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Improved delay-dependent stability criteria for systems with a delay varying in a range ☆

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ABSTRACT

This paper provides improved delay-dependent stability criteria for systems with a delay varying in a range. The criteria improve over some previous ones in that they have fewer matrix variables yet less conservatism, which is established theoretically. An example is given to show the advantages of the proposed results.

1. Introduction

Time delays are often encountered in various practical systems such as chemical processes, neural networks and long transmission lines in pneumatic systems (Hale, 1977; Richard, 2003; Shao, 2008a,b,c). It has been shown that the existence of time-delays may lead to oscillation, divergence or instability. This motivates the stability analysis problem for linear systems affected by time delays. Stability criteria for the system can be divided into two classes; that is, delay-independent ones and delay-dependent ones. Since delay-independent criteria tend to be more conservative especially for small size delays, considerable attention has been devoted to delay-dependent ones, see, e.g., Kim (2001), Lee, Moon, Kwon, and Park (2004), Niculescu, Neto, Dion, and Dugard (1995), Suplin, Fridman, and Shaked (2004), Wu, He, She, and Liu (2004) and Xu and Lam (2005).

As far as delay-dependent stability is concerned, there are roughly two approaches, namely the frequency-domain one and the time-domain one. The former can be found in Ebenbauer and Allgöwer (2006) where the sum of squares (SOS) technique was taken. As to the time-domain approach, Lyapunov functional is a powerful tool, which can deal with time-varying delays. A Lyapunov functional was constructed in Xie and de Souza (1997) based on the transformed model, and an inequality was proposed in Park (1999) or Moon, Park, Kwon, and Lee (2001) for bounding cross terms in the derivative of Lyapunov functional. The descriptor system approach together with Park's inequality in Fridman and Shaked (2002) resulted in a less conservative criterion. For the case of constant delay, even less conservative results were obtained in Gu (2000) by constructing a complete Lyapunov functional with discretized or piecewise methods. The complete Lyapunov functional is sufficient and necessary to the stability for time-invariant linear systems with a constant delay. Recently it was extended to interval time-varying delay.

Delay-dependent stability for systems with interval time-varying delay has been addressed and some stability results reported recently in the literature. In Han and Gu (2001), motivated by the complete Lyapunov functional for time-invariant delay, a more general one was introduced and discretized. The stability criterion derived is less conservative at the cost of more computation. Especially for constant delay it allows a stability bound approach the analytical solution as the discretization becomes finer. But it is only workable when the rate of delay is less than one. Papachristodoulou, Peet, and Niculescu (2007) extended the complete Lyapunov functional from constant delay to interval time-varying delay. They specialized function matrices as the sum of squares of polynomials, and obtained a stability condition. The stability result becomes less conservative as the order of the polynomials goes higher. But this means a greater

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computational requirement. Moreover, the stability result is not applicable to the case when the rate of delay is unknown. A simple stability criterion, however, can be found for this case in Kao and Lincoln (2004), where an input–output approach was employed. Unfortunately the simple criterion is only suitable for single-input–single-output control systems, and when the rate of delay is available it is conservative. The free weighting matrix method, by contrast, can keep a balance between the conservatism and the computational effort; see Jiang and Han (2005) and He, Wang, Lin, and Wu (2007). However there is still some conservatism in He et al. (2007), and the criteria can be simplified.

In this paper attention is focused on delay-dependent stability for systems with a delay varying in an interval. Via a different Lyapunov functional whose derivative is estimated using Jensen' Inequality, delay-dependent stability criteria are obtained. More importantly it is established theoretically that the criteria have less conservatism with fewer matrix variables than those in He et al. (2007). This implies that the latter can be simplified with less conservatism.

Consider the following system with a time-varying delay.

$$\dot{x}(t) = Ax(t) + A_1x(t-d(t)), \tag{1}$$
$$x(t) = \phi(t), \quad t \in [-h_2, 0],$$

where $x(t)$ is the state;A and A_1 are known real constant matrices; the time delay $d(t)$ is a continuous time-varying function satisfying

$$0 \le h_1 \le d(t) \le h_2, \tag{2}$$
$$\dot{d}(t) \le \mu; \tag{3}$$

$\phi(t)$ is a continuous real-valued initial function on $[-h_2, 0]$.

2. Main results

Now we provide a delay-dependent stability criterion for system (1).

Theorem 1. *System (1) subject to (2) and (3) is asymptotically stable for given $0 \le h_1 \le h_2$ and μ if there exist matrices $P > 0$, $Q_i > 0$, $i = 1, 2, 3$ and $Z_j > 0, j = 1, 2$, such that the following LMI holds*

$$\begin{bmatrix} \Upsilon & PA_1 & Z_1 & 0 & h_1A^TZ_1 & h_{12}A^TZ_2 \\ * & -(1-\mu)Q_3 - 2Z_2 & Z_2 & Z_2 & h_1A_1^TZ_1 & h_{12}A_1^TZ_2 \\ * & * & -Q_1 - Z_1 - Z_2 & 0 & 0 & 0 \\ * & * & * & -Q_2 - Z_2 & 0 & 0 \\ * & * & * & * & -Z_1 & 0 \\ * & * & * & * & * & -Z_2 \end{bmatrix} < 0 \tag{4}$$

