Automatica 48 (2012) 2007–2015

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Extended Kalman filtering with stochastic nonlinearities and multiple missing measurements[☆]

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ARTICLE INFO

Article history:
Received 15 September 2011
Received in revised form
31 January 2012
Accepted 22 March 2012
Available online 12 July 2012

Keywords:
Nonlinear systems
Extended Kalman filter
Stochastic nonlinearities
Multiple missing measurements
Recursive filter
Riccati-like difference equation

ABSTRACT

In this paper, the extended Kalman filtering problem is investigated for a class of nonlinear systems with multiple missing measurements over a finite horizon. Both deterministic and stochastic nonlinearities are included in the system model, where the stochastic nonlinearities are described by statistical means that could reflect the multiplicative stochastic disturbances. The phenomenon of measurement missing occurs in a random way and the missing probability for each sensor is governed by an individual random variable satisfying a certain probability distribution over the interval [0, 1]. Such a probability distribution is allowed to be any commonly used distribution over the interval [0, 1] with known conditional probability. The aim of the addressed filtering problem is to design a filter such that, in the presence of both the stochastic nonlinearities and multiple missing measurements, there exists an upper bound for the filtering error covariance. Subsequently, such an upper bound is minimized by properly designing the filter gain at each sampling instant. It is shown that the desired filter can be obtained in terms of the solutions to two Riccati-like difference equations that are of a form suitable for recursive computation in online applications. An illustrative example is given to demonstrate the effectiveness of the proposed filter design scheme.

1. Introduction

In the past few decades, the filtering or state estimation problems for stochastic systems have been extensively investigated. Accordingly, the filter theory has been successfully applied in many branches of practical domains such as computer vision, communications, navigation and tracking systems, econometrics and finance, etc. It is well known that the traditional Kalman filter (KF) serves as an optimal filter in the least mean square sense for *linear* systems with the assumption that the system model is exactly known. In the case that the system model is nonlinear and/or uncertain, there has been an increasing research effort to improve KF with hope to enhance their capabilities of handling nonlinearities and uncertainties. Along this direction, many alternative filtering schemes have been reported in the literature including the H_∞ filtering (Li, Lam, & Shu, 2010; Shi, Mahmoud, Nguang, & Ismail, 2006; Wu & Zheng, 2009; Xiong & Lam, 2006; Yue & Han, 2006), mixed H_2/H_∞ filtering (Rotstein, Sznaier, & Idan, 1994; Xie, Lu, Zhang, & Zhang, 2004), set-value estimation (Bishop, Savkin, & Pathirana, 2010; Calafiore, 2005; Cheng, Malyavej, & Savkin, 2010; Pathirana, Ekanayake, & Savkin, 2011) and robust extended Kalman filter (EKF) design (James & Petersen, 1998; Kallapur, Petersen, & Anavatti, 2009; Xiong, Liu, & Liu, 2011; Xiong, Wei, & Liu, 2010). Among them, the EKF has shown to be an effective way for tackling the nonlinear system estimation problems. In fact, EKF has recently gain particular research attention with promising application potentials in various engineering practices. For example, the EKF has been designed in James and Petersen (1998) and Kallapur et al. (2009) for uncertain systems with quadratic constraints. Moreover, the EKF algorithm has been successfully applied in Wang, Liu, Liu, Liang, and Vinciotti (2009) to identify the parameters and predict the states of a nonlinear stochastic biological network modeled by time series data.

☆ This work was supported in part by the National 973 Project under Grant 2009CB320600, National Natural Science Foundation of China under Grants 61028008, 61134009 and 60825303, the State Key Laboratory of Integrated Automation for the Process Industry (Northeastern University) of China, the Engineering and Physical Sciences Research Council (EPSRC) of the UK under Grant GR/S27658/01, the Royal Society of the UK, and the Alexander von Humboldt Foundation of Germany. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor James Lam under the direction of Editor Ian R. Petersen.

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doi:10.1016/j.automatica.2012.03.027

Apart from the stochasticity, the nonlinearity is another ubiquitous feature existing in almost all practical systems that contributes significantly to the complexity of system modeling. Since nonlinearities may cause undesirable dynamic behaviors such as oscillation or even instability, the analysis and synthesis problems for nonlinear systems have long been the main stream of research topics and much effort has been made to deal with the nonlinear stochastic systems, see e.g. Basin, Shi, and Calderon-Alvarez (2009), Chen and Zheng (2011), Li and Lam (2011), Reif, Günther, Yaz, and Unbehauen (1999) and Yaz (1987). It is worth pointing out that, in most literature, the nonlinearities are assumed to occur in a deterministic way. While this assumption is generally true especially for systems modeled according to physical laws, another kind of nonlinearities, namely, stochastic nonlinearities, deserve particular research attention since they occur randomly due probably to the high maneuverability of the tracked target, intermittent network congestion, random failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, modification of the operating point of a linearized model of nonlinear systems. In fact, such stochastic nonlinearities include the state-multiplicative noises as special cases. Recently, the filtering problem with stochastic nonlinearities described by statistical means has already stirred some research interests, and some latest results can be found in Wei, Wang, and Shu (2009) and Yaz and Yaz (2001) and the references therein. On the other hand, almost all real-time systems are time-varying and therefore the finite-horizon filtering problem is of practical significance. However, so far, there have been very few results in the literature regarding filtering problems with stochastic nonlinearities over a finite horizon due probably to the mathematical complexity and/or the computational difficulty.

