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Magnetohydrodynamic state estimation with boundary sensors[☆]

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ABSTRACT

We present a PDE observer that estimates the velocity, pressure, electric potential and current fields in a magnetohydrodynamic (MHD) channel flow, also known as Hartmann flow. This flow is characterized by an electrically conducting fluid moving between parallel plates in the presence of an externally imposed transverse magnetic field. The system is described by the inductionless MHD equations, a combination of the Navier–Stokes equations and a Poisson equation for the electric potential under the so-called inductionless MHD approximation in a low magnetic Reynolds number regime. We identify physical quantities (measurable on the wall of the channel) that are sufficient to generate convergent estimates of the velocity, pressure, and electric potential field away from the walls. Our observer consists of a copy of the linearized MHD equations, combined with linear injection of output estimation error, with observer gains designed using backstepping. Pressure, skin friction and current measurements from one of the walls are used for output injection. For zero magnetic field or nonconducting fluid, the design reduces to an observer for the Navier–Stokes Poiseuille flow, a benchmark for flow control and turbulence estimation. We show that for the linearized MHD model the estimation error converges to zero in the L^2 norm. Despite being a subject of practical interest, the problem of observer design for nondiscretized 3-D MHD or Navier–Stokes channel flow has so far been an open problem.

1. Introduction

Recent years have been marked by dramatic advances in active flow control, but developments have had little effect on conducting fluids moving in magnetic fields. There are some recent results in stabilization though, for instance using nonlinear model reduction (Baker & Christofides, 2002), open-loop control (Berger, Kim, Lee, & Lim, 2000) and optimal control (Debbagh, Cathalifaud, & Airiau, 2007). Some experimental results are available, showing that control of such flows is technologically feasible; actuators consist of magnets and electrodes (Breuer & Park, 2004; Pang & Choi, 2004; Thibault & Rossi, 2003). Mathematical studies of controllability of magnetohydrodynamic flows have been done, though they do not provide explicit controllers (Barbu, Popa, Havarneanu, & Sritharan, 2003; Sritharan, Barbu, Havarneanu, & Popa, 2005). Despite being a subject of obvious practical interest, there are no previous results focusing on estimation of the velocity and electromagnetic fields for conducting fluids.

In this paper, we consider an incompressible MHD channel flow, also known as the Hartmann flow, a benchmark model for applications such as cooling systems (computer systems, fusion reactors), hypersonic flight, propulsion and laser applications. In this flow, an electrically conducting fluid moves between parallel plates and is affected by an imposed transverse magnetic field. When a conducting fluid moves in the presence of a magnetic field, it produces an electric field due to charge separation and subsequently an electric current. The interaction between this created electric current and the imposed magnetic field originates a body force, called the Lorentz force, which acts on the fluid itself. The velocity and electromagnetic fields are mathematically described by the MHD equations (Muller & Buhler, 2001; Sermange & Temam, 1983), which are the Navier–Stokes equations coupled with the Maxwell equations.

Our observer obtains an estimate of the whole velocity, pressure, electric potential and current fields, derived only from wall measurements. Obtaining such an estimate can be of interest in itself, depending on the application. For example, the absence of effective state estimators modeling turbulent fluid flows is considered one of the key obstacles to reliable, model-based weather forecasting. In other engineering applications in which

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active control is needed, such as drag reduction (Pang & Choi, 2004) or mixing enhancement for cooling systems (Schuster & Krstic, 2003), designs usually assume unrealistic full state knowledge, therefore a state estimator is necessary for effective implementation.

This paper extends our previous work for estimation of the velocity field in a 2-D channel flow (Vazquez & Krstic, 2005). Our observer is designed for the continuum MHD model and consists of a copy of the plant together with output injection of measurement error. We identify which physical quantities (measurable on the wall of the channel) are sufficient to generate convergent estimates of the velocity, pressure, and electric potential field away from the walls. The main idea of the design is to apply the observer backstepping design method for parabolic PDEs (Smyshlyaev & Krstic, 2005) to the estimator error system; this system is similar to the Orr–Sommerfeld–Squire system of PDE's and presents the same difficulties (nonnormality leading to a large transient growth mechanism (Jovanovic & Bamieh, 2005; Schmid & Henningson, 2001)). Thus, applying the same ideas as in Cochran, Vazquez, and Krstic (2006), we use Fourier transform methods and some of the output injection gains to cast the system in a form where backstepping is applicable. Then, we design the remaining output injection gains not only to guarantee stability but also to decouple the system in order to prevent transients. The output injection gains can be computed solving linear hyperbolic PDEs—a much simpler task than, for instance, solving nonlinear Riccati equations (Smyshlyaev & Krstic, 2004). The observer needs measurements of pressure, skin friction and current at only one of the channel walls.