where $h_{12} = h_2 - h_1$, and

$$\Upsilon = PA + A^TP + Q_1 + Q_2 + Q_3 - Z_1.$$

Proof. Define a Lyapunov functional as

$$\begin{aligned} V(x_t) = {} & x(t)^TPx(t) + \int_{t-d(t)}^{t} x(\alpha)^TQ_3x(\alpha)\mathrm{d}\alpha \\ & + \sum_{i=1}^{2}\int_{t-h_i}^{t} x(\alpha)^TQ_ix(\alpha)\mathrm{d}\alpha \\ & + \int_{-h_1}^{0}\int_{t+s}^{t} h_1\dot{x}(\alpha)^TZ_1\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}s \\ & + \int_{-h_2}^{-h_1}\int_{t+s}^{t} h_{12}\dot{x}(\alpha)^TZ_2\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}s, \end{aligned} \tag{5}$$

where $x_t = x(t+\theta)$, $-2h_2 \le \theta \le 0$. Then along the trajectory of (1) we have

$$\begin{aligned} \dot{V}(x_t) \le {} & 2x(t)^TP(Ax(t) + A_1x(t-d(t))) + \sum_{i=1}^{3} x(t)^TQ_ix(t) \\ & - (1-\mu)x(t-d(t))^TQ_3x(t-d(t)) \\ & - \sum_{i=1}^{2} x(t-h_i)^TQ_ix(t-h_i) + (Ax(t) + A_1x(t-d(t)))^T \\ & \times (h_1^2Z_1 + h_{12}^2Z_2)(Ax(t) + A_1x(t-d(t))) \\ & - \int_{t-h_1}^{t} h_1\dot{x}(\alpha)^TZ_1\dot{x}(\alpha)\mathrm{d}\alpha - \int_{t-h_2}^{t-h_1} h_{12}\dot{x}(\alpha)^TZ_2\dot{x}(\alpha)\mathrm{d}\alpha. \end{aligned} \tag{6}$$

From Jensen's Inequality, it follows that

$$\begin{aligned} & -\int_{t-h_1}^{t} h_1\dot{x}(\alpha)^TZ_1\dot{x}(\alpha)\mathrm{d}\alpha \\ & \le -(x(t) - x(t-h_1))^TZ_1(x(t) - x(t-h_1)), \end{aligned} \tag{7}$$

and

$$\begin{aligned} & -\int_{t-h_2}^{t-h_1} h_{12}\dot{x}(\alpha)^TZ_2\dot{x}(\alpha)\mathrm{d}\alpha \\ & \le -\int_{t-h_2}^{t-d(t)} (h_2 - d(t))\dot{x}(\alpha)^TZ_2\dot{x}(\alpha)\mathrm{d}\alpha \\ & \quad - \int_{t-d(t)}^{t-h_1} (d(t) - h_1)\dot{x}(\alpha)^TZ_2\dot{x}(\alpha)\mathrm{d}\alpha \\ & \le -(x(t-d(t)) - x(t-h_2))^TZ_2(x(t-d(t)) - x(t-h_2)) \\ & \quad - (x(t-h_1) - x(t-d(t)))^TZ_2(x(t-h_1) - x(t-d(t))). \end{aligned} \tag{8}$$

Combining (6)–(8) yields

$$\begin{aligned} & \dot{V}(x_t) \\ & \le x(t)^T\left[PA + A^TP + \sum_{i=1}^{3} Q_i - Z_1 + A^T(h_1^2Z_1 + h_{12}^2Z_2)A\right]x(t) \\ & \quad + 2x(t)^T[PA_1 + A^T(h_1^2Z_1 + h_{12}^2Z_2)A_1]x(t-d(t)) \\ & \quad + 2x(t)^TZ_1x(t-h_1) + x(t-d(t))^T[-(1-\mu)Q_3 - 2Z_2 \\ & \quad + A_1^T(h_1^2Z_1 + h_{12}^2Z_2)A_1]x(t-d(t)) \\ & \quad + 2x(t-d(t))^TZ_2x(t-h_1) + 2x(t-d(t))^TZ_2x(t-h_2) \\ & \quad - x(t-h_1)^T(Q_1 + Z_1 + Z_2)x(t-h_1) \\ & \quad - x(t-h_2)^T(Q_2 + Z_2)x(t-h_2) \\ & = \zeta(t)^T\Phi\zeta(t), \end{aligned}$$

where

$$\zeta(t) = [x(t)^T \quad x(t-d(t))^T \quad x(t-h_1)^T \quad x(t-h_2)^T]^T,$$

$$\begin{aligned} \Phi = {} & \begin{bmatrix} \Upsilon & PA_1 & Z_1 & 0 \\ * & -(1-\mu)Q_3 - 2Z_2 & Z_2 & Z_2 \\ * & * & -Q_1 - Z_1 - Z_2 & 0 \\ * & * & * & -Q_2 - Z_2 \end{bmatrix} \\ & + \begin{bmatrix} A & A_1 & 0 & 0 \end{bmatrix}^T (h_1^2Z_1 + h_{12}^2Z_2) \begin{bmatrix} A & A_1 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{9}$$

With (9) and (4) it is concluded that system (1) is asymptotically stable. This ends the proof. □

When $h_1 = 0$, Theorem 1 reduces to the following corollary.

Corollary 1. *System (1) subject to (2) and (3) is asymptotically stable for given $h_2 > 0$, $h_1 = 0$ and μ if there exist matrices $P > 0$, $Q_i > 0, i = 2, 3$ and $Z_2 > 0$ such that the following LMI holds*