Automatica 48 (2012) 2079–2092

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Identification of spatiotemporally invariant systems for control adaptation☆

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ARTICLE INFO

Article history:
Received 17 April 2010
Received in revised form
5 March 2012
Accepted 16 April 2012
Available online 11 July 2012

Keywords:
Spatially invariant
Spatiotemporal systems
Identification
Interconnected systems
Local consensus
Complex systems
Distributed parameter estimation

ABSTRACT

We present a distributed projection algorithm for system identification of spatiotemporally invariant systems with the ultimate purpose of utilizing it in an indirect adaptive control scheme. Each subsystem communicates only with its immediate neighbors to share its current estimate along with a cumulative improvement index. On the basis of the cumulative improvement index, the best estimate available is picked in order to carry out the next iteration. For small estimation error, the scheme switches over to a "smart" averaging routine. The proposed algorithm guarantees to bring the local estimates arbitrarily close to one another, developing a "local consensus", which makes it amenable to control by the application of indirect distributed adaptive control schemes. It is also shown through simulations that the proposed algorithm has a clear advantage over the standard projection algorithm. Our proposed algorithm is also suitable for addressing the estimation problem in distributed networks that arise in a variety of applications, such as environment monitoring, target localization and potential sensor network problems.

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1. Introduction

With the advances in sensing and actuating techniques coupled with the incessant increase in computational power, the idea of developing more and more complex systems by putting together simpler smaller units is turning into a reality. Examples of such systems can be cited from various areas such as satellite constellations (Shaw, Miller, & Hastings, 1998), cross-directional control in paper processing applications (Stewart, 2000), airplane formation flying (Chichka & Speyer, 1998; Wolfe, Chichka, & Speyer, 1996), automated highway systems (Raza & Ioannou, 1996) and microcantilever array control for various nanorobotic applications (Sarwar, Voulgaris, & Salapaka, 2011a). Lumped approximations of partial differential equations (PDEs) can also be considered in this regard—examples include the deflection of beams, plates, and membranes, and the temperature distribution of thermally conductive materials (Taylor, 1996). Centralized control of such distributed systems becomes increasingly complicated and difficult to implement as the number of underlying subsystems or units increases, hence making distributed control inevitable. The control design of any system, however, is only as good as the system model. When the system model is not available upfront, system identification and control action have to be implemented in parallel. As the system model gets updated, the control law needs to adapt in order to guarantee stability/performance. Hence, adaptation as well as identification need to be carried out in a distributed manner for systems with large numbers of subsystems or units.

Control design of distributed systems is a daunting task in general, and is mostly dominated by the architectural and localization constraints. Such design problems are well known to be difficult, with now nearly three decades of research; see Siljak (1991) and the references therein. Several attempts have been made already to address the problem of distributed adaptive control of interconnected systems employing different approaches while assuming various structures. The most notable early work can be attributed to Ioannou and Kokotovic (1985) in this regard, where weakly interconnected subsystems were studied. Subsequent work includes the M-matrix approach in Ossman (1989), and a high-gain approach in Gavel and Siljak (1989) assuming a strict matching condition on the disturbances. A methodology for handling higher-order interconnections in a distributed adaptive control framework was developed in Shi and Singh (1992), whereas neural networks have been used to approximate unknown interconnections in Spooner

☆ This material is based upon work supported in part by the National Science Foundation under NSF Awards Nos CCR 03-25716 ITR, CNS 08-34409, and by AFOSR grant FA9950-06-1-0252. The material in this paper was partially presented at the American Control Conference (ACC 2010), June 30–July 2, 2010, Baltimore, Maryland, USA. This paper was recommended for publication in revised form by Associate Editor Xiaobo Tan under the direction of Editor Miroslav Krstic.

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0005-1098/$ – see front matter. Published by Elsevier Ltd
doi:10.1016/j.automatica.2012.06.047

Notation

$\mathbb{R}$	Set of reals
$\mathbb{Z}$	Set of integers
$\mathbb{Z}^+$	Set of non-negative integers
l_∞^e	Space of all real spatiotemporal sequences $f = \{f_i(t)\}$ with a two-sided spatial support ($-\infty \le i \le \infty$) and one-sided temporal support ($0 \le t \le \infty$)
LSTV	Linear spatiotemporally varying system
LSTI	Linear spatiotemporally invariant system
$\hat{R}(z, \lambda)$	z, λ transform of LSTI system R
$\mathscr{B}(a, \varepsilon)$	$\{x \in \mathbb{R} \mid \lvert a - x\rvert \le \varepsilon\}$
$Supp(m)$	Support of a spatiotemporal sequence $\{m_i(t)\} = \{[i, t] \in \mathbb{Z}^2 : m_i(t) \neq 0\}$
$\lVert \cdot \rVert$	The Euclidean norm for vectors or the system (induced) norm.

and Passino (1996) and Spooner and Passino (1999). Prior sharing of information amongst controllers about the reference model has been assumed in Mirkin and Gutman (2003), Narendra and Oleng (2002) and Hovakimyan, Lavretsky, Yang, and Calise (2005) for asymptotic tracking of desired outputs. In each of the above cited works, a very narrow class of systems has been addressed owing to the lack of any unified framework for distributed systems. Systems, therefore, have to be treated on a rather case by case basis for distributed control design.

