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Robust H_2 optimal filtering for continuous-time stochastic systems with polytopic parameter uncertainty☆

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A B S T R A C T

In this paper, robust H_2 optimal filtering is addressed for continuous-time stochastic systems with polytopic parameter uncertainty. A new robust stability condition is presented. A continuous-time robust H_2 optimal filter is obtained by solving a sufficient linear matrix inequality condition characterizing a solution of a robust minimum variance filtering problem which takes into account the polytopic type of model uncertainties. The proposed approach is demonstrated through numerical examples.

1. Introduction

A large amount of work has been carried out to search for optimal H_2 filtering, also called minimum variance filtering, with a large spectrum of practical applications in control engineering, signal processing, system failure detection, etc, since the seminal work of Kalman (Anderson & Moore, 1979). In recent years, enhancing robustness in filtering has attracted much attention because the consideration of uncertainty, which is present in most physical systems almost inevitably, is of a prime importance for applications to real systems. The purpose of robust H_2 filtering is to design a filter such that the worst case mean square estimation error is minimized for all admissible uncertainties under the assumption that the process noise input is a white noise with zero-mean and known covariance. In Petersen and McFarlane (1994), Shaked and de Souza (1995) and Sun and Packard (2005), the Riccati equation approaches were presented to deal with systems with norm-bounded parameter uncertainty. In Geromel (1999), Geromel and de Oliveira (2001),Yang and Hung (2002) and Palhares and Peres (2001), the linear matrix inequality (LMI) approaches were applied for systems with norm-bounded parameter uncertainty or convex polytopic type uncertainty.

As is well known, most robust filtering techniques rely on employing a single quadratic Lyapunov function overall uncertainty domain. Obviously there exists conservativeness in this type of design. Several attempts have been made in the past few years toward reducing conservativeness in existing robust filtering algorithms, particularly for uncertain discrete-time systems. In Geromel, de Oliveira, and Bernussou (2002) and Shaked, Xie, and Soh (2001), an LMI approach was applied to improve robustness of optimal H_2 filtering by using a robust stability condition established in de Oliveira, Bernussou and Geromel (1999), which enables the use of a parameter dependent Lyapunov function and consequently leads to a less conservative design for uncertain discrete-time systems. In Xie, Lu, Zhang, and Zhang (2004), a nonconvex bilinear matrix inequality optimization method with scaling parameters was proposed to solve the optimal H_2 filtering problem by using a robust stability condition established in Peaucelle, Arzelier, Bachelier, and Bernussou (2000), which offers extra degree-of-freedom in optimization. In Xie et al. (2004), a parameter search is necessary to find the best values for scaling parameters that lead to the best filter performance.

On the other hand, there have been few attempts for uncertain continuous-time systems. This is because of the difficulty in obtaining new LMI characterizations for robust stability in continuous-time and the problem still remains open. In Tuan, Apkarian, and Nguyen (2001), an LMI approach with a parameter

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dependent Lyapunov function was presented for continuous-time systems with convex polytopic uncertainty by applying an LMI characterization for robust stability found by Projection Lemma. In Barbosa, de Souza and Trofino (2005), a nonconvex matrix inequality method with searching parameters was proposed to produce a less conservative result as in Xie et al. (2004). In Gonçalves, Palhares, and Takahashi (2006), a domain search method using the branch and bound algorithm was applied directly in the space of filter parameters over a set of polytopic points in order to avoid conservativeness in robust filtering design at the expense of the computational effort. Note that the nonconvex optimization methods or the domain search methods, which were developed in Barbosa et al. (2005), Gonçalves et al. (2006) and Xie et al. (2004) at the expense of the computational cost, are not prohibitive for the design of robust filters with high order; however, in many applications, an LMI solution is preferred due to the less computational complexity. Therefore, it is of importance to find an efficient robust LMI solution in filter design.

In this paper, an LMI solution is proposed for robust H_2 filtering of continuous-time systems with polytopic parameter uncertainty. A new sufficient robust stability condition, which is expressed as LMIs, is presented for uncertain continuous-time stochastic systems. Using the new robust stability condition presented in this paper, a new continuous-time robust H_2 filter is obtained by solving a sufficient linear matrix inequality condition characterizing a solution of a minimum variance filtering problem which takes into account convex parameter uncertainty. It is shown that the proposed robust H_2 filter can significantly reduce conservatism in the existing LMI methods via numerical comparisons. In the case when there is no uncertainty, the proposed robust H_2 filter is reduced to the standard Kalman filter. In Section 2.1, the problem of interest is described. In Section 2.2, a new robust stability condition is developed for continuous-time stochastic systems with polytopic uncertainty. In Section 2.3, a new robust H_2 filter is proposed via a parameter dependent Lyapunov matrix procedure for uncertain continuous-time stochastic systems. In Section 3, illustrative numerical examples are included in order to demonstrate the less conservatism of the proposed robust H_2 filtering. Finally, conclusions are drawn in Section 4.

2. Main results

2.1. Problem description

Consider a plant described by

$$\Sigma : \dot{x} = Ax + Bv, \qquad y = Cx + Dv, \tag{1}$$

where $x \in \mathbf{R}^n$ is the state of the plant to be estimated, $v \in \mathbf{R}^m$ is the white noise input with zero mean and unit covariance matrix, and $y \in \mathbf{R}^p$ is the measured plant output. The system matrices in (1) are assumed to be unknown but belong to a known convex compact set of polytopic type, i.e.,

$$\mathscr{S} \triangleq (A, B, C, D) \in \Omega,$$
$$\Omega \triangleq \left\{ \mathscr{S} | \mathscr{S} = \sum_{i=1}^{r} \tau_i \mathscr{S}_i,\ \tau \triangleq (\tau_1, \ldots, \tau_r) \in \Gamma \right\}, \tag{2}$$

where $\Gamma \triangleq \left\{(\tau_1, \ldots, \tau_r) | \sum_{i=1}^{r} \tau_i = 1, \tau_i \geq 0\right\}$.

