Automatica 45 (2009) 1611–1618

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Asymptotic stabilization via control by interconnection of port-Hamiltonian systems ☆

Fernando Castaños [a,*], Romeo Ortega [a], Arjan van der Schaft [b], Alessandro Astolfi [c,d]

[a] Laboratoire des Signaux et Systèmes, Supélec 3 rue Joliot Curie, 91192 Gif-sur-Yvette, France

[b] Department of Mathematics, University of Groningen, PO Box 800, 9700 AV Groningen, The Netherlands

[c] Electrical Engineering Department, Imperial College London, Exhibition Road, London. SW7 2BT, UK

[d] DISP, University of Roma, Tor Vergata Via del Politecnico 1, 00133 Rome, Italy

ARTICLE INFO

Article history:
Received 10 April 2008
Received in revised form
15 October 2008
Accepted 5 March 2009
Available online 15 April 2009

Keywords:
Nonlinear control systems
Port-Hamiltonian systems
Asymptotic stability
Passivity-based control

ABSTRACT

We study the asymptotic properties of *control by interconnection*, a passivity-based controller design methodology for stabilization of port-Hamiltonian systems. It is well-known that the method, in its basic form, imposes some unnatural controller initialization to yield asymptotic stability of the desired equilibrium. We propose two different ways to overcome this restriction, one based on adaptation ideas, and the other one adding an extra damping injection to the controller. The analysis and design principles are illustrated through an academic example.

1. Introduction

Recently, port-Hamiltonian (PH) models (van der Schaft, 2000) have been a focus of attention in the control community (e.g. Cheng, Astolfi, and Ortega (2005), Fujimoto, Sukurama, and Sugie (2003), Ortega, van der Schaft, Maschke, and Escobar (2002) and Wang, Feng, and Cheng (2007)). There are, at least, two reasons for their appeal: first, that they describe a wide class of physical systems, including (but not limited to), systems described by Euler–Lagrange equations. Second, that PH models directly reveal the fundamental role of the physical concepts of energy, dissipation and interconnection—making passivity-based control (PBC) (Ortega & Spong, 1989; van der Schaft, 2000) a suitable candidate to regulate the behavior of PH systems.

In this paper, we are interested in the stabilization of PH systems using control by interconnection (CbI) (Ortega, van der Schaft, Mareels, & Maschke, 2001; Ortega et al., 2002). Similarly to other PBC techniques, the objective in CbI is to render the closed-loop passive with respect to a desired energy (storage) function. This is accomplished in CbI selecting the controller to also be a PH system which, connected to the plant through a power-preserving interconnection, results in a closed-loop that is again PH with energy function equal to the sum of the plant's and the controller's energies.

In its original formulation, applicability of CbI is stymied by the so-called dissipation obstacle (Ortega et al., 2001), a problem that appears when the dissipation of the open-loop is different from zero at the desired equilibrium. In Ortega, van der Schaft, Castaños, and Astolfi (2008), this problem has been solved, generating different passive outputs giving rise to the so-called power shaping CbI. Both methods, standard and power shaping CbI, rely on the creation of functions, called Casimirs, which are independent of the energy function. The existence of these invariants presents an obstruction to the *asymptotic* stabilization of the desired equilibrium. The main contribution of this paper is to propose two modifications to the existing CbI to overcome this problem. The first modification is motivated by adaptation principles, while the second one is based on the addition of an extra damping injection to the controller. As an additional by-product of the analysis performed, the two versions of CbI are unified.

☆ This paper was partially supported by CONACyT (Mexico) and European project HYCON with reference code FP6-IST-511368. The material in this paper was partially presented at XIII Congreso Latinoamericano de Control Automatico. Merida, Venezuela. Nov. 23–25 08. This paper was recommended for publication in revised form by Associate Editor Dragan Nešic under the direction of Editor Andrew R. Teel.

* Corresponding author. Tel.: +33 1 69 85 17 20; fax: +33 1 69 85 17 65.
E-mail addresses: castanos@lss.supelec.fr (F. Castaños), ortega@lss.supelec.fr (R. Ortega), a.j.van.der.schaft@math.rug.nl (A. van der Schaft), a.astolfi@ic.ac.uk (A. Astolfi).

0005-1098/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2009.03.015

To make the paper self-contained, we begin the following section with a brief description of CbI and refer the reader to Ortega et al. (2008) for more details. Section 3 contains specific guidelines to apply CbI for equilibrium stabilization. The modifications to achieve asymptotic stability are then presented in Section 4. Finally, we state some concluding remarks in Section 5.

Notation. The arguments of the functions are omitted once they are defined and there is no possibility of confusion. All vectors defined in the paper are column vectors, even the gradient of a scalar function, denoted with the operator $\nabla \triangleq \frac{\partial}{\partial x}$. We also define $\nabla^2 \triangleq \frac{\partial^2}{\partial x^2}$. Given a vector x and a matrix $K = K^\top > 0$, $\|x\|$ denotes the Euclidean norm and $\|x\|_K$ the norm $x^\top K x$.

2. Preliminaries

Although this note deals with PH systems (van der Schaft, 2000) only, it will be useful to consider first a general nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input and $y \in \mathbb{R}^m$ is the output, with $m \leq n$. The functions f, g and h are smooth and of appropriate dimensions and the matrix g is full rank, uniformly in x.