Systems that are either homogeneous or are made up of similar subsystems or units can be approximated by their infinite abstractions, i.e. they can be considered as spatially invariant (refer to Curtain, Iftime, and Zwart (2010) for instances where these abstractions are valid). Examples of such systems include arrays of identical microcantilevers for atomic force microscope applications (Sarwar et al., 2011a), large segmented telescopes (Jiang, Voulgaris, Holloway, & Thompson, 2006), temperature control of thermally conductive material (Taylor, 1996), and fluid flow control (Bamieh, Paganini, & Dahleh, 2002), to cite but a few. Spatial invariance is a strong property of a given system, which means that the dynamics of the system do not vary as we translate along some spatial axis. While the control design of spatially invariant systems has been worked out in some detail (see e.g. Bamieh et al. (2002) and D'Andrea and Dullerud (2003)), knowledge of the underlying system model is assumed a priori. A distributed identification scheme is, therefore, imperative for control design and implementation where the system model is not available upfront.

This article aims to develop a distributed identification scheme for spatially invariant systems that can be used for the adaptation of control laws that are designed for a known plant model. We focus on systems that are recursively computable within the class of spatially invariant systems. Recursibility is a property of certain difference equations, that allows one to iterate the equation by choosing an indexing scheme such that every output sample can be computed from outputs that have already been found from initial conditions and from samples of the input sequence. Systems that can be represented by a two-dimensional rational transfer functions are recursively computable, i.e. a system P is recursively computable if its transfer function has the form

$$\hat{P}(z, \lambda) = \frac{\hat{B}(z, \lambda)}{\hat{A}(z, \lambda)} \tag{1}$$

where $\hat{B}$ and $\hat{A}$ are polynomials in z (spatial domain) and λ (time domain) (Bose, 1982). If $\hat{A}$ and $\hat{B}$ or equivalent state space descriptions are known, current LSTI control design methods can be readily applied. Refer to Bose (1982) (Section 4.5) for a detailed discussion on the conversion of a transfer function formulation into the equivalent state space description. Recursive systems are guaranteed to be well defined and this class encompasses many systems of practical importance, such as discretized partial differential equations (PDEs; deflection of beams, plates, membranes, the temperature distribution of thermally conductive materials (Taylor, 1996)). Systems already discussed above in Sarwar et al. (2011a) and Jiang et al. (2006) are also recursively computable.

Results on parameter estimation for linear lumped systems have been well established (see Goodwin and Sin (1984) and Johnson (1988)). Distributed estimation/identification, on the other hand is still an active area of research. It finds applications in distributed optimization, network consensus, sensor fusion, dynamic systems characterized by PDEs, and wireless networks to name but a few examples. Each of the aforementioned areas brings its own flavor to the quest for distributed estimation/identification. The literature on system identification of distributed systems (assuming a centralized setting) is abundant, with the early attempts geared towards investigations dealing with the 'inverse problem' in heat transfer. For a thorough historic development in this regard see Banks and Kunish (1989), Kubrusly (1977) and Polis and Goodson (1976) and the references therein.

In principle, quite a few recently developed algorithms can be employed for the identification of spatially invariant systems. Diffusion techniques are proposed in Lopes and Syed (2007), where each subsystem combines its current estimate with the estimate of its neighbors, based on some performance criterion, to come up with an aggregate. This aggregate is then used for carrying out the next estimation update. A similar space–time diffusion approach can be found in Xiao, Boyd, and Lall (2006). An iterative optimization algorithm for a networked system is considered in Ram, Nedić, and Veeravalli (2009). Each subsystem (agent) obtains a weighted average of its own iterate with the iterates of its neighbors, and updates the average using the subgradient of its local function to generate the new iterate. Identification of circulant systems is considered in Massioni and Verhaegen (2008) by employing the spatial Fourier transform. The identified data available to each subsystem, however, should be processed centrally in order to construct the global system matrices.

From a control adaptation perspective, however, we are interested in ascertaining whether the following are achieved from a distributed system identification scheme for a spatially invariant system:

(a) the estimation error (the difference between the actual and predicted system output) goes to zero, regardless of the convergence of estimates to the true value;
(b) estimates get close to each other arbitrarily as time grows (at least locally).

While the requirement of (a) is quite clear, the requirement of having (b) is motivated from the fact that such estimated systems can be used for adaptive control using the results recently developed in Sarwar, Voulgaris, and Salapaka (2011b). The literature cited above on system identification, however, does not provide guarantees that we require in (a) and (b). No literature exists, to our knowledge, that addresses the system identification (or adaptive control for that matter) of spatially invariant systems from a control adaptation perspective that provides these guarantees. With this motivation, we develop a distributed projection algorithm for system identification of recursively computable spatiotemporally invariant systems that achieves both of the above mentioned objectives and can, therefore, be employed for adaptive control of spatially invariant systems as demonstrated in Sarwar, Voulgaris, and Salapaka