Automatica 47 (2011) 861–864

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical communique

An improved robust model predictive control design in the presence of actuator saturation ☆

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ARTICLE INFO

Article history:
Available online 23 February 2011

Keywords:
Robust model predictive control
Actuator saturation
Constrained control

ABSTRACT

A new robust model predictive control (RMPC) design algorithm is proposed for a linear uncertain system with a polytopic description and subject to actuator saturation. This algorithm involves the solution of an infinite horizon LQR problem for the uncertain system in the presence of actuator saturation at each time instant and the implementation of the first element of the resulting optimal control profile. By expressing a saturating linear feedback law on a convex hull of a group of auxiliary linear feedback laws and the actual linear feedback law, the LQR problem can be solved for a group of linear polytopic systems in the absence of saturation, with heavier weighting placed on the system corresponding to the actual linear feedback law. The additional design freedom in choosing the relative weighting on the actual and auxiliary feedback laws allows further improvement of the closed-loop system performance over those resulting from the existing algorithms. A numerical example illustrates the effectiveness of the proposed algorithm.

1. Introduction

Over the past years, model predictive control has been widely adopted as an effective method to deal with constrained control problems (Mayne, Rawlings, Rao, & Scokaert, 2000). Robust model predictive control (RMPC), as a significant branch of MPC because of its importance in practice, has attracted remarkable attention from the research community (see, *e.g.* Angeli, Casavola, Franzč, & Mosca, 2008; Casavola, Giannelli, & Mosca, 2000; Cheng & Jia, 2004; Kothare, Balakrishnan, & Morari, 1996).

In practical control systems, actuator saturation is a common nonlinearity. It is well known that when the actuator saturates, the performance of the closed-loop systems designed without considering actuator saturation may seriously deteriorate (Alamo, Cepeda, Limon, & Camacho, 2006; Hu & Lin, 2001). Hence, embedding actuator saturation consideration in an RMPC design is critically important and has been well addressed in the literature. Kothare et al. (1996) propose an RMPC design that guarantees asymptotic stability of the closed-loop system. The design in Kothare et al. (1996) deals with both the input constraints and the state constraints. Casavola et al. (2000) consider the RMPC design for systems with polytopic uncertainty and subject to input saturation. Several further improvements have been presented. Yu, Bohm, Chen, and Allgower (2009) assume that the system parameters are measurable online and improves the design in Kothare et al. (1996). Besselmann, Lofberg, and Morari (2008) adopt the H_1 and H_∞ index respectively as the optimal objective of RMPC to obtain the explicit RMPC. In addition, some off-line designs are also reported to reduce the online computational burden of RMPC (see, *e.g.* Angeli et al., 2008; Ding, Xi, & Li, 2004; Wan & Kothare, 2003).

By expressing a saturating linear feedback law on a convex hull of a group of auxiliary linear feedback laws and the actual linear feedback law (Hu & Lin, 2001), Cao and Lin (2005) propose an RMPC design method in the presence of actuator saturation. The uncertain system is assumed to be in the polytopic form. This design method leads to considerable improvement in the closed-loop performance. In the LQR problem formulated and solved in Cao and Lin (2005), a same weighting is assigned to the closed-loop systems resulting from both the auxiliary and actual linear feedback laws. This leaves room for further improvement. With this observation in mind, in this paper we will propose a new RMPC design that takes the relative weighting between the auxiliary and actual feedback laws into account. As will be shown in the numerical example, this additional design freedom in choosing the

☆ This work was supported in part by the National Natural Science Foundation of China (Grant No. 60934007, 61074060, and 60928008), China Postdoctoral Science Foundation Special Support (Grand No. 201003272), and a Zhiyuan Professorship at Shanghai Jiao Tong University. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Giancarlo Ferrari-Trecate under the direction of Editor André L. Tits.

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doi:10.1016/j.automatica.2011.01.045

relative weighting on actual and auxiliary feedback laws indeed leads to further performance improvement.

