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Brief paper

# Design of new fault diagnosis and fault tolerant control scheme for non-Gaussian singular stochastic distribution systems[☆]

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ABSTRACT

New fault diagnosis (FD) and fault tolerant control (FTC) algorithms for non-Gaussian singular stochastic distribution control (SDC) systems are presented in this paper. Different from general SDC systems, in singular SDC systems, the relationship between the weights and the control input is expressed by a singular state space model, which increases the difficulty in the FD and FTC design. The proposed approach relies on an iterative learning observer (ILO) for fault estimation. The fault may be constant, fast-varying or slow-varying. Based on the estimated fault information, the fault tolerant controller can be designed to make the post-fault probability density function (PDF) still track the given distribution. Simulations are given to show the effectiveness of the proposed FD and FTC algorithms.



## 1. Introduction

Reliability and stability are of paramount importance for practical processes. Fault diagnosis and fault tolerant control theory have attracted considerable academic interest and, as a result, a variety of techniques for FDD and FTC have been developed over the past two decades (see Basseville and Nikiforov (2002), Frank and Ding (1997), Isermann (2005), Jiang, Staroswiecki, and Cocquempot (2006), Patton and Chen (1996), Qu, Ihlefeld, Jin, and Saengdeejing (2003), Wang, Huang, and Steven (1997), Yang, Jiang, and Staroswiecki (2009) and Zhang, Polycarpou, and Parisini (2010) for surveys). For stochastic systems, the so-far obtained FDD approaches can be classified as

1 The system identification technique (Isermann, 2005);
2 The observer or filter-based method (Frank & Ding, 1997);
3 The statistical approach based on the Bayesian theorem, Monte Carlo approach, likelihood method, and hypothesis test technique (Basseville & Nikiforov, 2002).

The first approach uses an ARMAX model to represent the system and apply parameter identification, such as least square algorithms or stochastic gradient approaches, to estimate the unexpected changes in the system. In the second method, the residual can be generated using an observer or filter and the fault can be detected and estimated through the analysis and disposal of the residual. Furthermore, using the estimated fault information, the fault tolerant controller can be designed to guarantee the stability of the post-fault closed-loop system and maintain a certain performance. In terms of Kalman filter-based FDD methods, the innovation is obtained using the filter and the statistical information of the innovation is analyzed so that it can be decided whether the system has a fault or not. Generally, the observer-based or filter-based FDD methodologies have been developed along with the observer or filter design theory, and many of them have been applied to practical processes successfully.

For many practical systems, the system representation can be realized as a set of generalized dynamical "input–output" mathematical models between the input and the PDFs of the output, rather than the output itself (Crowley, Meadows, Kostoulas, & Doule III, 2000; Karny, 1996; Wang & Lin, 2000).

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These types of stochastic systems are much more general in their forms than the classical ones, which can be used to represent either Gaussian or non-Gaussian dynamical systems. Assuming the unexpected changes of the parameters in these mathematical models are regarded as faults in the system, then the FDD task should be performed in terms of detecting and diagnosing the faults by using the available output PDFs and the input. It is noted that if only output PDFs can be measured rather than the output vector itself, most existing observer-based FDD approaches are invalid. An observer-based fault detection algorithm was first developed in Wang and Lin (2000) to detect the fault in the non-Gaussian SDC system based on the linear $B$-spline approximation model, where signals that can be used are the input and the output PDFs. Subsequently, the FDD algorithms (Guo & Wang, 2005; Yao & Wang, 2008; Yin & Guo, 2009) have also been proposed for the non-Gaussian SDC systems based on the rational $B$-spline approximation model, the square-root $B$-spline approximation model and the rational square-root $B$-spline approximation model. In Guo and Wang (2005), a new observer-based fault detection approach is formulated in terms of linear matrix inequalities (LMIs) and the threshold is determined by the solution of LMIs and the bounds of uncertainties. Furthermore, an adaptive fault diagnostic observer is also designed to estimate the size of the fault. A nonlinear adaptive observer-based fault diagnosis algorithm is proposed in Yao and Wang (2008) to diagnose the fault in the dynamic part of the non-Gaussian SDC systems based on the rational square $B$-spline approximation model. Through the controller re-configuration, a good output PDF tracking can still be realized when the fault occurs.

In the above mentioned SDC systems, only dynamic links between the inputs and the weights are considered. However, in practice, some algebraic relations also exist between the input and the weights, leading to a singular state space model between the weights and the control input. Such systems are called singular stochastic distribution control systems. Only a few works have been reported on the FDD and FTC of such singular stochastic systems. This forms the main purpose of the work in this paper where the FDD algorithm of non-singular stochastic distribution systems can be extended to that of the singular stochastic distribution systems using an equivalent transformation. In this context, the present work considers FD and fault accommodation by proposing an iterative learning observer (ILO) based fault estimation algorithm using the estimated fault information. The fault tolerant controller is designed so as to make the post-fault PDF still track its given distribution.