2.1. Cyclo-passivity

Definition 1. System (1) is said to be *cyclo-passive* if there exists a differentiable function $H : \mathbb{R}^n \to \mathbb{R}$ (called the storage function) that satisfies the power balance inequality

$$\dot{H} \leq y^\top u \tag{2}$$

when evaluated along the trajectories of (1).

Recall that a system is passive if (2) holds and H is bounded from below. Because of this additional restriction, every passive system is cyclo-passive but the converse is not true. In terms of energy exchange, cyclo-passive systems exhibit a net absorption of energy along *closed* trajectories (Hill & Moylan, 1980), while passive systems absorb energy along *any* trajectory that starts from a state of minimal energy $x(0) = \arg\min H(x)$.

According to Hill–Moylan's Theorem (Hill & Moylan, 1980), system (1) is cyclo-passive (with storage function $H(x)$) if and only if, for some $q \in \mathbb{N}$, there exists a function $l : \mathbb{R}^n \to \mathbb{R}^q$ such that

$$\nabla H^\top f = -\|l\|^2 \tag{3a}$$

$$h = g^\top \nabla H. \tag{3b}$$

Setting the dissipation $d \triangleq \|l\|^2$ and differentiating H leads to the power balance

$$\dot{H} = y^\top u - d. \tag{4}$$

We now focus on PH systems

$$\Sigma : \begin{cases} \dot{x} = F\nabla H + gu \\ y = g^\top \nabla H \end{cases} \tag{5}$$

where $F : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, with $F + F^\top \leq 0$. It can be easily verified that (5) is cyclo-passive with storage function H and dissipation $d \triangleq -\nabla H^\top F \nabla H$.

For future reference let us compute the assignable equilibria of (5) as the elements of the set

$$\mathcal{E}_x \triangleq \left\{ x \mid g^\perp F \nabla H = 0 \right\}, \tag{6}$$

with $g^\perp : \mathbb{R}^n \to \mathbb{R}^{(n-m)\times n}$ a full rank left-annihilator of g, that is, $g^\perp g = 0$ and $\operatorname{rank} g = n - m$. Associated to each $x_\star \in \mathcal{E}_x$ there is a uniquely defined constant control given by

$$u_\star \triangleq -g^+(x_\star)F(x_\star)\nabla H(x_\star), \tag{7}$$

where g^+ is the Moore-Penrose pseudo-inverse of g, that is, $g^+ \triangleq [g^\top g]^{-1} g^\top$. Note that g^+ is well-defined since g is assumed full rank, implying that the inverse of $g^\top g$ always exists.

2.2. Example

The system described by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}x_1 + x_2 \\ -x_2^2 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} - x_2^2 \\ x_2^3 \end{pmatrix} u \tag{8}$$

can be written in the PH form (5) with

$$F = \begin{pmatrix} -\frac{1}{2} & x_2 \\ 0 & -x_2^2 \end{pmatrix}, \qquad H = \frac{1}{2}x_1^2 + x_2, \qquad g = \begin{pmatrix} \frac{1}{2} - x_2^2 \\ x_2^3 \end{pmatrix} \tag{9}$$

and output

$$y = g^\top \nabla H = x_1 \left(\frac{1}{2} - x_2^2 \right) + x_2^3.$$

Notice that Eq. (4) does not yield any information about the stability of the open-loop equilibrium $(0, 0)$, since H is not bounded from below. Actually, it can be readily seen that with $u = 0$ the equilibrium is unstable and that the trajectories of the open-loop system exhibit finite escape time. Moreover, the origin cannot be stabilized by any continuous feedback.

The set of assignable equilibria for this system is

$$\mathcal{E}_x = \left\{ (x_1, x_2) \mid x_2^2(1 - x_1 x_2) = 0 \right\}. \tag{10}$$

2.3. Control by interconnection

In CbI a PH controller of the form

$$\Sigma_c : \begin{cases} \dot{\xi} = u_c \\ y_c = \nabla H_c(\xi) \end{cases} \tag{11}$$

is proposed. $\xi \in \mathbb{R}^m$ is the state of the controller, u_c, y_c are the input and the output of the controller, respectively, and $H_c : \mathbb{R}^m \to \mathbb{R}$ is a to-be-designed controller storage function. See Ortega et al. (2008) and van der Schaft (2000) for a justification of this choice of controller structure.

Control by interconnection comes in two basic variants. In the standard version, Σ and Σ_c are coupled using the classical unitary feedback power-preserving interconnection

$$\Sigma_I : \left\{ \begin{pmatrix} u \\ u_c \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} y \\ y_c \end{pmatrix} + \begin{pmatrix} v \\ 0 \end{pmatrix} \right., \tag{12}$$

where v is a new virtual input.[1] It is well-known (van der Schaft, 2000) that the PH structure is invariant under power-preserving interconnection; this pattern leading to the interconnected PH system

$$\Sigma_{Ts} : \begin{cases} \begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} F & -g \\ g^\top & 0 \end{pmatrix} \nabla H_T + \begin{pmatrix} g \\ 0 \end{pmatrix} v \\ y_{Ts} = \begin{pmatrix} g^\top & 0 \end{pmatrix} \nabla H_T \end{cases} \tag{13}$$

[1] We recall that an interconnection of PH systems is power preserving if it satisfies $y^\top u + y_c^\top u_c = y^\top v$.