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Optimal transition maneuvers for a class of V/STOL aircraft☆

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ABSTRACT

This paper focuses on the problem of computing optimal *transition maneuvers* for a particular class of tail-sitter aircraft able to switch their flight configuration from hover to level flight and vice versa. Both minimum-time and minimum-energy optimal transition problems are formulated and solved numerically in order to compute reference maneuvers to be employed by the onboard flight control system to change the current flight condition. In order to guide the numerical computation and to validate its results, in a first stage approximated solutions are obtained as a combination of a finite number of *motion primitives* corresponding to analytical trajectories of approximated dynamic models. The approximated solution is then employed to generate an initial guess for the numerical computation applied to a more accurate dynamic model. Numerical trajectories computed for a small scale prototype of tail-sitter aircraft are finally presented, showing the effectiveness of the proposed methodology to deal with the complex dynamics governing this kind of systems.

1. Introduction

The problem of flying efficiently, in terms of the amount of onboard energy consumed, represents a very important issue in both the design and control of aerial vehicles. For this reason recent literature has focused on the topic of controlling Vertical/Short Take-Off and Landing (V/STOL) aircraft (see Ransone, 2002, for an overview on this class of vehicles) both in the efficient *level flight* and in the stable *hover flight*, which can be combined by performing the so-called *transition maneuver*. The transition maneuver consists of a certain flight trajectory changing the current flight configuration from the hover to the level flight and vice versa. In the efficient and fast level flight, the aircraft's configuration is similar to the one of an airplane, in which the force of gravity is compensated by the aerodynamic lift force generated by proper aerodynamic surfaces such as *wings* or *canards*—see Stengel (2004). During the hover flight, on the contrary, the aircraft configuration is more similar to the one of a helicopter, in which the force generated by the propeller is in charge of compensating for the gravity force, by consuming, as a main drawback, a large amount of the onboard fuel. On the other side hovering allows the aircraft to achieve a higher level of maneuverability and then to perform, for example, stable low speed flight next to infrastructures or obstacles, take-off and landing in small areas, etc. A typical flight plan for a V/STOL aircraft may indeed include several transition maneuvers, in order to enforce the flight behavior that is more convenient to perform the current operation. The problem of performing transition maneuvers has been recently under investigation in the control system literature by focusing on, for example, large scale V/STOL aircraft (Benosman and Lum (2007) and Oishi and Tomlin (1999)), miniature acrobatic airplanes (Frank, McGrew, Valenti, Levine, and How (2007) and Green and Oh (2005)), ducted-fan aerial vehicles (Guerrero, Londenberg, Gelhausen, and Myklebust (2003), Jadbabaie (0000) and Johnson and Turbe (2005)) and tail-sitter aircraft ((Stone & Clarke, 2001)). The main contribution of this paper is to develop a methodology suitable to address the complex optimal transition maneuver for the large class of tail-sitter V/STOL vehicles in which the transition is achieved through a change of the attitude configuration. Due to the complexity of the dynamic model governing the aircraft during the transition, largely depending on the nonlinearities in the expression of the aerodynamic forces, numerical optimization has been considered, by focusing on a *direct collocation* method (Betts, 2001) based upon pseudo-spectral approximation of the continuous states and inputs of the system (Ross & Fahroo, 2003). In order to guide the numerical computation and to validate its results, in the first part of the paper – drawing inspiration by motion planning approaches such as the ones proposed in Frazzoli (2001) – feasible solutions are obtained as a combination of a finite number of *motion primitives*.

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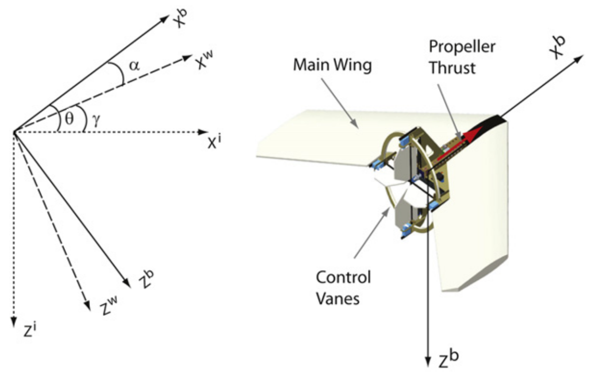

Fig. 1. A prototype of tail-sitter V/STOL aircraft.

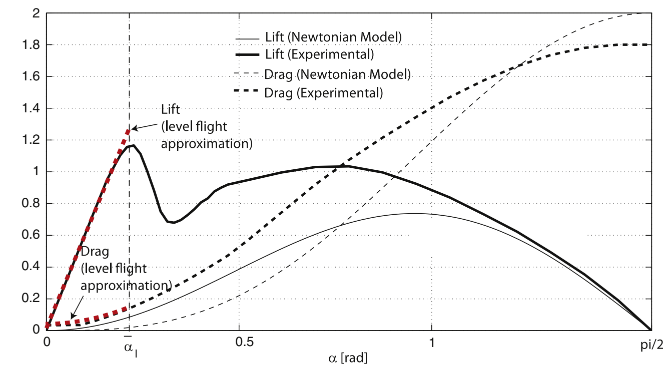

Fig. 2. Lift and drag coefficients versus angle of attack. The experimental values are taken from Sheldahl and Kilmas (1981), while Newtonian approximated model is described in Stengel (2004). Red dashed lines represent the approximations that will be used for the level flight.