The rest of this paper is organized as follows. Section 2 presents the model description. In Section 3, the iterative learning observer based fault diagnosis algorithm is proposed. Section 4 gives the design of the fault tolerant control. Simulation results of FD and FTC are presented in Section 5, followed by some concluding remarks in Section 6.

## 2. Model description

Denote $\gamma(y, u(t))$ as the probability density function of the system output with $y$ being defined on a known bounded interval $[a, b]$; the continuous singular stochastic distribution control (SDC) system can be expressed as follows:

$$E\dot{x}(t) = Ax(t) + Bu(t) + NF(t)$$
$$V(t) = Dx(t) \tag{1}$$

$$\gamma(y, u(t)) = C(y)V(t) + T(y) \tag{2}$$

where $x \in R^n$ is the state vector, $V(t) \in R^{n-1}$ is the output weight vector, $u(t) \in R^m$ is the control input vector and $F \in R^m$ is the fault vector. $A \in R^{n\times n}$, $B \in R^{n\times m}$, $D \in R^{(n-1)\times n}$, $E \in R^{n\times n}$ and $N \in R^{n\times m}$ are system parameter matrices with $rank(E) = q < n$ (i.e. $E$ is a singular matrix).

Indeed, system (1) represents the dynamic mode of the weight vector and (2) describes the static output PDF model using the $B$-spline network approximation. Eq. (2) comes from the linear $B$-spline approximation (Wang, 2000):

$$\gamma(y, u(t)) = \sum_{i=1}^{n} \omega_i(u(t))\phi_i(y) \tag{3}$$

where $\phi_i(y)(i = 1, \ldots, n)$ are the pre-specified basis functions defined on $[a, b]$ and $\omega_i(i = 1, \ldots, n)$ are the corresponding weights. $C(y) \in R^{1\times(n-1)}$, $T(y)$ in (2) are decided by the selected basis functions and $V(t) = [\omega_1(u(t))\omega_2(u(t)), \ldots, \omega_{n-1}(u(t))]^T$ is the independent weight vector shown in (1).

The following two assumptions are made:

**Assumption 1.** The system is regular, i.e. $|sE - A| \neq 0$.

**Assumption 2.** The system has no impulse, i.e. $rankE = \deg |sE - A|$.

**Remark 1.** Assumption 1 demonstrate that when system (1) is faultless, it is said to be regular if $\det(sE - A)$ is not identically zero. Assumption 2 shows that when system (1) is faultless, it is said to be impulse-free if $\deg(\det(sE - A)) = rankE$.

With these two assumptions, there exist two non-singular matrices $Q$ and $P$ such that

$$QEP = \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}, \qquad QAP = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-q} \end{bmatrix} \tag{4}$$

where $Q, P \in R^{n\times n}$, $A_1 \in R^{q\times q}$ and $I_i$ is the identity matrix of order $i$. By applying the following state coordinate transformation

$$x(t) = P \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{5}$$

where $x_1(t) \in R^{q\times 1}$, $x_2(t) \in R^{(n-q)\times 1}$, it can be obtained from Eqs. (1) and (2) that

$$\begin{aligned} &\dot{x}_1(t) = A_1x_1(t) + B_1u(t) + N_1F \\ &x_2(t) = -B_2u(t) - N_2F \\ &V(t) = D_1x_1(t) + D_2x_2(t) \\ &\gamma(y, u(t)) = C(y)[D_1x_1(t) + D_2x_2(t)] + T(y) \end{aligned} \tag{6}$$

where $B_1, N_1 \in R^{q\times m}$, $B_2, N_2 \in R^{(n-q)\times m}$, $D_1 \in R^{(n-1)\times q}$, $D_2 \in R^{(n-1)\times(n-q)}$ can be determined by the following equation

$$\begin{aligned} &QB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \qquad DP = \begin{bmatrix} D_1 & D_2 \end{bmatrix}, \\ &QN = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}. \end{aligned} \tag{7}$$

Under this state transformation, the static PDF model Eq. (2) remains unchanged. This means that the transformed system (6) is an equivalent representation of Eqs. (1) and (2). For system (6), it is also assumed that $\{A_1, D_1\}$ is observable.

## 3. Fault diagnosis algorithm

Since the ILO which is combined with an adaptive law will be used in this paper to estimate fault $F$, in this section we will present the feature of the ILO proposed in Ref. Chen and Saif (2006) for the