This paper is organized as follows. Section 2 describes the systems to be considered and the problem to be solved. The RMPC design algorithm of Cao and Lin (2005) is also briefly recalled in this section to motivate our new design algorithm to be presented in Section 3. Section 4 illustrates the effectiveness of the proposed design algorithm.

Throughout the paper, we will use $u(k+l|k)$ and $x(k+l|k)$ to denote respectively the control input and system state at time instant $k+l$, predicted at time instant k. For a matrix $Q>0$, $\|x\|_Q^2 = x^T Qx$. For two integers $r_1 \le r_2$, $I[r_1, r_2] = \{r_1, r_1+1, \dots, r_2\}$.

2. Problem formulation and motivation

2.1. Problem formulation

Consider the following polytopic uncertain system

$$x(k+1) = A(k)x(k) + B(k)\mathrm{sat}(u(k)), \tag{1}$$

$$y(k) = C(k)x(k), \tag{2}$$

where $x \in \mathbf{R}^n$ is the system state, $u \in \mathbf{R}^m$ is the control input, $y \in \mathbf{R}^s$ is the controlled output, $(A(k), B(k), C(k)) \in \Omega$, with $\Omega = \left\{(A, B, C) = \sum_{i=1}^q \alpha_i(A_i, B_i, C_i) : \alpha_i \ge 0,\ \sum_{i=1}^q \alpha_i = 1\right\}$, for some matrix triples (A_i, B_i, C_i), $i \in I[1, q]$, and sat : $\mathbf{R}^m \to \mathbf{R}^m$ is a vector-valued standard saturation function defined as $\mathrm{sat}(u) = [\mathrm{sat}(u_1), \mathrm{sat}(u_2), \dots, \mathrm{sat}(u_m)]^{\mathrm{T}}$, with $\mathrm{sat}(u_i) = \mathrm{sign}(u_i) \min\{|u_i|, 1\}$.

In addition, the constraints on the states are as follows,

$$|x_i(k)| \le \bar{x}_i, \quad \bar{x}_i > 0, \tag{3}$$

where x_i is the ith element of x and $\bar{x}_i$ is the limit on its absolute value.

In the RMPC problem to be solved in this paper, a state feedback gain F is designed at each time instant $k \ge 0$ by minimizing the following worst-case performance index,

$$\min_F \max_{(A,B,C)\in\Omega} J(k)$$

where $J(k) = \sum_{l=0}^{\infty} \left(\|x(k+l|k)\|_{\mathcal{M}}^2 + \|u(k+l|k)\|_{\mathcal{R}}^2\right)$, with $\mathcal{M} \ge 0$ and $\mathcal{R} > 0$ being weighting matrices.

2.2. Preliminaries and motivation

For a given $F \in \mathbf{R}^{m\times n}$, let $\mathcal{L}(F) = \{x \in \mathbf{R}^n : |f_i x| \le 1,\ i \in I[1, m]\}$, where f_i is the ith row of F. For a positive definite matrix $P \in \mathbf{R}^{n\times n}$ and a nonnegative scalar ρ, $\mathcal{E}(P, \rho) = \{x \in \mathbf{R}^n : x^{\mathrm{T}}Px \le \rho\}$. Let $\mathcal{V}$ be the set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. There are 2^m elements in $\mathcal{V}$. Label the elements of $\mathcal{V}$ as E_j, $j \in I[1, 2^m]$, with $E_1 = I$. Denote $E_j^- = I - E_j$. Clearly, $E_j^- \in \mathcal{V}$ if $E_j \in \mathcal{V}$.

Lemma 1 (*Hu & Lin, 2001*). *Let $F, H \in \mathbf{R}^{m\times n}$ be given. For an $x \in \mathbf{R}^n$, if $x \in \mathcal{L}(H)$, then*

$$\mathrm{sat}(Fx) \in co\,\{E_jFx + E_j^- Hx : j \in I[1, 2^m]\}, \tag{4}$$

where co stands for convex hull.

