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Brief paper

Control for discrete singular hybrid systems☆

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ABSTRACT

The problems of stability, state feedback control and static output feedback control for a class of discrete-time singular hybrid systems are investigated in this paper. A new sufficient and necessary condition for a class of discrete-time singular hybrid systems to be regular, causal and stochastically stable is proposed in terms of a set of coupled strict linear matrix inequalities (LMIs). Sufficient conditions are proposed for the existence of state feedback controller and static output feedback controller in terms of a set of coupled strict LMIs, respectively. Finally, two illustrative examples are provided to demonstrate the applicability of the proposed approach.

1. Introduction

Many physical systems can have different structures due to random abrupt changes, which may be caused by random failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, modification of the operating point of a linearized model of a nonlinear system, etc. Such systems can be modeled as hybrid ones with two components in the state vector. One is system state, the other is a discrete variable called system mode. A special class of hybrid systems is the so-called Jump Linear System (JLS). JLS is a hybrid system with many operation modes. In the JLS, each mode corresponds to a deterministic dynamical behavior, and the random jumps in system parameters are governed by a Markov process which takes values in a finite set. A number of control problems related to continuous- or discrete-time JLS has been analyzed by several authors since the mid 1960s, see, e.g. Aberkane, Ponsart, and Sauter (2006), Blair and Sworder (1975), Boukas, Liu, and Liu (2001), Boukas and Shi (1998), de Souza (2006), Shi and Boukas (1997), Shi, Boukas, and Agarwal (1999), Xie, Ogai, Inoe, and Ohata (2006) and Zhang, Basin, and Skliar (2006) and the references therein.

As far as we know, a singular system is also a natural representation of dynamic systems and describes a larger class of systems than the normal linear system model. The singular form is useful to represent and handle systems such as mechanical systems, electric circuits, interconnected systems, and so on. In the past decades, control for singular systems has been extensively studied and many notions and results in state-space systems have been extended to singular systems, such as stability and stabilization (see, e.g. Boukas, Xu, and Lam (2005), Dai (1989), Xu and Lam (2004) and Xu and Yang (1999)), H_∞ control problem (see, e.g. Masubuchi, Kamime, Ohara, and Suda (1997) and Xia and Jia (2003)), mixed H_2/H_∞ control (see, e.g. Xia, Shi, Liu, and Rees (2005) and Zhang, Huang, and Lam (2003)), filtering problem (see, e.g. Xu, Lam, and Zou (2003) and Yue and Han (2004)), and so on. In recent years, more and more attention has been paid to deriving strict LMI condition for stability analysis and controller design, see, e.g. Uezato and Ikeda (1999), Xu, Van Dooren, Stefan, and Lam (2002) and Zhang et al. (2003) for continuous singular system, and Xu and Lam (2004) and Zhang, Xia, and Shi (2008) for discrete singular system. The strict LMI conditions, that is, definite LMIs without equality constraints, are highly tractable and reliable when we use recent popular softwares for solving LMIs. More recently, continuous singular systems with Markovian switching have been extensively studied, see for example Boukas (2005, 2006a,b, 2007) and references therein, and the LMI conditions are not in the strict LMI settings. Moreover, to date and to the best of our knowledge, for a discrete singular system with Markovian jump parameters, the problem of stability, stabilization

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and feedback control has not been fully investigated yet Lam, Shu, Xu, and Boukas (2007). This problem is important and challenging in both theory and practice, which motivated us for this study.

In this paper, firstly, we consider the problems of stability for discrete-time singular hybrid systems. A sufficient and necessary condition for a discrete-time singular hybrid system to be regular, causal and stochastically stable is proposed in terms of strict linear matrix inequalities (LMIs). Next, the state feedback controller and static output feedback controller for discrete-time singular hybrid systems are proposed. Finally, two illustrative examples are given to show the effectiveness of the proposed approach.

2. Problem formulation

Let the dynamics of the class of discrete-time singular systems with Markovian jumps be described by the following:

$$Ex_{k+1} = A(r_k)x_k + B(r_k)u_k \tag{1}$$

$$y_k = C(r_k)x_k \tag{2}$$

where, for $k \in \mathcal{Z}$, $x_k \in \mathbb{R}^n$ is the descriptor variable, $u_k \in \mathbb{R}^m$ is the control input and $y_k \in \mathbb{R}^q$ is the controlled output. $\{r_k, k \in \mathcal{Z}\}$ is a time homogeneous Markov chain taking values in a finite set $\mathcal{S} = \{1, 2, \ldots, N\}$, with stationary transition probability matrix $\Pi = [\pi_{ij}]_{N\times N}$, where $\pi_{ij} = \Pr\{r_{k+1} = j | r_k = i\}$ with $\pi_{ij} \geq 0$, for $i, j \in \mathcal{S}$, and $\sum_{j=1}^{N} \pi_{ij} = 1$. The matrix $E \in \mathbb{R}^{n\times n}$ is supposed to be singular with $\operatorname{rank}(E) = r < n$. $A(r_k) \in \mathbb{R}^{n\times n}$, $B(r_k) \in \mathbb{R}^{n\times m}$ and $C(r_k) \in \mathbb{R}^{q\times n}$, for $r_k = i$, $i \in \mathcal{S}$ are known real-valued constant matrices of appropriate dimensions that describe the nominal system and $C(r_k)$ for $r_k = i$, $i \in \mathcal{S}$ are assumed to be of full row rank for simplicity.

Definition 1 (*Xu & Lam, 2006*).