In the proposed approach a motion primitive corresponds to an analytical trajectory of an approximated low order dynamic model of the aircraft. A finite number of primitives can be then combined in order to reach the final desired flight configuration (hover or level) and, in turn, to solve the original optimal transition problem as a reduced complexity Mixed-Integer Nonlinear Programming (MINLP) problem. In the latter the analytical trajectories computed for each primitive are combined as a function of a small number of optimization parameters. Feasible solutions of the proposed MINLP are finally employed to generate the initial guess for the numerical computation carried out on a more accurate numerical dynamic model. The paper is organized as follows. Section 2 introduces the dynamic model of the system and the approximations introduced to compute analytical trajectories. Section 3 details the four main optimal control problems addressed in the paper. Section 4 shows how approximated optimal trajectories can be generated as a combination of suitably defined motion primitives, introducing the MINLP underlying the optimal transition problem. Section 5 shows the numerical results obtained using the numerical tool and taking advantage of the solutions of the MINLP previously defined. Finally, in Section 6, some final considerations are presented.

2. Dynamic model

2.1. General model

The V/STOL aircraft considered in this paper (see also the prototype sketched in Fig. 1) consists of a miniature vehicle capable of both stable hover flight and, by means of proper aerodynamic surfaces such as wings or canards, level flight.

The equations of motion can be derived by defining three Euclidean reference frames: the inertial earth-fixed reference frame, denoted as F_i, the body-fixed reference frame, denoted as F_b, and finally the so-called wind frame, which is a reference frame sharing the origin with the body-fixed frame and with the x-axis aligned with the flight speed vector of the vehicle (see Fig. 1). We denote by the superscript i, b, w vectors belonging respectively to F_i, F_b and F_w. For sake of simplicity, a reduced-order planar dynamic model of the system, describing the longitudinal, vertical and pitch dynamics of the vehicle, is considered. Namely, lateral and yaw dynamics of the vehicle are neglected in the computation of the optimal transition maneuver. The ratio behind this assumption is that yaw and lateral reference trajectories can be trivially taken zero during the transition without affecting the optimal solution. Put into a control perspective, this simplification is justified if the flight configurations underlying the transition maneuver are characterized by sufficient control authority as far as the yaw and lateral degree-of-freedom are concerned. For this reason a particular attention will be paid, in the forthcoming analysis, to generate longitudinal, vertical and pitch transition references that do not affect the control authority on the lateral and yaw direction of the vehicle.

In order to model the planar dynamics, we consider the following Newton–Euler rigid body equations in the configuration manifold $\mathbb{R}^2 \times S_1$

$$M \begin{bmatrix} \ddot{x}^i \\ \ddot{z}^i \end{bmatrix} = R_{ib}(\theta) \begin{bmatrix} T \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ Mg \end{bmatrix} + R_{ib} R_{bw} \begin{bmatrix} -D(\cdot) \\ -L(\cdot) \end{bmatrix} \tag{1}$$

where x^i and z^i denote, respectively, the longitudinal and vertical position of the center of gravity, M denotes the mass of the vehicle and T the propeller thrust. The two rotation matrices $R_{ib}(\theta)$ and $R_{bw}(\alpha)$, which define, respectively, the transformation from the body frame to the inertial frame and from the wind frame to the body frame, are given by

$$R_{ib}(\theta) := \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \qquad R_{bw}(\alpha) := \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

in which θ denotes the *pitch* angle and α the *angle of attack*. The expression of the angle of attack (see, among others, Stengel, 2004), by neglecting, for sake of simplicity, the presence of wind, is a function of the vehicle velocity expressed in the body-fixed axis, i.e. $\alpha := \arctan(\dot{z}^b/\dot{x}^b)$. The aerodynamic forces have been modeled by considering the presence of a main wing as the most important airfoil on the vehicle. In this respect, the lift, $L(\cdot)$, and the drag, $D(\cdot)$, aerodynamic forces are given by the following expressions

$$L(\alpha, v) := \frac{1}{2}\rho S C_L(\alpha) v^2, \qquad D(\alpha, v) := \frac{1}{2}\rho S C_D(\alpha) v^2 \tag{2}$$

where $v = \sqrt{\dot{x}^{i2} + \dot{z}^{i2}}$ is the flight speed, S the airfoil's surface, ρ the air density and where $C_L(\alpha) : (-\pi, \pi] \rightarrow \mathbb{R}, C_D(\alpha) : (-\pi, \pi] \rightarrow \mathbb{R}_{\geq 0}$ are, for the large flight envelope considered in this paper, nonlinear functions of the angle of attack to be properly estimated or measured experimentally (see also Randall, Hoffmann, & Shkarayev, 2011). Representative approximated lift and drag coefficients, which have been employed to derive some of the numerical results described in Section 5, are shown in Fig. 2 (obtained by the experimental data presented in Sheldahl & Kilmas, 1981). The reader is referred, for example, to Stengel (2004) for more details in this direction.

System (1) can be rewritten by means of the following *longitudinal aircraft dynamics* equations (see for example Etkin, 2005; Stengel, 2004):

$$\begin{aligned} M\dot{v} &= -D(\alpha, v) + T\cos\alpha - Mg\sin\gamma \\ Mv\dot{\gamma} &= L(\alpha, v) + T\sin\alpha - Mg\cos\gamma \end{aligned} \tag{3}$$

where the *flight path angle* γ denotes the direction of the motion of the aircraft (as depicted in Fig. 1) and it is given by $\gamma := \theta - \alpha$. By construction, we have $\dot{x}^i = v\cos\gamma$ and $\dot{z}^i = -v\sin\gamma$. The above longitudinal aircraft dynamics will be employed in