By using the convex hull expression of a saturating linear feedback law as described in Lemma 1, Ref. Cao and Lin (2005) formulates the RMPC design, in the absence of state constraints, into the following optimization problem

$$\min_{Q>0,Y,Z} \gamma,$$

$$\text{s.t. (a)} \begin{bmatrix} Q & * & * & * \\ A_iQ + B_i(E_jY + E_j^- Z) & Q & 0 & 0 \\ \mathcal{M}^{1/2}Q & 0 & \gamma & 0 \\ \mathcal{R}^{1/2}(E_jY + E_j^- Z) & 0 & 0 & \gamma \end{bmatrix} > 0,$$

$$j \in I[1, 2^m],\ i \in I[1, q], \tag{5}$$

$$\text{(b)} \begin{bmatrix} 1 & z_i \\ z_i^{\mathrm{T}} & Q \end{bmatrix} \ge 0, \quad i \in I[1, m],$$

$$\text{(c)} \begin{bmatrix} 1 & x^{\mathrm{T}}(k|k) \\ x(k|k) & Q \end{bmatrix} \ge 0,$$

where z_i is the ith row of matrix Z. Once Problem (5) is solved, the feedback gain at time instant k is computed as $F = YQ^{-1}$.

In deriving the above LMI optimization problem, the auxiliary feedback $(E_jF + E_j^- H)x(k+l|k)$, $j \ne 1$, and the actual feedback $Fx(k+l|k)$ are treated equally. This observation motivates that possible improvement in the closed-loop performance could be achieved by placing a heavier weighting on the actual feedback in the optimization problem.

3. An improved RMPC design

At time instant $k \ge 0$, systems (1)–(2) can be written as

$$x(k+l+1|k) = A(k+l)x(k+l|k) + B(k+l)\mathrm{sat}(u(k+l|k)), \tag{6}$$

$$y(k+l|k) = C(k+l)x(k+l|k). \tag{7}$$

Let the linear state feedback law be denoted as $u(k+l|k) = Fx(k+l|k)$. We note here that the feedback gain F is a function of k and, for simplicity, we have not written it as $F(k)$. Similar notational simplifications will also be made for other matrices and variables that arise in the design at each time instant k.

Consider a Lyapunov function $V(x) = x^{\mathrm{T}}Px$, with $P > 0$. Then,

$$\begin{aligned} \Delta V(k+l|k) &= V(x(k+l|k)) - V(x(k+l+1|k)) \\ &= \|x(k+l|k)\|_P^2 - \|x(k+l+1|k)\|_P^2. \end{aligned} \tag{8}$$

Imposing the robust stability condition on (8), we obtain

$$\Delta V(k+l|k) > \|x(k+l|k)\|_{\mathcal{M}}^2 + \|u(k+l|k)\|_{\mathcal{R}}^2, \tag{9}$$

which, by Lemma 1, is implied by

$$\Delta V(k+l|k) > \frac{1}{a}\|x(k+l|k)\|_{\mathcal{M}+F^{\mathrm{T}}\mathcal{R}F}^2, \quad a \in (0, 1], \tag{10}$$

$$\Delta V(k+l|k) > \|x(k+l|k)\|_{\mathcal{M}+(E_jF+E_j^-H)^{\mathrm{T}}\mathcal{R}(E_jF+E_j^-H)}^2,$$

$$j \in I[2, 2^m], \tag{11}$$

provided that $x(k+l|k) \in \mathcal{L}(H)$. We note here that the additional parameter a assigns a heavier weighting to the actual feedback law than to the auxiliary feedback laws. This parameter also provides extra design freedom in comparison with the algorithms of Cao and Lin (2005) and Kothare et al. (1996).

Lemma 2. *Consider systems (1)–(2). At time instant k, let a positive definite matrix $P \in \mathbf{R}^{n\times n}$ and $F, H \in \mathbf{R}^{m\times n}$ satisfy the following conditions,*

$$\begin{bmatrix} P & * & * & * \\ A_i + B_iF & P^{-1} & * & * \\ \mathcal{M}^{1/2} & 0 & aI & * \\ \mathcal{R}^{1/2}F & 0 & 0 & aI \end{bmatrix} > 0, \quad i \in I[1, q], \tag{12}$$