I. The discrete singular hybrid system in (1) is said to be regular if, for each $i \in \mathcal{S}$, $\det(sE - A(i))$ is not identically zero.
II. The discrete singular hybrid system in (1) is said to be causal if, for each $i \in \mathcal{S}$, $\deg(\det(sE - A(i))) = \operatorname{rank}(E)$.
III. The discrete singular hybrid system in (1) is said to be stochastically stable if for any $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathcal{S}$, there exists a scalar $M(x_0, r_0) > 0$ such that

$$\lim_{N\to\infty} \mathbb{E}\left\{\sum_{k=0}^{N} \|x(k, x_0, r_0)\|^2 | x_0, r_0\right\} \leq M(x_0, r_0),$$

where $x(k, x_0, r_0)$ denote the solution to system (1) at time k under the initial conditions x_0 and r_0.
IV. The discrete singular hybrid system in (1) is said to be stochastically admissible if it is regular, causal and stochastically stable.

Definition 2. System (1) is said to be regular, causal and stochastically stabilizable via state feedback (static output feedback) if there exists a control

$$u_k = K(r_k)x_k \tag{3}$$

or

$$u_k = K(r_k)y_k \tag{4}$$

with $K(i)$, when $r_k = i$, is a constant matrix, such that the closed-loop system is stochastically admissible.

The objective of this paper is to:

I. develop LMI-based conditions for system (1) with $u(t) \equiv 0$ to check if system (1) is stochastically admissible;
II. design a state feedback controller of the form (3) that renders the closed-loop system to be stochastically admissible; and
III. design a static output feedback controller of the form (4) that makes the closed-loop system stochastically admissible.

3. Stability analysis

In this section, we analyze the stochastic stability of system (1). Our attention will be paid to establishing strict LMI conditions to check the regularity, causality and stochastic stability of system (1). Firstly, we recall the stability results based on nonstrict conditions, then change them into strict ones.

Lemma 3 (*Xu & Lam, 2006*). *System (1) is stochastically admissible if and only if there exist symmetric matrices $P(i)$, $i \in \mathcal{S}$, such that the following coupled LMIs hold for each $i \in \mathcal{S}$:*

$$E^{\mathrm{T}}P(i)E \geq 0, \tag{5}$$

$$A^{\mathrm{T}}(i)\bar{P}(i)A(i) - E^{\mathrm{T}}P(i)E < 0, \tag{6}$$

where $\bar{P}(i) = \sum_{j=1}^{N} \pi_{ij}P(j)$.

Define $R \in \mathbb{R}^{n\times n}$ as the matrix with the properties of $E^{\mathrm{T}}R^{\mathrm{T}} = 0$ and $\operatorname{rank} R = n - r$, which is used in all the subsequent results.

Theorem 4. *System (1) is stochastically admissible if and only if there exist a set of symmetric-positive-definite matrices $P(i)$, $i \in \mathcal{S}$ and a symmetric and nonsingular matrix Φ, such that the following coupled LMIs hold for each $i \in \mathcal{S}$:*

$$A^{\mathrm{T}}(i)\left(\bar{P}(i) - R^{\mathrm{T}}\Phi R\right)A(i) - E^{\mathrm{T}}P(i)E < 0, \tag{7}$$

where $\bar{P}(i) = \sum_{j=1}^{N} \pi_{ij}P(j)$.

Proof. *Sufficiency*. Let $Y(i) = P(i) - R^{\mathrm{T}}\Phi R$ in (7), we can get

$$E^{\mathrm{T}}Y(i)E = E^{\mathrm{T}}(P(i) - R^{\mathrm{T}}\Phi R)E = E^{\mathrm{T}}P(i)E \geq 0, \tag{8}$$

$$A^{\mathrm{T}}(i)\bar{Y}(i)A(i) - E^{\mathrm{T}}Y(i)E < 0, \tag{9}$$

where $\bar{Y}(i) = \sum_{j=1}^{N} \pi_{ij}Y(j)$.

Necessity. Suppose that system (1) is stochastically admissible. Now, we choose two nonsingular matrices M and N such that

$$E = M\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}N, \quad A(i) = M\begin{bmatrix} A_1(i) & A_2(i) \\ A_3(i) & A_4(i) \end{bmatrix}N. \tag{10}$$

From Xu and Lam (2006), we know that the regularity and causality of (1) imply that $A_4(i)$ is nonsingular for any $i \in \mathcal{S}$. Then, select a nonsingular matrix as

$$\mathcal{L}(i) = \begin{bmatrix} I & -A_3^{\mathrm{T}}(i)A_4^{-\mathrm{T}}(i) \\ 0 & I \end{bmatrix},$$

and let $\tilde{N}(i) = N^{-1}\mathcal{L}^{\mathrm{T}}(i)$. Then, it can be verified that

$$\tilde{E} = M^{-1}E\tilde{N}(i) = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \tag{11}$$

$$\tilde{A}(i) = M^{-1}A(i)\tilde{N}(i) = \begin{bmatrix} \tilde{A}_1(i) & A_2(i) \\ 0 & A_4(i) \end{bmatrix}, \tag{12}$$

where $\tilde{A}_1(i) = A_1(i) - A_2(i)A_4^{-1}(i)A_3(i)$.

Therefore, the stochastic stability of system (1) implies that the discrete Markovian jump system

$$\xi(k+1) = \tilde{A}_1(r_k)\xi(k),$$

is stochastically stable. It follows that there exist matrices $\tilde{P}(i) > 0$, $i \in \mathcal{S}$, such that

$$\tilde{A}_1^{\mathrm{T}}(i)\bar{\tilde{P}}(i)\tilde{A}_1(i) - \tilde{P}(i) < 0$$

where $\bar{\tilde{P}}(i) = \sum_{j=1}^{N} \pi_{ij}\tilde{P}(i)$.