Automatica 48 (2012) 2194–2200

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

On an intrinsic formulation of time-variant Port Hamiltonian systems[☆]

Markus Schöberl [1], Kurt Schlacher

Johannes Kepler University Linz, Institute of Automatic Control and Control Systems Technology, 4040 Linz, Austria

ARTICLE INFO

Article history:
Received 9 February 2010
Received in revised form
6 February 2012
Accepted 14 March 2012
Available online 30 June 2012

Keywords:
Nonlinear control systems
Differential geometric methods
Mathematical systems theory
Tracking applications
Mechanical systems

ABSTRACT

In this contribution we present an intrinsic description of time-variant Port Hamiltonian systems as they appear in modeling and control theory. This formulation is based on the splitting of the state bundle and the use of appropriate covariant derivatives, which guarantees that the structure of the equations is invariant with respect to time-variant coordinate transformations. In particular, we will interpret our covariant system representation in the context of control theoretic problems. Typical examples are time-variant error systems related to trajectory tracking problems which allow for a Hamiltonian formulation. Furthermore we will analyze the concept of collocation and the balancing/interaction of power flows in an intrinsic fashion.

1. Introduction

Hamiltonian systems have been the object of analysis for a long period and they have been investigated from many different points of view and in many different scientific areas. In the last two decades, in mathematical physics especially field theoretic aspects of Hamiltonian systems without control input are of importance, see Giachetta, Sardanashvily, and Mangiarotti (1997), Gotay (1991) and Kanatchikov (1998). In field theory the use of bundles to distinguish dependent and independent coordinates is commonly used and since time-variant lumped parameter systems can be seen as a special case of field theory with only one independent coordinate the use of bundles also applies to time-variant systems where the fibration is accomplished with respect to the time coordinate. Besides field theory, also in classical mechanics, especially in the time-invariant setting, the geometric interpretation of the Hamiltonian picture is well established, see for example Abraham and Marsden (1978) for many details concerning this subject.

From the control theoretic point of view, the class of Port Hamiltonian systems is a well-analyzed class, see for example Ortega, van der Schaft, Maschke, and Escobar (2002) and van der Schaft (2000) and references therein, where both the theoretical point of view and, of course, the physical applications play a prominent role. Roughly speaking, the main idea of many passivity based control approaches is to maintain the Hamiltonian structure of the system by feedback since this structure has some pleasing properties concerning the stability proof also in the nonlinear scenario.

In the literature most of these approaches for the lumped parameter scenario concerning control theoretic aspects present system analysis, modeling and control for time-invariant systems, whereas the time-variant case is analyzed very rarely. We believe that the main difficulty in the time-variant scenario is the fact that the geometric picture of the equations changes considerably. In the time-invariant setting the role of time is solely to be the curve parameter, which is not true in the time-variant scenario. Contributions which treat the time-variant case especially with regard to control theoretic problems are for example Fujimoto and Sugie (2001, 2003) where the authors consider what they call generalized Hamiltonian systems and canonical transformations which might be time-dependent.

Two important applications where time-variant systems arise quite naturally based on a time-variant change of coordinates should be mentioned at this stage: Firstly, the introduction of displacement coordinates with respect to a system trajectory as it arises for instance when the analysis of the tracking error is the objective, see for example Fujimoto and Sugie (2003). And secondly, in mechanics/robotics floating/accelerated frames of reference are commonly used with respect to an inertial one.

The main contributions of this paper are that: (i) an intrinsic definition of time-variant (Port) Hamiltonian systems is given

[☆] The material in this paper was partially presented at the IFAC Symposium on Nonlinear Control Systems (NOLCOS 2007), August 22–24 2007, Pretoria, South Africa. This paper was recommended for publication in revised form by Associate Editor Henri Huijberts under the direction of Editor Andrew R. Teel.

E-mail addresses: markus.schoeberl@jku.at (M. Schöberl), kurt.schlacher@jku.at (K. Schlacher).

[1] Tel.: +43 650 2055173; fax: +43 732 2468 6322.

0005-1098/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2012.06.014

based on a covariant derivative induced by a connection; (ii) this intrinsic description is analyzed in a differential geometric way; (iii) for the system class of time-variant (controlled) Hamiltonian mechanics, a covariant version of the power balance relation including collocation is developed; and (iv) for the special case where (beside a possible feed-forward) the connection can be expressed as an additive Hamiltonian the results of Fujimoto and Sugie (2001) are recovered.