In recent years, networked systems have become very prevalent and, accordingly, much work has been done in the literature on the network-induced problems such as missing measurements (also called packet loss or dropout) and random communication delays, see e.g. Basin, Shi, and Calderon-Alvarez (2010), Hounkpevi and Yaz (2007a,b), Sahebsara, Chen, and Shah (2007), Sun, Xie, Xiao, and Soh (2008) and Yaz and Yaz (1997). To be more specific, the optimal estimation problems have been investigated in Hounkpevi and Yaz (2007a) and Sun et al. (2008) for linear systems with multiple packet dropouts and the random sensor delays have been taken into account in Hounkpevi and Yaz (2007b) and Yaz and Yaz (1997). It is worth mentioning that, in most reported results, the measurement signal has been assumed to be either completely lost or successfully transferred, and a typical way is to model the missing measurements by the Bernoulli distribution. However, in practical applications, owing to the sensors aging, sensor temporal failure or some of the data coming from a highly noisy environment, the measurement missing might be partial and individual sensor could have different missing probability in the data transmission process (Wei et al., 2009). It is noted that most available results with respect to the filtering problem with missing measurements have been concentrated on linear systems only, and the corresponding results for nonlinear systems have been very few. It is worth mentioning that, in Kluge, Reif, and Brokate (2010), the stochastic stability has been analyzed for EKF with intermittent observations. Up to now, to the best of the authors' knowledge, the finite-horizon extended Kalman filtering problem with both stochastic nonlinearities and multiple missing measurements has not been addressed yet, which still remains as a challenging research issue. It is, therefore, the purpose of this paper to shorten such a gap by resorting to a recursive Riccati-like equation approach.

Motivated by the above discussion, in this paper, we make a major effort to design the EKF for a class of discrete time-varying systems with stochastic nonlinearities and multiple missing measurements. The considered stochastic nonlinearities are governed by zero mean Gaussian noises. The multiple missing measurements are included to model the randomly intermittent behaviors of the individual sensors. The description of the multiple missing measurements is more general than the commonly used one modeled by the Bernoulli distribution. The probability distribution governing the missing measurements from individual sensor is allowed to be any discrete distribution taking values over the interval [0, 1] with known occurrence probability. A recursive approach is developed here to deal with the EKF design problem. An optimized upper bound is guaranteed on the filtering error covariance for both the stochastic nonlinearities and multiple missing measurements. The main contributions of this paper can be summarized (from the aspects of model, problem and algorithm) as follows: (1) the system model is comprehensive that covers stochastic nonlinearities and multiple missing measurements, thereby better reflecting the reality; (2) the addressed extended Kalman filtering problem over a finite horizon is new especially when multiple missing measurements are presented; and (3) the developed filter design algorithm is of a form suitable for recursive computation in online applications.

The remainder of this paper is organized as follows. Section 2 briefly introduces the problem under consideration. In Section 3, the linearization is first enforced to facilitate the filter design. Then, the evolutions of one-step prediction error covariance and filtering error covariance are derived for the addressed model. In the same section, an upper bound of the filtering error covariance is obtained and the filter gain is then designed to minimize such an upper bound at each sampling instant. An illustrative example is utilized in Section 4 to show the effectiveness of the proposed algorithm. The paper is concluded in Section 5.

Notation. The notations used throughout the paper are standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n \times m$ matrices, respectively. For a matrix P, P^T and P^{-1} represent its transpose and inverse, respectively. $P > 0$ means that the matrix P is real symmetric and positive definite. $\circ$ is the Hadamard product with this product being defined as $[A \circ B]_{ij} = A_{ij} \cdot B_{ij}$. $\mathrm{tr}(\cdot)$ stands for the trace of a matrix. $\mathbb{E}\{x\}$ stands for the expectation of random variable x. I and 0 represent the identity matrix and the zero matrix with appropriate dimensions, respectively. $\mathrm{diag}\{X_1, X_2, \ldots, X_n\}$ stands for a block-diagonal matrix with matrices $X_1, X_2, \ldots, X_n$ on the diagonal. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation and preliminaries

In this paper, we consider the filtering problem for a general class of discrete time-varying systems with stochastic nonlinearities and multiple missing measurements, where the schematic diagram is shown in Fig. 1. The plant under consideration is of the following form:

$$x_{k+1} = f(x_k) + g(x_k, \eta_k) + D_k\omega_k \tag{1}$$

$$y_k = \Xi_k h(x_k) + s(x_k, \zeta_k) + \nu_k \tag{2}$$

where k is the sampling instant, $x_k \in \mathbb{R}^n$ is the state vector to be estimated, $y_k \in \mathbb{R}^q$ is the measurement output, η_k and ζ_k are zero-mean Gaussian noise sequences, D_k is a known matrix with appropriate dimension, $\omega_k \in \mathbb{R}^m$ is the process noise, and $\nu_k \in \mathbb{R}^q$ is the measurement noise. $\Xi_k := \mathrm{diag}\{\alpha_k^1, \alpha_k^2, \ldots, \alpha_k^q\}$ where α_k^i $(i = 1, 2, \ldots, q)$ are q independent random variables in k as well as i and are independent of all noise signals. It is assumed that α_k^i has the probability density function $p_k^i(s)$ on the interval [0, 1] with mathematical expectation μ_k^i and variance