with non-delay ones step by step, from the top of the cascade structure down. Controller gains are computed for the proxy system without delays, while the construction guarantees the same closed loop poles for the delay system and the proxy one. The observer is designed by applying the duality argument and the separation principle is also shown to apply.

1. Introduction

In this paper we develop a recursive method to design controllers for linear, block-feedforward systems with input, output, and state delays. The method is a generalization of the well-known approach to control systems with input, but no state delay of the form

$$\dot{x}(t) = Ax(t) + \sum_{i=0}^{l} B_i u(t-\tau_i) \tag{1.1}$$

where $x \in R^n$ and $u \in R^m$. This problem can be reduced to control design for a system without delay,

$$\dot{x}(t) = Ax(t) + B_d u(t), \qquad B_d = \sum_{i=0}^{l} e^{-A\tau_i} B_i. \tag{1.2}$$

The feedback controller $u = -Kx$ for (1.2) can now be obtained by a control design method of choice, assuming that the pair (A, B_d) is stabilizable. The fact that the spectrum of $A - B_d K$ coincides with that of (1.1) with the control

$$u(t) = -K\left(x(t) + \sum_{i=0}^{l} \int_0^{\tau_i} e^{-A\theta} B_i u(t+\theta-\tau_i)\mathrm{d}\theta\right) \tag{1.3}$$

provides the stabilizing feedback for (1.1) and a finite spectrum assignment for the closed loop system (see Manitius and Olbrot (1979)). This method applies even if there is a distributed delay in u and/or the matrices A and B_i are time varying (Artstein, 1982; Manitius & Olbrot, 1979). Note that when there is only one delay τ, a simple manipulation shows that $u(t) = -K_A\hat{x}(t+\tau|t)$, where $K_A = Ke^{-A\tau}$ and $\hat{x}(t+\tau|t)$ is the predicted value of the state x at time $t+\tau$, based on the information up to time t (values of u applied after t do not impact $x(t+\tau)$ because of the input delay). For this reason, the control law of the form (1.3) is often referred to as "predictor-like" or "predictor-type" while the method is known as the model reduction (see Gu and Niculescu (2003), Section 4.2 and the references therein). We note that multiple-delays and multiple-inputs are completely transparent from the control design point of view.

The model reduction technique does not work if state delay is also present. Indeed, finite spectrum assignment (FSA) methods for systems that include state delays follow different approaches (see, for example, Loiseau (2000), Manitius and Olbrot (1979) and Watanabe, Nobuyama, Kitamori, and Ito (1992)). The FSA designs are progressively more complex for systems with multiple-inputs and multiple-delays (depending on the ratio of the delays) and stop working if the delays are non-commensurate. In this paper we apply a model reduction like method to a class of systems with state, input, and output delays, which may be non-commensurate, under a structural constraint on the system. That is, we consider MIMO systems having the following block-feedforward structure:

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$$\dot{z} = \begin{bmatrix} A_1 & * & * & \dots & * \\ 0 & A_2 & * & \dots & * \\ 0 & 0 & A_3 & \dots & * \\ \vdots & \vdots & & & \vdots \\ 0 & 0 & 0 & \dots & A_p \end{bmatrix} z + \begin{bmatrix} * \\ * \\ * \\ \vdots \\ * \end{bmatrix} u \qquad (1.4)$$
$$y = \begin{bmatrix} * & * & * & \dots & * \end{bmatrix} z$$

where $z \in R^n$, $n = n_1 + \cdots + n_p$, $u \in R^m$, and $y \in R^r$. The entries "$*$" designate delayed terms. In other words, if we denote delay operators by μ_i (that is, $\mu_i z(t) = z(t-\tau_i)$, $i = 0, \dots, l$),[1] a "$*$" in (1.4) denotes a term of the form $\sum_{i=0}^{l} S_i \mu_i$ for some matrices S_i of appropriate dimensions. Just like the model reduction method, our generalized predictor imposes no restrictions on the delays τ_i. It would also work for distributed delays, but, to reduce notational complexity, only discrete delays are considered. The recent Lyapunov based result, the cross-term forwarding of Jankovic (2009), applies to feedforward linear and nonlinear systems with the same structure as (1.4), but does not allow matrices A_i to have unstable modes. It is also computationally more complex. Another method that uses a transformation into a non-delay system, proposed in Fiagbedzi and Pearson (1990), is based on eigenvalues and eigenvectors of the characteristic quasi-polynomial matrix. For the system (1.4) it would provide a control law that places the complete set of eigenvalues at prescribed locations. The recursive method proposed here does not use the system eigenstructure, but, as pointed out by a reviewer, it does implicitly solve the characteristic matrix equation for (1.4). In doing so, the method avoids certain complexity and additional assumptions associated with repeated eigenvalues and related Jordan forms (see Fiagbedzi (1996), Fiagbedzi and Pearson (1990) and Zheng, Cheng, and Gao (1994)).

We first consider the control design for (1.4) assuming the full state is available for feedback. For this, we propose a recursive method, based on the single step spectral equivalence result (observation) from Jankovic (2009). The result is reinterpreted here in a form that allows removal of delays from subsystems that grow larger at each recursion step. The delays are replaced by a matrix exponent factor in a fashion similar to the model reduction technique. The matrix exponent at each step depends on the matrix exponents from previous steps. After p steps, all the delays will have been removed and a "proxy" non-delay system obtained. A controller for the proxy system can be designed using one of the standard techniques (pole placement, *LQR*, H_∞, etc). The spectra for the original delay system and the delay-free proxy system are made the same by augmenting the proxy controller with predictor integrals similar to those in (1.3). Certain robustness properties of the delay and the proxy systems turned out to be equivalent as shown in Remark 3 and Section 5.