In this paper, a new robust H_2 filter will be developed for the stochastic system (1) and (2).

2.2. Robust stability condition

A robust H_2 performance condition is suggested first for continuous-time systems. Our LMI-based robust condition will play an important role in implementing a robust continuous-time H_2 filter.

Lemma 1 (*Boyd, Ghaoui, Feron, & Balakrishnan, 1994*). *Given a system $\Sigma_1 : \dot{x} = Ax + Bv$, $y = Cx$, where $\mathscr{P} \triangleq (A, B, C)$, $\mathscr{P} \in \Omega$, the following statements are equivalent:*

(i) *A is stable and $\|T(\tau) \triangleq C(sI - A)^{-1}B\|_2^2 < \nu$, $\forall \tau \in \Gamma$, where $\nu > 0$.*

(ii) *$\exists P(\tau) = P^{\mathrm{T}}(\tau)$, $W(\tau) = W^{\mathrm{T}}(\tau)$ such that*

$$\begin{pmatrix} A^{\mathrm{T}}P(\tau) + P(\tau)A & P(\tau)B \\ B^{\mathrm{T}}P(\tau) & -\nu I_m \end{pmatrix} < 0, \tag{3}$$

$$\begin{pmatrix} P(\tau) & C^{\mathrm{T}} \\ C & W(\tau) \end{pmatrix} > 0, \tag{4}$$

$$\mathrm{tr}(W(\tau)) < 1 \tag{5}$$

for all $\tau \in \Gamma$ such that $\mathscr{P} \in \Omega$. ◇

A new sufficient condition for Lemma 1 is proposed based on linear matrix inequalities (LMIs) in the following theorem:

Theorem 1. *Given system Σ_1, if there exist matrices $P_i = P_i^{\mathrm{T}}$, Y, and $W_i = W_i^{\mathrm{T}}$ satisfying the following LMIs*

$$\begin{pmatrix} \frac{1}{2}P_i - \frac{1}{2}(I - A_i^{\mathrm{T}})Y - \frac{1}{2}Y^{\mathrm{T}}(I - A_i) & Y^{\mathrm{T}}B_i & \frac{1}{2}Y^{\mathrm{T}}(I + A_i) \\ B_i^{\mathrm{T}}Y & -\nu I_m & 0 \\ \frac{1}{2}(I + A_i^{\mathrm{T}})Y & 0 & -\frac{1}{2}P_i \end{pmatrix} < 0, \tag{6}$$

$$\begin{pmatrix} P_i & C_i^{\mathrm{T}} \\ C_i & W_i \end{pmatrix} > 0, \tag{7}$$

$$\mathrm{tr}(W_i) < 1 \tag{8}$$

for all $i = 1, 2, \ldots, r$, then the system Σ_1 is robustly stable and $\|T(\tau)\|_2^2 < \nu$ for all $\tau \in \Gamma$. Moreover, for any $\mathscr{P} \in \Omega$, $P(\tau)$ given by $P(\tau) \triangleq \sum_{i=1}^{r} \tau_i P_i$ is a parameter dependent positive-definite Lyapunov function such that (3)–(5) hold.

Proof. ($\Rightarrow$) Assume that there exists a solution $\{P_i, Y, W_i\}$ by (6)–(8) for all $i = 1, 2, \ldots, r$. Let $P(\tau) \triangleq \sum_{i=1}^{r} \tau_i P_i$, i.e. $P(\tau)$ is a linear parameter dependent function. Multiplying each LMI in (3) by $\tau_i > 0$ and adding them to get a convex combination for τ_i, $i = 1, 2, \ldots, r$, we have the following inequality

$$\begin{pmatrix} \left(\begin{matrix} \frac{1}{2}P(\tau) - \frac{1}{2}\left(I - \sum_{i=1}^{r}\tau_i A_i^{\mathrm{T}}\right)Y \\ -\frac{1}{2}Y^{\mathrm{T}}\left(I - \sum_{i=1}^{r}\tau_i A_i\right) \end{matrix}\right) & Y^{\mathrm{T}}\sum_{i=1}^{r}\tau_i B_i & \frac{1}{2}Y^{\mathrm{T}}\left(I + \sum_{i=1}^{r}\tau_i A_i\right) \\ \sum_{i=1}^{r}\tau_i B_i^{\mathrm{T}}Y & -\nu I_m & 0 \\ \frac{1}{2}\left(I + \sum_{i=1}^{r}\tau_i A_i^{\mathrm{T}}\right)Y & 0 & -\frac{1}{2}P(\tau) \end{pmatrix} < 0, \tag{9}$$

where $\sum_{i=1}^{r} \tau_i A_i = A(\tau)$ and $\sum_{i=1}^{r} \tau_i B_i = B(\tau)$ according to the definition of the convex set (2). Rewrite inequality (9) in the following form

$$\mathscr{A}(\tau) + \mathscr{B}(\tau)Y\mathscr{C}^T + \mathscr{C}Y^T\mathscr{B}^T(\tau) < 0, \tag{10}$$