It is worth mentioning that in our opinion a time-variant (Port) Hamiltonian system has to be introduced using covariant derivatives, which differs significantly from the definition in Fujimoto and Sugie (2001, 2003). We identify 'covariant' with the fact that system properties do not depend on the chosen coordinate chart, i.e., we formulate systems in an intrinsic way. The key idea is the use of a connection which induces a covariant derivative, see Giachetta et al. (1997). Partially, results in this paper have been presented preliminarily in Schöberl and Schlacher (2006) and Schöberl, Stadlmayr, and Schlacher (2007).

2. The time-invariant case

This introductory section is a reminder of time-invariant Port Hamiltonian systems (Maschke, Ortega, & van der Schaft, 2000; Ortega et al., 2002; van der Schaft, 2000) including also the arising matching conditions when state transformations and affine input transformations are considered. It serves as a basis for the generalization to the time-variant case and is also used to introduce the differential geometric language which is then extensively exploited in the time-variant scenario. The notation is similar to the one in Giachetta et al. (1997), where the interested reader can find much more details about this geometric machinery.

To keep the formulas short and readable we will use tensor notation and especially Einstein's convention on sums where we will not indicate the range of the indices used when they are clear from the context. We use the standard symbol $\otimes$ for the tensor product, d is the exterior derivative, $\rfloor$ the natural contraction between tensor fields and $\circ$ denotes the composition of maps. By ∂_A^B are meant the partial derivatives with respect to coordinates with the indices ${}^A_B$.

To study the time-invariant case of Port Hamiltonian systems in a geometric fashion we introduce the state manifold $\mathcal{X}$ equipped with coordinates (x^α), where $\alpha = 1, \ldots, \dim(\mathcal{X})$ and we consider diffeomorphisms (in the following also called transition functions) of the type $\bar{x} = \varphi(x)$ where $\bar{x}$ denotes the states in the transformed coordinate system. Standard differential geometric constructions, see Abraham and Marsden (1978), Giachetta et al. (1997), Nijmeijer and van der Schaft (1990) and Saunders (1989), lead to the tangent bundle $\mathcal{T}(\mathcal{X})$ and the cotangent bundle $\mathcal{T}^*(\mathcal{X})$, which possess the induced coordinates $(x^\alpha, \dot{x}^\alpha)$ and $(x^\alpha, \dot{x}_\alpha)$ with respect to the holonomic bases ∂_α and dx^α. Typical elements of $\mathcal{T}(\mathcal{X})$ (vector fields) and $\mathcal{T}^*(\mathcal{X})$ (1-*forms*) read in local coordinates as $w = \dot{x}^\alpha(x)\partial_\alpha$ and $\omega = \dot{x}_\alpha(x)dx^\alpha$, respectively. To introduce inputs and outputs we consider the vector bundle $\mathcal{U} \to \mathcal{X}$ with the coordinates (x^α, u^i) for $\mathcal{U}$ and the base e_i for the fibers where $i = 1, \ldots, \dim(\mathcal{U}_\mathcal{F})$, where $\mathcal{U}_\mathcal{F}$ denotes the fibers of the input bundle (vector spaces) as well as the dual output vector bundle $\mathcal{Y} \to \mathcal{X}$ possessing the coordinates (x^α, y_i) and the fiber base e^i. Greek indices will correspond to the components of the coordinates of the state manifold and induced structures. Latin indices correspond to the components of the input and the output variables (fibers of the dual bundles $\mathcal{U} \to \mathcal{X}$ and $\mathcal{Y} \to \mathcal{X}$). Let us consider the maps $J, R : \mathcal{T}^*(\mathcal{X}) \to \mathcal{T}(\mathcal{X})$ which are contravariant tensors that are given by the local coordinate expressions

$$J = J^{\alpha\beta}\partial_\alpha \otimes \partial_\beta, \qquad R = R^{\alpha\beta}\partial_\alpha \otimes \partial_\beta \tag{1}$$

with $J^{\alpha\beta}, R^{\alpha\beta} \in \mathcal{C}^\infty(\mathcal{X})$ where J is skew-symmetric, i.e. $J^{\alpha\beta} = -J^{\beta\alpha}$ and R is symmetric $R^{\alpha\beta} = R^{\beta\alpha}$ and positive-semidefinite. Furthermore we introduce the bundle map $G : \mathcal{U} \to \mathcal{T}(\mathcal{X})$ which is a tensor that has the local coordinate expression $G = G_i^\alpha e^i \otimes \partial_\alpha$ with $G_i^\alpha \in C^\infty(\mathcal{X})$. Having the maps J, R and G at our disposal a time-invariant Port Hamiltonian system (with dissipation), see Maschke et al. (2000), Ortega et al. (2002) and van der Schaft (2000), can be constructed as

