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Brief paper

Exponential stabilization controller design for interconnected time delay systems ☆

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ABSTRACT

Decentralized robust control problem is investigated for a class of large scale systems with time varying delays. The considered systems have mismatches in time delay functions. A state coordinate transformation is first employed to change the original system into a cascade system. Then the virtual linear state feedback controller is developed to stabilize the first subsystem. Based on the virtual controller, a memoryless state feedback controller is constructed for the second subsystem. By choosing new Lyapunov Krasovskii functional, we show that the designed decentralized continuous adaptive controller makes the solutions of the closed-loop system exponentially convergent to a ball, which can be rendered arbitrary small by adjusting design parameters. Finally, a numerical example is given to show the feasibility and effectiveness of the proposed design techniques.

1. Introduction

Many practical systems are of large scale systems and consist of a set of interconnected subsystems in the real world, such as power systems, digital communication networks, economic systems and urban traffic networks. Robust control for large-scale systems have been one of the focused study topics in the past years, and a lot of achievements have been made, see Jiang (2004), Siljak (1991), Wen (1994), Zhang, Wen, and Soh (2000), and the references therein. However, the systems investigated in the above quoted literature are free of the time delays.

It is well known that time delays are frequently encountered in various engineering systems and can be cause of instability (Gu, Kharitonov, & Chen, 2003). The information transmission among the subsystems often induces appearance of time delays in the interconnected systems. For large scale systems with linear interconnection, the stability analysis and control problem have been extensively investigated. Ikeda and Siljak (1980) first introduced time delay into decentralized control of large scale systems and investigated the exponential stabilization problem. In Almi and Derbel (1995) and Lee and Radovic (1988), the control problem was considered for a class of time invariant large scale interconnected systems free of uncertainties subject to multiple constant delays. In Mahmoud and Bingulac (1998), the robust control problem was considered for a class of interconnected systems with interconnections free of time delays. The stabilization problem of large-scale stochastic systems with time delays was studied in Xie and Xie (2000), while stabilization of a class of time-varying large scale systems subject to multiple time-varying delays in the interconnections was investigated in Oucheriah (2000). Shyua, Liu, and Hsu (2005) proposed variable structure control method for large scale systems with known linear time delay interconnection. Based on linear matrix inequality approach, the stability criteria were presented in De Souza (2001) and Lin, Wang, and Lee (2006). For the case that the uncertain interconnections are bounded by linear functions with unknown coefficients, Wu (2002) presented the adaptive controller design methodology. Oucheriah (2005) further considered the input nonlinearity case and the closed-loop system was shown to be exponentially stable. By analysis on the above cited literatures, there are the following restrictions: (i) The subsystems should be linearly interconnected and (ii) The systems considered often satisfy the matching condition. Obviously, these conditions will limit the application of the achievements of the former literatures. With the interconnections bounded by polynomial functions, Hua, Guan, and Shi (2005) proposed the decentralized control design method.

In this paper, the above restricted conditions are removed on the interconnected time delay systems. We consider a class of

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interconnected time delay systems with mismatched time delay functions and general nonlinear interconnections. By changing the subsystem into a cascade system, we successfully dispose of the mismatched function. With the help of proposed novel nonlinear Lyapunov Krasovskii functional, the uncertain nonlinear time delay interconnections are well dealt with. The decentralized memoryless state feedback controller is designed such that the solutions of the resulting closed-loop system are uniformly ultimately bounded and exponentially convergent towards a ball with adjustable radius. Finally, numerical simulation is presented to show the potential of the proposed techniques.

2. System formulation and preliminaries

Consider an interconnected system with the ith subsystem described by

$$S_i:\ \dot{x}_i(t)=A_ix_i(t)+A_{di}x_i(t-\tau_i(t)) + B_i(u_i+H_i(t,x_1(t),x_2(t),\ldots,x_N(t), x_1(t-d_{i1}(t)),x_2(t-d_{i2}(t)),\ldots,x_N(t-d_{iN}(t)))), \quad (1)$$