If the fluid is not conductive, or there is no magnetic field, the problem reduces to the Poiseuille channel flow problem and our observer design still holds. Frequently cited as a paradigm for transition to turbulence (Schmid & Henningson, 2001), the Poiseuille flow is a prototypical problem for flow control and turbulence estimation. There are many results in channel flow stabilization, for instance, using optimal control (Hogberg, Bewley, & Henningson, 2003), backstepping (Vazquez & Krstic, 2007), spectral decomposition/pole placement (Barbu, 2006; Triggiani, 2007), Lyapunov design/passivity (Aamo & Krstic, 2002; Balogh, Liu, & Krstic, 2001), or nonlinear model reduction/in-domain actuation (Baker, Armaou, & Christofides, 2000). Observer designs are more scarce; apart from the continuum backstepping approach (Vazquez & Krstic, 2005), previous works were in the form of an Extended Kalman Filter for the spatially discretized Navier–Stokes equations, employing high-dimensional algebraic Riccati equations for computation of observer gains (Chevalier, Hoepffner, Bewley, & Henningson, 2006; Hoepffner, Chevalier, Bewley, & Henningson, 2005).

The paper is organized as follows. Section 2 introduces the governing equations of our system. The equilibrium profile is presented in Section 3 and the observer structure and measurements are introduced in Section 4. Section 5 presents the design of the output injection gains to guarantee convergence of the observer estimates. In Section 6 we present a nonlinear estimator based on the linear design. We finish the paper with some concluding remarks in Section 7.

2. Model of the Hartmann flow

Consider an incompressible conducting fluid enclosed between two plates, separated by a distance L, under the influence of a pressure gradient ∇P and a magnetic field B_0 normal to the walls, as shown in Fig. 1. Under the assumption of a very small magnetic Reynolds number

$$Re_M = \nu\rho\sigma U_0 L \ll 1, \tag{1}$$

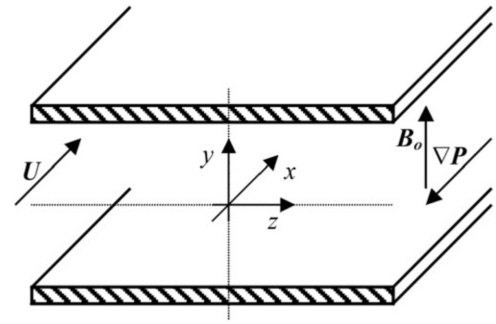

Fig. 1. Hartmann flow.

where ν is the viscosity of the fluid, ρ the density of the fluid, σ the conductivity of the fluid, and U_0 the reference velocity (maximum velocity of the equilibrium profile), the dynamics of the magnetic field can be neglected and the dimensionless velocity and electric potential field is governed by the inductionless MHD equations (Lee & Choi, 2001).

We set nondimensional coordinates (x, y, z), where x is the streamwise direction (parallel to the pressure gradient), y the wall normal direction (parallel to the magnetic field), z the spanwise direction, and where $(x, y, z) \in (-\infty, \infty) \times [0, 1] \times (-\infty, \infty)$. Let f_t denote the time derivative of a function f, similarly f_x, f_y and f_z the derivatives with respect to x, y and z, and define the 3-D Laplacian operator as $\triangle = \partial_{xx} + \partial_{yy} + \partial_{zz}$. The governing equations of the Hartmann flow are

$$U_t = \frac{\triangle U}{Re} - UU_x - VU_y - WU_z - P_x + N\phi_z - NU, \tag{2}$$

$$V_t = \frac{\triangle V}{Re} - UV_x - VV_y - WV_z - P_y, \tag{3}$$

$$W_t = \frac{\triangle W}{Re} - UW_x - VW_y - WW_z - P_z - N\phi_x - NW, \tag{4}$$

$$\triangle\phi = U_z - W_x, \tag{5}$$

where $U(t, x, y, z)$, $V(t, x, y, z)$ and $W(t, x, y, z)$ denote, respectively, the streamwise, wall-normal and spanwise velocities, $P(t, x, y, z)$ the pressure, $\phi(t, x, y, z)$ the electric potential, $Re = \frac{U_0 L}{\nu}$ is the Reynolds number and $N = \frac{\sigma L B_0^2}{\rho U_0}$ the Stuart number. Since the fluid is incompressible, the continuity equation is verified

$$U_x + V_y + W_z = 0. \tag{6}$$

The boundary conditions for the velocity field are

$$U(t, x, 0, z) = U(t, x, 1, z) = 0, \tag{7}$$

$$V(t, x, 0, z) = V(t, x, 1, z) = 0, \tag{8}$$

$$W(t, x, 0, z) = W(t, x, 1, z) = 0, \tag{9}$$

and assuming perfectly conducting walls, the electric potential must verify

$$\phi(t, x, 0, z) = \phi(t, x, 1, z) = 0. \tag{10}$$

The nondimensional electric current, $j(t, x, y, z)$, is a vector field that can be directly computed from the electric potential and velocity fields as follows,

$$j^x(t, x, y, z) = -\phi_x - W, \tag{11}$$

$$j^y(t, x, y, z) = -\phi_y, \tag{12}$$

$$j^z(t, x, y, z) = -\phi_z + U, \tag{13}$$

where j^x, j^y, and j^z denote the components of j.

Remark 1. If we set $N = 0$ (zero magnetic field, or nonconducting fluid) in Eqs. (2)–(5), they reduce to the classical Navier–Stokes equations without body forces. Then Eqs. (2)–(4) and (6)–(9) describe a pressure driven channel flow, the so-called Poiseuille flow.