An asymptotic observer that provides state estimates for (1.4) is designed using the duality argument. As a result, the state estimation error is governed by an infinite dimensional system that has a finite spectrum. The separation principle is shown to apply as well. That is, when the estimated states are used instead of the actual ones in the feedback law, the closed loop system remains stable and the closed loop spectrum is the sum of the spectra for the observer and the full state feedback system.

The paper is organized as follows. Section 2 provides the spectral equivalence result that will be used in the recursive method. The recursive predictor controller design is presented in Section 3. Section 4 contains the observer design and the separation principle. Section 5 contains an example with simulation results.

[1] We shall use $\mu = (\mu_0, \mu_1, \dots, \mu_p)$ and also apply the same notations in the (Laplace) s-domain with $\mu_i = e^{-s\tau_i}$.

2. Spectral equivalence result

The method proposed in this paper is based on a result (observation) from Jankovic (2009), which itself is a version of the result from Section IV of Manitius and Olbrot (1979). In its original form the result applies to linear systems of the form

$$\begin{aligned} \dot{x}(t) &= Fx(t) + \sum_{i=0}^{l} H_i \xi(t-\tau_i) \\ \dot{\xi}(t) &= A\xi(t) + Bu(t) \end{aligned} \qquad (2.1)$$

where $x \in R^{n_x}$, $\xi \in R^{n_\xi}$, $u \in R^m$ and $0 = \tau_0 < \tau_1 < \cdots < \tau_l$. In Jankovic (2009) it was shown that any stabilizing control $v_0 = -K_x x - K_\xi \xi$ for the cascade system with no delay

$$\begin{aligned} \dot{x} &= Fx + \sum_{0}^{l} e^{-F\tau_i} H_i \xi \\ \dot{\xi} &= A\xi + Bv_0 \end{aligned} \qquad (2.2)$$

provides a control law

$$u = -K_\xi \xi - K_x \left(x + \sum_{i=0}^{l} \int_0^{\tau_i} e^{-F\theta} H_i \xi(t+\theta-\tau_i) d\theta \right) \qquad (2.3)$$

that stabilizes (2.1). Moreover, the two systems – (2.2) with the control v_0 and (2.1) with the control (2.3) – have the same closed loop poles.

For this paper we need the result extended to the class of systems

$$\begin{aligned} \dot{x}(t) &= Fx(t) + \sum_{i=0}^{l} H_i \xi(t-\tau_i) \\ \dot{\xi}(t) &= \mathcal{A}\xi_d(t) + Bu(t) \end{aligned} \qquad (2.4)$$

where $\mathcal{A}$ is a linear functional acting on the present and past values of ξ ($\xi_d(t)$ denotes the state trajectory over the interval $[t-r, t]$ for some $r \geq \tau_l$) of the form

$$\mathcal{A}\xi_d(t) = \sum_{i=0}^{l} A_i \xi(t-\tau_i) + \int_0^r Q(\theta)\xi(t-\theta) d\theta.$$

The control law we consider is also modified — instead of the simple gain matrix K_ξ in (2.3) we employ a functional $\mathcal{K}_\xi$ having the same form as $\mathcal{A}$:

$$\mathcal{K}_\xi \xi_d(t) = \sum_{i=0}^{l} K_{\xi i} \xi(t-\tau_i) + \int_0^r \Upsilon_\xi(\theta)\xi(t-\theta) d\theta.$$

With these changes, the closed loop system takes the form

$$\begin{aligned} \dot{x}(t) &= Fx(t) + \sum_{i=0}^{l} H_i \xi(t-\tau_i) \\ \dot{\xi}(t) &= -BK_x \left(x(t) + \sum_{i=0}^{l} \int_0^{\tau_i} e^{-F\theta} H_i \xi(t+\theta-\tau_i) d\theta \right) \\ &\quad + (\mathcal{A} - B\mathcal{K}_\xi)\xi_d(t). \end{aligned} \qquad (2.5)$$

With the Laplace transform of $\int_0^{\tau_i} e^{-F\theta} H_i \xi(t+\theta-\tau_i) d\theta$ given by $(sI-F)^{-1}(e^{-F\tau_i} - e^{-s\tau_i} I) H_i \xi(s)$, the characteristic quasi-polynomial of (2.5) is

$$\chi(s) = \det \begin{bmatrix} sI - F & -\sum_0^l H_i e^{-s\tau_i} \\ BK_x & \begin{array}{c} sI - \mathcal{A}(s) + B\mathcal{K}_\xi(s) + BK_x \\ \times (sI-F)^{-1} \sum_0^l (e^{-F\tau_i} - e^{-s\tau_i} I) H_i \end{array} \end{bmatrix} \qquad (2.6)$$

where $\mathcal{A}(s) = \sum_0^l A_i e^{-\tau_i s} + \int_0^r Q(\theta) e^{-s\theta} d\theta$ and $\mathcal{K}_\xi(s) = \sum_0^l K_{\xi i} e^{-\tau_i s} + \int_0^r \Upsilon_\xi(\theta) e^{-s\theta} d\theta$. Using the well-known identity for determinants of block matrices