where N is the total number of subsystems in the large scale system, $x_i\in\Re^{n_i}$ and $u_i\in\Re^{m_i}$ represent the state and control vectors of the i-th subsystem, respectively. $A_i, A_{di}\in\Re^{n_i\times n_i}$ and $B_i\in\Re^{n_i\times m_i}$ are known constant matrices. Without loss of generality, we assume $\text{Rank}(B_i)=m_i$. $H_i(\cdot)$ are uncertain nonlinear interconnections, which indicate the coupling among the current state and the delayed state of subsystem S_i and other subsystems. The time delay $\tau_i(t)$ represents the delay time of state x_i in x_i subsystem, while $d_{ij}(t)$ denotes the delay time of state $x_j(t)$ in $x_i(t)$ subsystem. $\tau_i(t)$ and $d_{ij}(t)$ are bounded and differentiable satisfying

$$0\le\tau_i(t)\le\overline{\tau}_i<\infty,\qquad \dot{\tau}_i(t)\le\tau_i^*<1,$$
$$0\le d_{ij}(t)\le\overline{d}_{ij}<\infty,\qquad \dot{d}_{ij}(t)\le d_{ij}^*<1, \quad (2)$$

where $\overline{\tau}_i$, τ_i^*, $\overline{d}_{ij}$ and d_{ij}^* are known positive scalars. The initial condition of i-th subsystem is given as follows

$$x_i(t)=\Upsilon_i(t),\quad t\in[t_0-h_i,t_0],\ i=1,2,\ldots N,$$

where $h_i=\max\{\overline{\tau}_i,\overline{d}_{1i},\overline{d}_{2i},\ldots,\overline{d}_{Ni}\}$, $\Upsilon_i(t)$ are continuous functions.

Assumption 1. The nonlinear interconnected functions H_i satisfy

$$\|H_i\|\le\sum_{j=1}^{N}\left(\overline{\theta}_{ij}^{\mathrm{T}}\overline{\alpha}_{ij}\left(\|x_j(t)\|\right)+\overline{\vartheta}_{ij}^{\mathrm{T}}\overline{\beta}_{ij}\left(\|x_j(t-d_{ij}(t))\|\right)\right), \quad (3)$$

where $\overline{\theta}_{ij}\in\Re^{r_{ij}}$ and $\overline{\vartheta}_{ij}\in\Re^{s_{ij}}$ are unknown parameter vectors, r_{ij} and s_{ij} are known positive integers, $\overline{\alpha}_{ij}(\cdot)=[\overline{\alpha}_{ij1}(\cdot),\overline{\alpha}_{ij2}(\cdot),\ldots,\overline{\alpha}_{ijr_{ij}}(\cdot)]^{\mathrm{T}}$, $\overline{\beta}_{ij}(\cdot)=[\overline{\beta}_{ij1}(\cdot),\overline{\beta}_{ij2}(\cdot),\ldots,\overline{\beta}_{ijs_{ij}}(\cdot)]^{\mathrm{T}}$ where $\overline{\alpha}_{ijl}(\cdot)$ and $\overline{\beta}_{ijp}(\cdot)$ are smooth class-κ functions with known structure and there exist functions $\alpha_{ijl}(\cdot)$ and $\beta_{ijp}(\cdot)$ such that $\overline{\alpha}_{ijl}(\chi)=\chi\alpha_{ijl}(\chi)$ and $\overline{\beta}_{ijp}(\cdot)=\chi\beta_{ijp}(\chi)$.

Remark 1. With functions $\overline{\alpha}_{ijl}(\chi)=\overline{\beta}_{ijl}(\chi)=\chi$ (linear interconnection), the robust control problem of system (1) were investigated extensively. With the coefficients of the bound functions unknown and $A_{di}=0$, the control problem has been considered in Oucheriah (2005), Shyua et al. (2005) and Wu (2002). However, the proposed methods of above cited literature are not suitable for the nonlinear interconnection case. To the authors' best knowledge, there is no method reported on designing memoryless controller for system (1). In this paper, a new methodology is proposed to deal with the decentralized control problem of the system.