$$\begin{aligned} \dot{x} &= (J - R)\rfloor dH + G\rfloor u \\ y &= G^*\rfloor dH \end{aligned} \tag{2}$$

where the function $H \in C^\infty(\mathcal{X})$ denotes the Hamiltonian and $G^* : \mathcal{T}^*(\mathcal{X}) \to \mathcal{Y}$ the adjoint (dual) map of G. The local coordinate expression of (2) reads as

$$\begin{aligned} \dot{x}^\alpha &= \left(J^{\alpha\beta} - R^{\alpha\beta}\right)\partial_\beta H + G_i^\alpha u^i \\ y_i &= G_i^\alpha \partial_\alpha H. \end{aligned} \tag{3}$$

We want to analyze structure preserving transformations for the system (2). To allow for affine input transformations we can replace the input bundle by an affine one $\mathcal{Z} \to \mathcal{X}$ (with underlying vector bundle $\mathcal{U} \to \mathcal{X}$), for the geometric properties of affine bundles see for example Giachetta et al. (1997) and references therein. The transition functions for the vector bundle and the affine bundle read as

$$\bar{u} = Mu, \qquad \bar{u}^{\bar{j}} = M_i^{\bar{j}} u^i \tag{4}$$

$$\bar{u} = Mu + g, \qquad \bar{u}^{\bar{j}} = M_i^{\bar{j}} u^i + g^{\bar{j}} \tag{5}$$

with $M_i^{\bar{j}}, g^{\bar{j}} \in C^\infty(\mathcal{X})$ where $\bar{u}$ denotes the transformed input coordinates and we restrict ourselves to regular transformations (i.e. M is invertible). The geometric representation of the system leads to the observation that the structure of (2) is preserved by a diffeomorphism of the type $\bar{x} = \varphi(x)$ together with (4). The case of an affine input bundle is more challenging since the preservation of the structure demands to solve a partial differential equation. See also Cheng, Astolfi, and Ortega (2005) in this context, where the problem of general feedback equivalence of nonlinear systems to Port Hamiltonian systems is discussed and so-called *matching conditions* appear.

Lemma 1. *Consider the system (2) together with the diffeomorphism $\bar{x} = \varphi(x)$ and (5). The structure of (2) is preserved if and only if we can find a solution $\breve{H} \in C^\infty(\bar{\mathcal{X}})$ of the partial differential equations*

$$(\bar{J}^{\bar{\alpha}\bar{\beta}} - \bar{R}^{\bar{\alpha}\bar{\beta}})\partial_{\bar{\beta}}\breve{H} - \left(\partial_\alpha \varphi^{\bar{\alpha}} G_i^\alpha \hat{M}_{\bar{j}}^i g^{\bar{j}}\right) \circ \hat{\varphi} = 0. \tag{6}$$

Here $\bar{J}^{\bar{\alpha}\bar{\beta}}$ and $\bar{R}^{\bar{\alpha}\bar{\beta}}$ are the components of the transformed tensors (1) with respect to $\bar{x} = \varphi(x)$. The inverse maps are denoted by $x = \hat{\varphi}(\bar{x})$ and $M_i^{\bar{j}}\hat{M}_{\bar{j}}^k = \delta_i^k$ where δ is the Kronecker delta.

Remark 2. The partial differential equations (6) are written in the coordinates $\bar{x}$ but it is readily observed that they can be formulated in the original coordinates x, as well.

The proof of this lemma is a straightforward calculation in local coordinates. If in Lemma 1 a solution for $\breve{H}$ can be obtained, then the following corollary is an immediate consequence.

Corollary 3. *Suppose (6) is met, then the system (3) in the new coordinates reads as*

$$\begin{aligned} \dot{\bar{x}}^{\bar{\alpha}} &= (\bar{J}^{\bar{\alpha}\bar{\beta}} - \bar{R}^{\bar{\alpha}\bar{\beta}})\partial_{\bar{\beta}}(\bar{H} - \breve{H}) + \bar{G}_{\bar{i}}^{\bar{\alpha}}\bar{u}^{\bar{i}} \\ \bar{y}_{\bar{i}} &= \bar{G}_{\bar{i}}^{\bar{\alpha}}\partial_{\bar{\alpha}}(\bar{H} - \breve{H}) \end{aligned}$$