In view that $\text{Rank}(B_i)=m_i$, there always exists a nonsingular matrix $\Gamma_i\in\Re^{n_i\times n_i}$ such that

$$\Gamma_iB_i=\begin{bmatrix}O_{i((n_i-m_i)\times m_i)}\\ \overline{B}_{i(m_i\times m_i)}\end{bmatrix}, \quad (4)$$

where $\overline{B}_i$ is a nonsingular matrix. For simplicity, we assume $\overline{B}_i$ is the identity matrix. Choosing the coordinate transformation $z_i(t)=\Gamma_ix_i(t)$ gives

$$\begin{aligned}\dot{z}_{i1}(t)&=A_{i11}z_{i1}(t)+A_{i12}z_{i2}(t)\\&\quad+A_{di11}z_{i1}(t-\tau_i(t))+A_{di12}z_{i2}(t-\tau_i(t))\\ \dot{z}_{i2}(t)&=A_{i21}z_{i1}(t)+A_{i22}z_{i2}(t)+A_{di21}z_{i1}(t-\tau_i(t))\\&\quad+A_{di22}z_{i2}(t-\tau_i(t))+u_i(t)+H_i,\end{aligned} \quad (5)$$

where $z_{i1}(t)\in\Re^{n_i-m_i}$, $z_{i2}(t)\in\Re^{m_i}$ and

$$\begin{bmatrix}A_{i11}&A_{i12}\\A_{i21}&A_{i22}\end{bmatrix}=\Gamma_iA_i\Gamma_i^{-1},\qquad \begin{bmatrix}A_{di11}&A_{di12}\\A_{di21}&A_{di22}\end{bmatrix}=\Gamma_iA_{di}\Gamma_i^{-1}.$$

By choosing the state transformation, we obtain the cascade nonlinear time delay system (5). In the following, we first construct a linear virtual feedback controller to stabilize the z_{i1}-subsystem, then the decentralized state feedback controller is further designed based on the linear virtual controller.

For system (5), we choose the following state transformation

$$\begin{cases}y_{i1}(t)=z_{i1}(t),\\ y_{i2}(t)=z_{i2}(t)-K_iy_{i1}(t)\end{cases} \quad (6)$$

where $K_iy_{i1}(t)$ is the virtual controller to be designed for stabilizing the z_{i1}-subsystem. With (6), the new system arises

$$\begin{aligned}\dot{y}_{i1}(t)&=(A_{i11}+A_{i12}K_i)\,y_{i1}(t)+(A_{di11}+A_{di12}K_i)\,y_{i1}(t-\tau_i(t))\\&\quad+A_{i12}y_{i2}(t)+A_{di12}y_{i2}(t-\tau_i(t))\\ \dot{y}_{i2}(t)&=A_{i21}y_{i1}(t)+A_{i22}y_{i2}(t)+A_{di21}y_{i1}(t-\tau_i(t))\\&\quad+A_{di22}y_{i2}(t-\tau_i(t))+A_{i22}K_iy_{i1}(t)\\&\quad+A_{di22}K_iy_{i1}(t-\tau_i(t))-K_i\dot{y}_{i1}(t)+u_i(t)+H_i.\end{aligned} \quad (7)$$

Our aim of this paper is to construct a memoryless controller such that the solutions of the closed-loop system exponentially converge to an adjustable bounded region.

3. Controller design

First, we show how to determine the virtual control law $K_iy_{i1}(t)$. For y_{i1}-subsystem, choose the following Lyapunov functional

$$V_i=y_{i1}^{\mathrm{T}}(t)P_iy_{i1}(t)+W_i, \quad (8)$$

with

$$\begin{aligned}W_i&=\frac{1}{1-\tau_i^*}\int_{t-\tau_i(t)}^{t}\mathrm{e}^{-\gamma_i(t-\xi)}y_{i1}^{\mathrm{T}}(\xi)\,Q_iy_{i1}(\xi)\,\mathrm{d}\xi\\&\quad+\frac{\varepsilon_{i2}^{-1}\mathrm{e}^{\gamma_i\overline{\tau}_i}}{1-\tau_i^*}\int_{t-\tau_i(t)}^{t}\mathrm{e}^{-\gamma_i(t-\xi)}\,\|y_{i2}(\xi)\|^2\,\mathrm{d}\xi,\end{aligned} \quad (9)$$

where P_i and Q_i are positive matrices, γ_i and ε_{i2} are positive scalars.

With Lyapunov functional (8), we have the following preliminary result:

Lemma 1. *For system (7), if there exist positive matrices M_i, L_i and matrix N_i such that the following LMI holds for $i=1,2,\ldots,N$*

$$\Psi_i=\begin{bmatrix}\Psi_{i11}&A_{di11}M_i+A_{di12}N_i\\(A_{di11}M_i+A_{di12}N_i)^{\mathrm{T}}&-\mathrm{e}^{-\gamma_i\overline{\tau}_i}L_i\end{bmatrix}<0, \quad (10)$$